Life Tables

Life table: is a life history of a hypothetical group or cohort of people which diminishes gradually by pre-set schedule of mortality. Generally, the cohort begins from a standard number, say 100,000 called radix or root of the life table. All columns of the life table are inter-related therefore if information of one column is known it is possible to construct the remaining part of the table. Usually age specific mortality rates from any national survey are available for construction of the life table. It is commonly being used in estimation of life expectancy by age and sex, projection of population, fromulation of health policy and in many statistical techniques and in models.

⇒ **Assumptions of Life tables:**

A life table is constructed based on the following assumptions:

- The cohort starts from a standard number, usually 100,000 called the radix or root of the life table. The philosophy of standard number is to facilitate in comparison of two different life tables.
- The cohort is closed against migration, thus there will be no change in population except losses through deaths.
- The member of the cohort died at each age according to pre-set schedule of mortality.
- Excepting first few years of life deaths are evenly distributed between one birthday to next birthday.
- Cohort normally consists of members of one sex. However it is possible to construct a life table for both sexes together, but the differences in male and female mortality pattern at different ages are sufficient to justify construction of life

tables separately.

⇒ Types of Life table:

A Life table can be a complete life table or abridged life table:

Complete life table: It is generally one in which the values of the life table functions are given in single years of age.

Abridged life table: It is one in which the functions are given only for certain age groups and intermediate values have to be obtained by interpolation.

⇒ Functions of life tables:

Survivors (I_x) are the number of persons surviving at exact age "x"

Number of deaths ($_nd_x$) between age "x" and age "x+n", where "n" is the age interval. For complete life table it is 1 and for abridged life table it could be 5 or 10 depending upon the class interval. It is the difference in survivors at exact age "x+n" from survivors at exact age "x". Symbolically:

$$\boxed{_nd_x = I_x - I_{x+n}}$$

also known as mortality rate

Probability of dying ($_nq_x$) is the chances of death of population between age "x" and "x+n". It is estimated by dividing the number of deaths occurring between ages "x" and "x+n" by survivors at exact age "x". Symbolically:

$$\boxed{_nq_x = \frac{_nd_x}{I_x}}$$

As we know $_nd_x = I_x - I_{x+n}$ so

$$\boxed{_nq_x = \frac{I_x - I_{x+n}}{I_x}}$$

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also Known as Survival rate

Probability of Surviving ($_nP_x$) is the chances of surviving of population between age "x" and "x+n". It is estimated by dividing the number of survivors at exact age "x+n" by survivors at exact age "x"

$$\boxed{{}_nP_x = \frac{I_{x+n}}{I_x} = 1 - {}_nq_x}$$

because

$$1 - {}_nq_x = 1 - \frac{I_x - I_{x+n}}{I_x}$$

$$1 - {}_nq_x = \frac{\cancel{I_x} - \cancel{I_x} + I_{x+n}}{I_x}$$

$$1 - {}_nq_x = \frac{I_{x+n}}{I_x} = {}_nP_x$$

Life table population or Persons years lived ($_nL_x$) is the number of years lived by persons between ages "x" and "x+n". It is obtained by multiplying the sum of I_x and I_{x+n} by $n/2$. Symbolically:

$$\boxed{{}_nL_x = \frac{n(I_x + I_{x+n})}{2}}$$

$\Rightarrow$ with the assumption that number of death evenly distributed.

But initial age (0-1) and end of life table (60^+ ~~or~~ 70^+, 80^+ or 90^+ etc) the deaths are not distributed evenly. Hence the values are estimated for this age groups by:

$$\boxed{{}_1L_0 = 0.3I_0 + 0.7I_1}$$

$$\boxed{{}_1L_1 = 0.4I_0 + 0.6I_1}$$

for complete life table

$$\boxed{{}_\infty L_\infty = \frac{{}_\infty d_\infty}{{}_\infty M_\infty}}$$

where M_∞ is the age specific mortality rate of the last age group.

For the the abridged life table $_nL_x$ is different for $_3L_2$

$$\boxed{_3L_2 = -0.021\, I_0 + 1.384\, I_2 + 1.637\, I_5}$$

Same formula for L_∞.

Total persons years lived (T_x) is the total number of person year's lived by a cohort from exact age "x" till death of all member of the cohort. Total number of years lived by a cohort from age "x" till death of all members are equivalent to $\sum_{x=x}^{\infty} {_nL_x}$

Life expectancy (e_x) is the average number of years lived by cohort from exact age "x" till death of all members of the cohort. It is computed by dividing T_x with I_x. Symbolically

$$\boxed{e_x = \frac{T_x}{I_x}}$$

Survival Ratio (S)

It is obtained by multiplying the division of $_nL_{x+n}$ and $_nL_x$ by $1/n$.

$$S = \boxed{\frac{_nL_{x+n}}{n \times {_nL_x}}}$$

⇒ Interrelationship of Life table functions.

$${}_nd_x = I_x - I_{x+n} = I_x \cdot {}_nq_x$$

$$I_{x+n} = I_x - {}_nd_x = I_x \cdot {}_nP_x$$

$${}_nq_x = \frac{I_x - I_{x+n}}{I_x} = 1 - {}_nP_x$$

$${}_nP_x = \frac{I_{x+n}}{I_x} = 1 - {}_nq_x$$

$${}_nL_x = \frac{n(I_x + I_{x+n})}{2}$$ except ${}_1L_0$, ${}_1L_1$, ${}_3L_2$ and L_∞ which are equal to.

$${}_1L_0 = 0.3 I_0 + 0.7 I_1$$

$${}_1L_1 = 0.4 I_0 + 0.6 I_1$$

$${}_3L_1 = -0.021 I_0 + 1.384 I_2 + 1.687 I_5$$

$${}_\infty L_\infty = \frac{{}_\infty d_\infty}{{}_\infty M_\infty}$$ where M_∞ is the age specific mortality rate from actual population of the last age group.

Construction of Abridged life table

If you calculate this column firstly

Age	ASMR	Survivors	Number of deaths during the time interval	Probability of dying during the time interval	Probability of surviving during the time interval	Number of persons living in interval	Number of persons lived beyond age x
x	${}_nM_x$	I_x	${}_nd_x$	${}_nq_x$	${}_np_x$	${}_nL_x$	T_x
0		100,000	$I_x - I_{x+n}$	$\left(\frac{{}_nd_x}{I_x}\right)$ or $\left(\frac{I_x - I_{x+n}}{I_x}\right)$	$\left(\frac{I_{x+n}}{I_x}\right)$ or $\left(1-{}_nq_x\right)$	${}_1L_0 = 0.3I_0 + 0.7I_1$	
1						${}_4L_1 = \frac{4(I_1 + I_{1+4})}{2}$	
5						${}_5L_5 = \frac{5(I_5 + I_{5+5})}{2}$	Cumulative sum
10							
15		given					
20							
⋮							
80		I_{80}					${}_\infty L_{85} + {}_5L_{80}$
85	${}_\infty M_{85}$	I_{85}	$d_{85} = I_{85} \cdot {}_\infty q_{85}$	$\boxed{1} = {}_\infty q_{85}$	$\boxed{0} = {}_\infty p_{85}$	${}_\infty L_{85} = \frac{{}_\infty d_{85}}{{}_\infty M_{85}}$	${}_\infty L_{85}$

Sometimes given otherwise just given for last age interval

number alive at the begining of time x

assumed that no one survive after age 85

assumed that no one will survive after age 85

Remaining table

Average number of life remaining (life expectancy)	Survival Ratio
e_x	S
	${}_nL_{x+n} / {}_nx{}_nL_x$
T_x / I_x	${}_4L_1 / 4 \times {}_1L_0$
	${}_5L_5 / 5 \times {}_4L_1$
	${}_5L_{10} / 5 \times {}_5L_5$
	⋮
	$I_{85} / (I_{80} + I_{85})$

If you calculate this column firstly

Construction of Complete life table

x	I_x	nq_x	nd_x	nP_x	nL_x	T_x	e_x	S
0	100,000	$I_x - I_{x+n} / I_x$	$I_x \cdot nq_x$	$\left(\frac{I_{x+n}}{I_x}\right)$ or $(1 - nq_x)$	${}_1L_0 = 0.3 I_0 + 0.7 I_1$		T_x / I_x	${}_1L_1 / {}_1L_0$
1		⋮	⋮	⋮	${}_1L_1 = 0.4 I_0 + 0.6 I_1$	↑	⋮	${}_1L_2 / {}_1L_1$
2		⋮	⋮	⋮	${}_1L_2 = \frac{1(I_2 + I_{2+1})}{2}$	cumulative sum	⋮	${}_1L_3 / {}_1L_2$
3	given	⋮	⋮	⋮	⋮		⋮	
⋮		⋮	⋮	⋮	⋮		⋮	
84	I_{84}					${}_\infty L_{85} + {}_1L_{84}$		
85	I_{85}	${}_\infty q_{85} = 1$		${}_\infty P_{85} = 0$	${}_\infty L_{85} = \frac{{}_\infty d_{85}}{{}_\infty M_{85}}$	${}_\infty L_{85}$		$I_{85} / (I_{84} + I_{85})$

if ${}_\infty M_{85}$ is given

Example of Abridge Life Table

x	l_x	$_nd_x$ $(l_x - l_{x+n})$	$_nq_x$ $(_nd_x/l_x)$	$_np_x$ $(1-{_nq_x})$	$_nL_x$	T_x	e_x (T_x/l_x)	SR
0	100,000	100,000 − 81152 = 18848	18848/100,000 = 0.18848	1 − 0.18848 = 0.81152	0.3(100,000) + 0.7(81,152) = 86806	4207733 + 86806 = 4294539	4294539/100,000 = 42.95	$\frac{307930}{4\times 86806}$ = 0.88683
1	81,152	81152 − 72813 = 8339	8339/81,152 = 0.10276	1 − 0.10276 = 0.89724	4(81152 + 72813)/2 = 307930	3899803 + 307930 = 4207733	4207733/81152 = 51.85	$\frac{360993}{5\times 307930}$ = 0.23446
5	72,813	72813 − 71584 = 1229	1229/72813 = 0.01688	1 − 0.01688 = 0.98312	5(72813 + 71584)/2 = 360993	3538810 + 360993 = 3899803	3899803/72813 = 53.56	$\frac{355010}{5\times 360993}$ = 0.19669
10	71,584	71584 − 70420 = 1164	1164/71584 = 0.01626	1 − 0.01626 = 0.98374	5(71584 + 70420)/2 = 355010	3183800 + 355010 = 3538810	3538810/71584 = 49.44	$\frac{346275}{5\times 355010}$ = 0.19508
15	70,420	70420 − 68090 = 2330	2330/70420 = 0.03309	1 − 0.03309 = 0.96691	5(70420 + 68090)/2 = 346275	2837525 + 346275 = 3183800	3183800/70420 = 45.21	$\frac{333040}{5\times 346275}$ = 0.19236
20	68,090	68,090 − 65126 = 2964	2964/68090 = 0.04352	1 − 0.04352 = 0.95648	5(68090 + 65126)/2 = 333040	2504485 + 333040 = 2837525	2837525/68,090 = 41.67	$\frac{318175}{5\times 333040}$ = 0.19107
25	65,126	65126 − 62144 = 2982	2982/65126 = 0.04579	1 − 0.04579 = 0.95421	5(65126 + 62144)/2 = 318175	2186310 + 318175 = 2504485	2504485/65126 = 38.46	$\frac{303383}{5\times 318175}$ = 0.19070
30	62,144	62144 − 59210 = 2934	2934/62144 = 0.04721	1 − 0.04721 = 0.95279	5(62144 + 59210)/2 = 303383	1882927 + 303383 = 2186310	2186310/62144 = 35.18	$\frac{288554}{5\times 303383}$ = 0.19022
35	59,210	59210 − 56212 = 2998	2998/59210 = 0.05063	1 − 0.05063 = 0.94937	5(59210 + 56212)/2 = 288554	1594373 + 288554 = 1882927	1882927/59210 = 31.80	$\frac{273249}{5\times 288554}$ = 0.18939
40	56,212	56212 − 53088 = 3124	3124/56212 = 0.05558	1 − 0.05558 = 0.94442	5(56212 + 53088)/2 = 273249	1321124 + 273249 = 1594373	1594373/56212 = 28.36	$\frac{257181}{5\times 273249}$ = 0.18824
45	53,088	53088 − 49785 = 3303	3303/53088 = 0.06222	1 − 0.06222 = 0.93778	5(53088 + 49785)/2 = 257181	1063943 + 257181 = 1321124	1321124/53088 = 24.89	$\frac{239193}{5\times 257181}$ = 0.18601
50	49,785	49785 − 45892 = 3893	3893/49785 = 0.07818	1 − 0.07818 = 0.92182	5(49785 + 45892)/2 = 239193	824750 + 239193 = 1063943	1063943/49785 = 21.37	$\frac{217418}{5\times 239193}$ = 0.18179
55	45,892	45892 − 41075 = 4817	4817/45892 = 0.10498	1 − 0.10498 = 0.89502	5(45892 + 41075)/2 = 217418	607332 + 217418 = 824750	824750/45892 = 17.97	$\frac{190815}{5\times 217418}$ = 0.17553
60	41,075	41075 − 35251 = 5824	5824/41075 = 0.14179	1 − 0.14179 = 0.85821	5(41075 + 35251)/2 = 190815	416517 + 190815 = 607332	607332/41075 = 14.79	$\frac{158138}{5\times 190815}$ = 0.16575
65	35,251	35251 − 28004 = 7247	7247/35251 = 0.20558	1 − 0.20558 = 0.79442	5(35251 + 28004)/2 = 158138	258379 + 158138 = 416,517	416517/35251 = 11.82	$\frac{119999}{5\times 158138}$ = 0.15177
70	28,004	28004 − 19996 = 8008	8008/28004 = 0.28596	1 − 0.28596 = 0.71404	5(28004 + 19996)/2 = 119999	138380 + 119999 = 258379	258379/28004 = 9.23	$\frac{19996}{(28004 + 19996)}$ = 0.41658
75	19,996	19996(1) = 19996	19996/19996 = 1	1 − 1 = 0	19996/0.14450 = 138380	138380	138380/19996 = 6.92	

Where M_{75^+} is equal to 0.14450

An rough example of Complete life table.

x	l_x	ASMR	$_nq_x \left(\frac{l_x - l_{x+n}}{l_x}\right)$	$_nd_x (l_x \cdot {}_nq_x)$	$_nP_x \left(\frac{l_{x+n}}{l_x}\right)$	$_nL_x$	T_x
0	100000	0.18758	$\frac{100000 - 83419}{100000} = 0.16581$	100000 × 0.16581 = 16581	83419/100000 = 0.83419	0.3(100,000) + 0.7(83419) = 88393	11734504 + 88393 = 11822897
1	83419	0.07003	$\frac{83419 - 77813}{83419} = 0.067203$	83419 × 0.067203 = 5606	77813/83419 = 0.932797	0.4(100,000) + 0.6(83419) = 90051	11644453 + 90051 = 11734504
2	77813	0.03719	$\frac{77813 - 74972}{77813} = 0.036511$	77813 × 0.036511 = 2841	74972/77813 = 0.963489	1(77813 + 74972)/2 = 76392	11568061 + 76392 = 11644453
3	74972	0.02281	$\frac{74972 - 73281}{74972} = 0.022555$	74972 × 0.022555 = 1691	73281/74972 = 0.977445	1(74972 + 73281)/2 = 74127	11493934 + 74127 = 11568061
4	73281	0.01511	$\frac{73281 - 72182}{73281} = 0.014997$	73281 × 0.014997 = 1099	72182/73281 = 0.985003	1(73281 + 72182)/2 = 72732	11421202 + 72732 = 11493934
5	72182	0.00632	1	72182 × 1 = 72182	0	72182/0.00632 = 11421202	11421202

$e_x (T_x/l_x)$	S
11822897/1000000 = 118.23	90051/1 × 88393 = 1.0188
11734504/83419 = 140.67	76392/1 × 90051 = 0.8483
11644453/77813 = 149.65	74127/1 × 76392 = 0.9704
11568061/74972 = 152.30	72732/1 × 74127 = 0.9812
1493934/73281 = 156.85	11421202/1 × 72732 = 157.03
11421202/72182 = 158.23	72182/(73281 + 72182) = 0.4962

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