Solve the following Fredholm Integral Equation by using Adomian Decomposition Method

Example #1. $u(x) = \sin x - x + x\int_0^{\pi/2} u(t)\,dt$

Sol: By ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots \quad (1)$$

$$u_0(x) = f(x) \qquad \because [f(x) = \sin x - x]$$

$$\boxed{u_0(x) = \sin x - x}$$

$$u_{n+1}(x) = \lambda \int_0^{\pi/2} K(x,t)\,u_n(t)\,dt$$

where $\lambda = x$, $K(x,t) = 1$

$$u_{n+1}(x) = x\int_0^{\pi/2} u_n(t)\,dt$$

Put $n=0$

$$u_1(x) = x\int_0^{\pi/2} (\sin t - t)\,dt$$

$$u_1(x) = x\int_0^{\pi/2} (\sin t - t)\,dt$$

$$u_1(x) = x\left\{\left|-\cos t\right|_0^{\pi/2} - \left|\frac{t^2}{2}\right|_0^{\pi/2}\right\}$$

$$u_1(x) = x\left[-\cos\frac{\pi}{2} + \cos 0 - \frac{\pi^2}{8} - 0\right]$$

$$\boxed{u_1(x) = x\left[1 - \frac{\pi^2}{8}\right]}$$

Put $n=1$

$$u_2(x) = x\int_0^{\pi/2} t\left(1 - \frac{\pi^2}{8}\right)dt$$

$$u_2(x) = x\left\{\left|\frac{t^2}{2}\left(1 - \frac{\pi^2}{8}\right)\right|_0^{\pi/2}\right\}$$

$$u_2(x) = x\left\{\frac{\pi^2}{8}\left(1 - \frac{\pi^2}{8}\right)\right\}$$

$$\boxed{u_2(x) = x\left[\frac{\pi^2}{8} - \frac{\pi^4}{64}\right]}$$

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Put n=2

$$u_3(x) = x\int_0^{\pi/2} u_2(t)\,dt$$

$$u_3(x) = x\int_0^{\pi/2} t\left(\frac{\pi^2}{8}-\frac{\pi^4}{64}\right)dt$$

$$u_3(x) = x\left[\left|\frac{t^2}{2}\left(\frac{\pi^2}{8}-\frac{\pi^4}{64}\right)\right|_0^{\pi/2}\right]$$

$$u_3(x) = x\left[\frac{\pi^2}{8}\left(\frac{\pi^2}{8}-\frac{\pi^4}{64}\right)\right]$$

$$\boxed{u_3(x) = x\left[\frac{\pi^4}{64}-\frac{\pi^6}{512}\right]}$$

From ① Eq/ $\Rightarrow$

$$u(x) = u_0(x)+u_1(x)+u_2(x)+u_3(x)+\ldots$$

$$u(x) = \sin x - \cancel{x} + \cancel{x} - \cancel{\frac{x\pi^2}{8}} + \cancel{\frac{x\pi^2}{8}} - \cancel{\frac{x\pi^4}{64}} + \cancel{\frac{x\pi^4}{64}} - \frac{x\pi^6}{512}+\ldots$$

$$\boxed{u(x) = \sin x}$$ Ans.

Example #2: $u(x) = x+e^x-\frac{4}{3}+\int_0^1 t\,u(t)\,dt$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x)+u_1(x)+u_2(x)+\ldots \quad -①$$

$$u_0(x) = f(x) \qquad \because \left[f(x) = x+e^x-\frac{4}{3}\right]$$

$$\boxed{u_0(x) = x+e^x-\frac{4}{3}}$$

$$u_{n+1}(x) = \lambda\int_0^1 K(x,t)\,u_n(t)\,dt$$

where $\lambda=1$, $K(x,t)=t$

$$u_{n+1}(x) = \int_0^1 t\,u_n(t)\,dt$$

Put $n=0$

$$u_1(x) = \int_0^1 t\, u_0(t)\, dt$$

$$u_1(x) = \int_0^1 t\left(t + e^t - \frac{4}{3}\right) dt$$

$$u_1(x) = \int_0^1 \left(t^2 + te^t - \frac{4}{3}t\right) dt$$

$$u_1(x) = \left|\frac{t^3}{3} + te^t\right|_0^1 - e^t\Big|_0^1 - \left(\frac{4}{3}\right)\frac{t^2}{2}\Bigg|_0^1$$

$$u_1(x) = \frac{1}{3} + \cancel{e} - \cancel{e} + e^0 - \frac{4}{6}$$

$$u_1(x) = \frac{1}{3} + 1 - \frac{2}{3}$$

$$u_1(x) = \frac{1+3-2}{3}$$

$$\boxed{u_1(x) = \frac{2}{3}}$$

Put $n=1$

$$u_2(x) = \int_0^1 t\, u_1(t)\, dt$$

$$u_2(x) = \int_0^1 t\left(\frac{2}{3}\right) dt$$

$$u_2(x) = \left|\frac{t^2}{2}\left(\frac{2}{3}\right)\right|_0^1$$

$$\boxed{u_2(x) = \frac{1}{3}}$$

Put $n=2$

$$u_3(x) = \int_0^1 t\, u_2(t)\, dt$$

$$u_3(x) = \int_0^1 t\left(\frac{1}{3}\right) dt$$

$$u_3(x) = \left|\frac{t^2}{2}\left(\frac{1}{3}\right)\right|_0^1$$

$$\boxed{u_3(x) = \frac{1}{6}}$$

From Eq ① $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots$$

$$u(x) = x + e^x - \frac{4}{3} + \frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \cdots$$

$$\left[r = \frac{a_2}{a_1} \quad S = \frac{1}{a_1 - r} \right]$$

$$u(x) = x + e^x - \frac{4}{3} + \frac{2}{3}\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \right]$$

$$u(x) = x + e^x - \frac{4}{3} + \frac{2}{3}\left[\frac{1}{1 - \frac{1}{2}} \right]$$

$$S = \frac{1}{1 - \frac{1}{2}} \Rightarrow 2$$

$$u(x) = x + e^x - \frac{4}{3} + \frac{2}{3}\left[\frac{1}{\frac{2-1}{2}} \right]$$

$$u(x) = x + e^x - \frac{4}{3} + \frac{2}{3}(2)$$

$$u(x) = x + e^x - \cancel{\frac{4}{3}} + \cancel{\frac{4}{3}}$$

$$\boxed{u(x) = x + e^x} \quad \text{Ans.}$$

Example #3: $u(x) = 2 + \cos x + \int_0^{\pi} t\, u(t)\, dt$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \quad ①$$

$$u_0(x) = f(x) \qquad \because \{ f(x) = 2 + \cos x \}$$

$$\boxed{u_0(x) = 2 + \cos x}$$

$$u_{n+1}(x) = \lambda \int_0^{a} k(x,t)\, u_n(t)\, dt$$

where $\lambda = 1$, $k(x,t) = t$

$$u_{n+1}(x) = \int_0^{a} t\, u_n(t)\, dt$$

Put $n=0$

$$u_1(x) = \int_0^x t\, u_0(t)\, dt$$

$$u_1(x) = \int_0^x t(2+\cos t)\, dt$$

$$u_1(x) = \int_0^x (2t + t\cos t)\, dt$$

$$u_1(x) = 2\frac{t^2}{2}\Big|_0^x + t\sin t\Big|_0^x - \int_0^x \sin t\, dt$$

$$u_1(x) = x^2 + 0 - \left|-\cos t\right|_0^x$$

$$u_1(x) = x^2 - \left[-\cos x + \cos(0)\right]$$

$$u_1(x) = x^2 - \left[-(-1) + 1\right]$$

$$\boxed{u_1(x) = x^2 - 2}$$

Put $n=1$

$$u_2(x) = \int_0^x t\, u_1(t)\, dt$$

$$u_2(x) = \int_0^x t\,(x^2-2)\, dt$$

$$u_2(x) = \left|\frac{t^2}{2}(x^2-2)\right|_0^x$$

$$u_2(x) = \frac{x^2}{2}(x^2-2)$$

$$\boxed{u_2(x) = \frac{x^4}{2} - x^2}$$

Put $n=2$

$$u_3(x) = \int_0^x t\, u_2(t)\, dt$$

$$u_3(x) = \int_0^x t\left(\frac{x^4}{2} - x^2\right) dt$$

$$u_3(x) = \left|\frac{t^2}{2}\left(\frac{x^4}{2} - x^2\right)\right|_0^x \Rightarrow \frac{x^2}{2}\left(\frac{x^4}{2} - x^2\right)$$

$$\boxed{u_3(x) = \frac{x^6}{4} - \frac{x^4}{2}}$$

From Eq ① ⇒

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \dots$$

$$u(x) = \cancel{x} + \cos x + \cancel{x^2} - \cancel{x} + \cancel{\frac{x^4}{2}} - \cancel{x^2} + \cancel{\frac{x^6}{4}} - \cancel{\frac{x^4}{2}} + \dots$$

$$\boxed{u(x) = \cos x}$$ Ans.

Example #4: $u(x) = 1 + \frac{1}{2}\int_0^{\pi/4} \sec^2 x \, u(t)\,dt$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots \quad ①$$

$$u_0(x) = f(x) \qquad [f(x) = 1]$$

$$\boxed{u_0(x) = 1}$$

$$u_{n+1}(x) = \lambda \int_0^{\pi/4} K(x,t)\, u_n(t)\,dt$$

where $\lambda = \frac{1}{2}$; $K(x,t) = \sec^2 x$

$$u_{n+1}(x) = \frac{1}{2}\int_0^{\pi/4} \sec^2 x \, u_n(t)\,dt$$

Put $n=0$

$$u_1(x) = \frac{1}{2}\sec^2 x \int_0^{\pi/4} u_0(t)\,dt$$

$$u_1(x) = \frac{1}{2}\sec^2 x \int_0^{\pi/4} (1)\,dt$$

$$u_1(x) = \frac{1}{2}\sec^2 x \,\left| t \right|_0^{\pi/4}$$

$$u_1(x) = \frac{1}{2}\sec^2 x \left(\frac{\pi}{4}\right)$$

$$\boxed{u_1(x) = \frac{\pi}{8}\sec^2 x}$$

Put n=1

$$u_1(x) = \frac{1}{2}\int_0^{\pi/4} \sec^2 x\ u_0(t)\,dt$$

$$u_1(x) = \frac{1}{2}\sec^2 x \int_0^{\pi/4} \frac{\pi}{8}\sec^2 t\ dt$$

$$u_1(x) = \frac{1}{2}\sec^2 x \left|\frac{\pi}{8}\ \tan t\right|_0^{\pi/4}$$

$$u_1(x) = \frac{1}{2}\sec^2 x \left(\frac{\pi}{8}\ \tan\frac{\pi}{4}\right)$$

$$u_1(x) = \frac{1}{2}\sec^2 x \left[\frac{\pi}{8}(1)\right]$$

$$\boxed{u_1(x) = \frac{\pi}{16}\sec^2 x}$$

Put n=2

$$u_2(x) = \frac{1}{2}\int_0^{\pi/4} \sec^2 x\ u_1(t)\,dt$$

$$u_2(x) = \frac{1}{2}\sec^2 x \int_0^{\pi/4} u_1(t)\,dt$$

$$u_2(x) = \frac{1}{2}\sec^2 x \int_0^{\pi/4} \frac{\pi}{16}\ \sec^2 t\ dt$$

$$u_2(x) = \frac{1}{2}\sec^2 x \left[\frac{\pi}{16}\left|\tan t\right|_0^{\pi/4}\right]$$

$$= \frac{1}{2}\sec^2 x \left[\frac{\pi}{16}\ \tan\left(\frac{\pi}{4}\right)\right]$$

$$u_2(x) = \frac{1}{2}\sec^2 x \left(\frac{\pi}{16}\right)$$

$$\boxed{u_2(x) = \frac{\pi}{32}\sec^2 x}$$

From ① Eq $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots$$

$$u(x) = 1 + \frac{\pi}{8}\sec^2 x + \frac{\pi}{16}\sec^2 x + \frac{\pi}{32}\sec^2 x + \cdots$$

$$u(x) = 1 + \frac{\pi}{8}\sec^2 x\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right)$$

$r = \frac{a_2}{a_1} \Rightarrow \frac{1/2}{1} \Rightarrow r = \frac{1}{2}$

$S = \frac{1}{a_1 - r} \Rightarrow \frac{1}{1-\frac{1}{2}} \Rightarrow \frac{1}{1/2}$

$S = 2$

So

$u(x) = 1 + \frac{\pi}{8} \sec^2 x \,(2)$

$\boxed{u(x) = 1 + \frac{\pi}{4} \sec^2 x}$ Ans.

Example #5: $u(x) = \pi x + \sin 2x + x \int_{-\pi}^{\pi} t\, u(t)\, dt$

Sol: By using ADM

$u(x) = \sum_{n=0}^{\infty} u_n(x)$

$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots$ —①

$u_0(x) = f(x)$ $\{f(x) = \pi x + \sin 2x\}$

$\boxed{u_0(x) = \pi x + \sin 2x}$

$u_{n+1}(x) = \lambda \int_{-\pi}^{\pi} K(x,t)\, u_n(t)\, dt$

where $\lambda = x$, $K(x,t) = t$

$u_{n+1}(x) = x \int_{-\pi}^{\pi} t\, u_n(t)\, dt$

Put $n=0$

$u_1(x) = x \int_{-\pi}^{\pi} t\, u_0(t)\, dt$

$u_1(x) = x \int_{-\pi}^{\pi} t\, (\pi t + \sin 2t)\, dt$

$u_1(x) = x \int_{-\pi}^{\pi} (\pi t^2 + t \sin 2t)\, dt$

$u_1(x) = x \left[\pi \frac{t^3}{3} \Big|_{-\pi}^{\pi} + t \frac{-\cos 2t}{2} \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{-\cos 2t}{2}\, dt \right.$

$$u_1(x) = x\left[\frac{\lambda^3}{3} - \left(-\frac{\lambda^3}{3}\right) + \lambda\left(-\frac{\sin 2\lambda}{2}\right) - (\lambda)(\cos 2\lambda) + \frac{\sin 2\lambda}{4}\Big|_{-\lambda}^{\lambda}\right] \quad \cos(-\theta) = \cos\theta$$

$$u_1(x) = x\left[\frac{\lambda^3}{3} + \frac{\lambda^3}{3} - \lambda - \lambda\right]$$

$$\boxed{u_1(x) = \frac{2}{3}\lambda^3 x - \lambda x}$$

Put $n=1$

$$u_2(x) = x\int_{-\lambda}^{\lambda} t\left(\frac{2}{3}\lambda^3 t - \lambda t\right)dt$$

$$u_2(x) = x\int_{-\lambda}^{\lambda}\left(\frac{2}{3}\lambda^3 t^2 - \lambda t^2\right)dt$$

$$u_2(x) = x\left[\frac{2}{3}\lambda^3 \frac{t^3}{3}\Big|_{-\lambda}^{\lambda} - \lambda\frac{t^3}{3}\Big|_{-\lambda}^{\lambda}\right]$$

$$u_2(x) = x\left[\frac{2}{3}\lambda^3\left(\frac{\lambda^3}{3} + \frac{\lambda^3}{3}\right) - \lambda\left(\frac{\lambda^3}{3} + \frac{\lambda^3}{3}\right)\right]$$

$$u_2(x) = x\left[\frac{4}{9}\lambda^7 - \frac{2\lambda^4}{3}\right]$$

$$\boxed{u_2(x) = \frac{4}{9}\lambda^7 x - \frac{2}{3}\lambda^4 x}$$

Put $n=2$

$$u_3(x) = x\int_{-\lambda}^{\lambda} t\left(\frac{4}{9}\lambda^7 t - \frac{2}{3}\lambda^4 t\right)dt$$

$$u_3(x) = x\int_{-\lambda}^{\lambda}\left(\frac{4}{9}\lambda^7 t^2 - \frac{2}{3}\lambda^4 t^2\right)dt$$

$$u_3(x) = x\left[\frac{4}{9}\lambda^7\frac{t^3}{3}\Big|_{-\lambda}^{\lambda} - \frac{2}{3}\lambda^4\frac{t^3}{3}\Big|_{-\lambda}^{\lambda}\right]$$

$$u_3(x) = x\left[\frac{4}{9}\lambda^7\left(\frac{\lambda^3}{3} + \frac{\lambda^3}{3}\right) - \frac{2}{3}\lambda^4\left(\frac{\lambda^3}{3} + \frac{\lambda^3}{3}\right)\right]$$

$$u_3(x) = x\left[\frac{8}{27}\lambda^{10} - \frac{4}{9}\lambda^7\right]$$

$$\boxed{u_3(x) = \frac{8}{27}\lambda^{10}x - \frac{4}{9}\lambda^7 x}$$

$\vdots$

From Eq. (1) $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \dots$$

$$u(x) = 3x + \sin 2x - \tfrac{2}{3}\cancel{x} - 6x + \tfrac{4}{9}\cancel{x} - \tfrac{2}{3}\cancel{x} + \tfrac{2}{3}\cancel{x} - \tfrac{4}{9}\cancel{x} + \dots$$

$$\boxed{u(x) = \sin 2x} \quad \text{Ans.}$$

(Exercise 4.2.1)

Solve the following Fredholm Integral Equation by using ADM:

(2) $u(x) = e^x + e^{-1}\int_0^1 u(t)\,dt$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots \quad -(1)$$

$$u_0(x) = f(x) \qquad \because [f(x) = e^x]$$

$$\boxed{u_0(x) = e^x}$$

$$u_{n+1}(x) = \lambda \int_0^1 K(x,t)\, u_n(t)\,dt$$

where $\lambda = e^{-1}$, $K(x,t) = 1$

$$u_{n+1}(x) = e^{-1}\int_0^1 u_n(t)\,dt$$

Put $n=0$

$$u_1(x) = e^{-1}\int_0^1 u_0(t)\,dt$$

$$u_1(x) = e^{-1}\int_0^1 e^t\,dt$$

$$u_1(x) = e^{-1}\left| e^t \right|_0^1$$

$$u_1(x) = e^{-1}\left[e - e^0\right]$$

$$\boxed{u_1(x) = 1 - e^{-1}}$$

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Put $n=1$

$$u_2(x) = e^{-1}\int_0^1 u_1(t)\,dt$$

$$u_2(x) = e^{-1}\int_0^1 (1-e^{-1})\,dt$$

$$u_2(x) = e^{-1}\left[t|_0^1 - e^{-1}t|_0^1\right]$$

$$u_2(x) = e^{-1}\left[1 - e^{-1}(1)\right]$$

$$u_2(x) = e^{-1}\left[1-e^{-1}\right]$$

$$\boxed{u_2(x) = e^{-1} - e^{-2}}$$

Put $n=2$

$$u_3(x) = e^{-1}\int_0^1 u_2(t)\,dt$$

$$= e^{-1}\int_0^1 (e^{-1}-e^{-2})\,dt$$

$$= e^{-1}\left[te^{-1}|_0^1 - e^{-2}t|_0^1\right]$$

$$u_3(x) = e^{-1}\left[e^{-1}-e^{-2}\right]$$

$$\boxed{u_3(x) = e^{-2} - e^{-3}}$$

From Eq ① $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots$$

$$u(x) = e^{x} + 1 \cancel{-e^{-1}} \cancel{+e^{-1}} \cancel{-e^{-2}} \cancel{+e^{-2}} - e^{-3} + \cdots$$

$$\boxed{u(x) = e^{x}+1}$$ Ans.

③ $u(x) = \cos x + 2x + \int_0^{\pi} xt\,u(t)\,dt$

Sol. By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots \quad \text{--- (1)}$$

$$u_0(x) = f(x) \qquad \because \left[f(x) = \cos x + 2x \right]$$

$$\boxed{u_0(x) = \cos x + 2x}$$

$$u_{n+1}(x) = \lambda \int_0^{\pi} K(x,t)\, u_n(t)\, dt$$

where $\lambda = 1$, $K(x,t) = xt$

$$u_{n+1}(x) = \int_0^{\pi} xt\, u_n(t)\, dt$$

Put $n=0$

$$u_1(x) = \int_0^{\pi} xt\, u_0(t)\, dt$$

$$u_1(x) = \int_0^{\pi} xt\,(\cos t + 2t)\, dt$$

$$u_1(x) = x \int_0^{\pi} (t\cos t + 2t^2)\, dt$$

$$u_1(x) = x \left[t \sin t \Big|_0^{\pi} - \int_0^{\pi} \sin t\, dt + \frac{2t^3}{3} \Big|_0^{\pi} \right]$$

$$u_1(x) = x \left[0 - \left| -\cos t \right|_0^{\pi} + \frac{2}{3}\pi^3 \right]$$

$$u_1(x) = x \left[+(\cos \pi - \cos(0)) + \frac{2}{3}\pi^3 \right]$$

$$u_1(x) = x \left[-1 - 1 + \frac{2}{3}\pi^3 \right]$$

$$\boxed{u_1(x) = -2x + \frac{2}{3}\pi^3 x}$$

Put $n=1$

$$u_2(x) = \int_0^{\pi} xt\, u_1(t)\, dt$$

$$u_2(x) = \int_0^{\pi} xt \left(-2t + \frac{2}{3}\pi^3 t \right) dt$$

$$u_2(x) = x\int_0^x \left(-2t^2 + \frac{2}{3}x^3 t^2\right) dt$$

$$u_2(x) = x\left[-\frac{2}{3}t^3\Big|_0^x + \frac{2}{3}x^3\frac{t^3}{3}\Big|_0^x\right]$$

$$u_2(x) = x\left[-\frac{2}{3}x^3 + \frac{4}{9}x^6\right]$$

$$\boxed{u_2(x) = -\frac{2}{3}x^3 x + \frac{2}{9}x^6 x}$$

Put $n=2$

$$u_3(x) = \int_0^x xt\, u_2(t)\,dt$$

$$u_3(x) = x\int_0^x t\left(-\frac{2}{3}x^3 t + \frac{4}{9}x^6 t\right)dt$$

$$u_3(x) = x\int_0^x \left(-\frac{2}{3}x^3 t^2 + \frac{4}{9}x^6 t^2\right)$$

$$u_3(x) = x\left[-\frac{2}{3}x^3\frac{t^3}{3}\Big|_0^x + \frac{4}{9}x^6\frac{t^3}{3}\Big|_0^x\right]$$

$$u_3(x) = x\left[-\frac{2}{9}x^6 + \frac{4}{27}x^9\right]$$

$$\boxed{u_3(x) = -\frac{2}{9}x^6 x + \frac{4}{27}x^9 x}$$

$\vdots$

From ① Eq $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots$$

$$u(x) = \cos x + \cancel{2x} - \cancel{2x} + \cancel{\frac{2}{3}x^3x} - \cancel{\frac{2}{3}x^3x} + \cancel{\frac{2}{9}x^6x} - \cancel{\frac{2}{9}x^6x} + \cancel{\frac{4}{27}x^9x} \ldots$$

$\boxed{u(x) = \cos x}$ Ans.

(4) $u(x) = \sin x - x + \int_0^{\pi/2} xt\, u(t)\, dt$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \quad (1)$$

$$u_0(x) = f(x) \qquad \because [f(x) = \sin x - x]$$

$$\boxed{u_0(x) = \sin x - x}$$

$$u_{n+1}(x) = \lambda \int_0^{\pi/2} k(x,t)\, u_n(t)\, dt$$

where $\lambda = 1$, $K(x,t) = (xt)$

$$u_{n+1}(x) = \int_0^{\pi/2} xt\, u_n(t)\, dt$$

Put $n=0$

$$u_1(x) = \int_0^{\pi/2} xt\, u_0(t)\, dt$$

$$u_1(x) = x \int_0^{\pi/2} t\,(\sin t - t)\, dt$$

$$u_1(x) = x \int_0^{\pi/2} (t \sin t - t^2)\, dt$$

$$u_1(x) = x \left[t\,|-\cos t|_0^{\pi/2} - \int_0^{\pi/2} -\cos t\, dt - \frac{t^3}{3}\Big|_0^{\pi/2} \right]$$

$$u_1(x) = x \left[0 + |\sin t|_0^{\pi/2} - \frac{\pi^3}{24} \right]$$

$$u_1(x) = x\left(1 - \frac{\pi^3}{24}\right)$$

$$\boxed{u_1(x) = x - \frac{\pi^3}{24} x}$$

Put $n=1$

$$u_2(x) = \int_0^{\pi/2} xt\, u_1(t)\, dt$$

$$u_2(x) = x\int_0^{\pi/2} t\left(t - \frac{\pi^3}{24}t\right)dt$$

$$u_2(x) = x\int_0^{\pi/2} \left(t^2 - \frac{\pi^3}{24}t^2\right)dt$$

$$u_2(x) = x\left[\frac{t^3}{3}\Big|_0^{\pi/2} - \frac{\pi^3}{24}\frac{t^3}{3}\Big|_0^{\pi/2}\right]$$

$$u_2(x) = x\left[\frac{\pi^3}{24} - \frac{\pi^6}{576}\right]$$

$$\boxed{u_2(x) = \frac{\pi^3}{24}x - \frac{\pi^6}{576}x}$$

Put $n=2$

$$u_3(x) = \int_0^{\pi/2} xt\, u_2(t)\,dt$$

$$u_3(x) = x\int_0^{\pi/2} t\left(\frac{\pi^3}{24}t - \frac{\pi^6}{576}t\right)dt$$

$$u_3(x) = x\int_0^{\pi/2} \left(\frac{\pi^3}{24}t^2 - \frac{\pi^6}{576}t^2\right)dt$$

$$u_3(x) = x\left[\frac{\pi^3}{24}\frac{t^3}{3}\Big|_0^{\pi/2} - \frac{\pi^6}{576}\frac{t^3}{3}\Big|_0^{\pi/2}\right]$$

$$u_3(x) = x\left[\frac{\pi^6}{576} - \frac{\pi^9}{13824}\right]$$

$$\boxed{u_3(x) = \frac{\pi^6}{576}x - \frac{\pi^9}{13824}x}$$

⋮

From ① Eq $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \dots$$

$$u(x) = \sin x - \cancel{x} + \cancel{x} - \cancel{\frac{\pi^3}{24}x} + \cancel{\frac{\pi^3}{24}x} - \cancel{\frac{\pi^6}{576}x} + \cancel{\frac{\pi^6}{576}x} - \frac{\pi^9}{13824}x + \dots$$

$$\boxed{u(x) = \sin x}$$ Ans.

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(9) $u(x) = xe^{x} - \frac{1}{2} + \frac{1}{2}\int_0^1 u(t)\,dt$

Sol. By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots \quad -①$$

$$u_0(x) = f(x) \qquad \because \left(f(x) = xe^{x} - \frac{1}{2}\right)$$

$$\boxed{u_0(x) = xe^{x} - \frac{1}{2}}$$

$$u_{n+1}(x) = \lambda \int_0^1 K(x,t)\, u_n(t)\, dt$$

where $\lambda = \frac{1}{2}$, $K(x,t) = 1$

$$u_{n+1}(x) = \frac{1}{2}\int_0^1 u_n(t)\, dt$$

Put $n=0$

$$u_1(x) = \frac{1}{2}\int_0^1 u_0(t)\,dt$$

$$u_1(x) = \frac{1}{2}\int_0^1 \left(te^{t} - \frac{1}{2}\right) dt$$

$$u_1(x) = \frac{1}{2}\left[t\,e^{t}\Big|_0^1 - \int_0^1 e^{t}\,dt - \frac{1}{2}\,t\Big|_0^1\right]$$

$$u_1(x) = \frac{1}{2}\left[e - e^{t}\Big|_0^1 - \frac{1}{2} + 0\right]$$

$$u_1(x) = \frac{1}{2}\left[\cancel{e} - \cancel{e} + e^{0} - \frac{1}{2}\right]$$

$$u_1(x) = \frac{1}{2}\left[1 - \frac{1}{2}\right]$$

$$u_1(x) = \frac{1}{2}\left[\frac{1}{2}\right]$$

$$\boxed{u_1(x) = \frac{1}{4}}$$

Put $n=1$

$$u_2(x) = \frac{1}{2}\int_0^1 u_1(t)\,dt$$

$$= \frac{1}{2}\int_0^1 \frac{1}{4}\,dt$$

$u_1(x) = \frac{1}{2}\left[\frac{1}{4} t\Big|_0^1\right]$

$\boxed{u_1(x) = \frac{1}{8}}$

Put $n=2$

$u_3(x) = \frac{1}{2}\int_0^1 u_2(t)\,dt$

$u_3(x) = \frac{1}{2}\int_0^1 \frac{1}{8}\,dt$

$u_3(x) = \frac{1}{2}\left[\frac{1}{8} t\Big|_0^1\right]$

$u_3(x) = \frac{1}{2}\left(\frac{1}{8}\right)$

$\boxed{u_3(x) = \frac{1}{16}}$

⋮

From ① Eq $\Rightarrow$

$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots$

$u(x) = xe^x - \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

$u(x) = xe^x - \frac{1}{2} + \frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{4} + \cdots\right]$

$r = \frac{a_2}{a_1} \Rightarrow \frac{1}{2}$

$S = \frac{1}{a_1 - r} \Rightarrow \frac{1}{1-\frac{1}{2}} \Rightarrow \frac{1}{\frac{2-1}{2}}$

$S = 2$

$u(x) = xe^x - \frac{1}{2} + \frac{1}{4_2}(2)$

$u(x) = xe^x - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}}$

$\boxed{u(x) = xe^x}$ Ans.

$$u_2(x) = \frac{1}{2}\left[\frac{\pi}{4} - \frac{\pi^2}{16}\right]$$

(5) $$\boxed{u_2(x) = \frac{\pi}{8} - \frac{\pi^2}{32}}$$

Put $n=2$

$$u_3(x) = \frac{1}{2}\int_0^{\pi/2} u_2(t)\,dt$$

$$= \frac{1}{2}\int_0^{\pi/2}\left(\frac{\pi}{8} - \frac{\pi^2}{32}\right)dt$$

$$u_3(x) = \frac{1}{2}\left[\frac{\pi}{8}\,t\Big|_0^{\pi/2} - \frac{\pi^2}{32}\,t\Big|_0^{\pi/2}\right]$$

$$u_3(x) = \frac{1}{2}\left[\frac{\pi^2}{16} - \frac{\pi^3}{64}\right]$$

$$\boxed{u_3(x) = \frac{\pi^2}{32} - \frac{\pi^3}{128}}$$

$\vdots$

From Eq (1) $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$$

$$u(x) = x\sin x - \cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{2}} - \cancel{\tfrac{\pi}{8}} + \cancel{\tfrac{\pi}{8}} - \cancel{\tfrac{\pi^2}{32}} + \cancel{\tfrac{\pi^2}{32}} - \cancel{\tfrac{\pi^3}{128}} + \cdots$$

$$\boxed{u(x) = x\sin x}$$ Ans.

---

(11) $$u(x) = x\cos x + 1 + \frac{1}{2}\int_0^{\pi} u(t)\,dt.$$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \quad -(1)$$

$$u_0(x) = f(x) \qquad \because [f(x) = x\cos x + 1]$$

$$\boxed{u_0(x) = x\cos x + 1}$$

$$u_{n+1}(x) = \lambda\int_0^{\pi} k(x,t)\,u_n(t)\,dt$$

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where $\lambda = \frac{1}{2}$, $K(x,t) = 1$

$$u_{n+1}(x) = \frac{1}{2}\int_0^{\pi} u_n(t)\,dt$$

Put $n=0$

$$u_1(x) = \frac{1}{2}\int_0^{\pi} u_0(t)\,dt$$

$$u_1(x) = \frac{1}{2}\int_0^{\pi} (t\cos t + 1)\,dt$$

$$u_1(x) = \frac{1}{2}\left[t\,|\sin t|_0^{\pi} - \int_0^{\pi} \sin t\,dt + t|_0^{\pi} \right]$$

$$u_1(x) = \frac{1}{2}\left[0 - |-\cos t|_0^{\pi} + \pi \right]$$

$$u_1(x) = \frac{1}{2}\left[+ \cos(\pi) - \cos(0) + \pi \right]$$

$$u_1(x) = \frac{1}{2}\left[-1 - 1 + \pi \right]$$

$$u_1(x) = \frac{1}{2}\left[-2 + \pi \right]$$

$$\boxed{u_1(x) = -1 + \frac{\pi}{2}}$$

Put $n=1$

$$u_2(x) = \frac{1}{2}\int_0^{\pi} u_1(t)\,dt$$

$$u_2(x) = \frac{1}{2}\int_0^{\pi} \left(-1 + \frac{\pi}{2}\right) dt$$

$$u_2(x) = \frac{1}{2}\left[-t|_0^{\pi} + \frac{\pi}{2} t|_0^{\pi} \right]$$

$$u_2(x) = \frac{1}{2}\left[-\pi + \frac{\pi^2}{2} \right]$$

$$\boxed{u_2(x) = -\frac{\pi}{2} + \frac{\pi^2}{4}}$$

Put $n=2$

$$u_3(x) = \sin x \int_0^{\pi/2} \cos t \; u_2(t)\,dt$$

$$u_3(x) = \sin x \int_0^{\pi/2} \cos t \; \frac{\sin t}{4}$$

$$u_3(x) = \frac{\sin x}{4} \int_0^{\pi/2} (\sin t \cos t)\,dt$$

$$u_3(x) = \frac{\sin x}{4} \left[\left| \frac{\sin^2 t}{2} \right|_0^{\pi/2} \right]$$

$$u_3(x) = \frac{\sin x}{8} \left[1 - 0 \right]$$

$$\boxed{u_3(x) = \frac{\sin x}{8}}$$

From Eq ① $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \ldots$$

$$u(x) = \sin x + \frac{\sin x}{2} + \frac{\sin x}{4} + \frac{\sin x}{8} + \ldots$$

$$u(x) = \sin x + \frac{\sin x}{2} \left[\overset{a_1}{1} + \overset{a_2}{\frac{1}{2}} + \overset{a_3}{\frac{1}{4}} + \ldots \right]$$

$$r = \frac{a_2}{a_1} \Rightarrow r = \frac{1}{2}$$

$$S = \frac{1}{a_1 - r} \Rightarrow \frac{1}{1 - \frac{1}{2}} \Rightarrow \frac{1}{1/2}$$

$$S = 2$$

So

$$u(x) = \sin x + \frac{\sin x}{2}(2)$$

$$\boxed{u(x) = 2\sin x} \quad \text{Ans.}$$

(b) $u(x) = x\sin x - \frac{1}{2} + \frac{1}{2}\int_0^{\pi/2} u(t)\,dt$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots \quad (1)$$

$$u_0(x) = f(x) \qquad \because \left[f(x) = x\sin x - \frac{1}{2}\right]$$

$$\boxed{u_0(x) = x\sin x - \frac{1}{2}}$$

$$u_{n+1}(x) = \lambda\int_0^{\pi/2} K(x,t)\,u_n(t)\,dt$$

where $\lambda = \frac{1}{2}$, $K(x,t) = 1$

$$u_{n+1}(x) = \frac{1}{2}\int_0^{\pi/2} u_n(t)\,dt$$

Put $n=0$

$$u_1(x) = \frac{1}{2}\int_0^{\pi/2} u_0(t)\,dt$$

$$u_1(x) = \frac{1}{2}\int_0^{\pi/2}\left(t\sin t - \frac{1}{2}\right)dt$$

$$u_1(x) = \frac{1}{2}\left[t(-\cos t)\Big|_0^{\pi/2} - \int_0^{\pi/2} -\cos t\,dt - \frac{1}{2}t\Big|_0^{\pi/2}\right]$$

$$u_1(x) = \frac{1}{2}\left[0 + (\sin t)\Big|_0^{\pi/2} - \frac{1}{2}\left(\frac{\pi}{2}\right)\right]$$

$$u_1(x) = \frac{1}{2}\left[\sin\left(\frac{\pi}{2}\right) - 0 - \frac{\pi}{4}\right]$$

$$u_1(x) = \frac{1}{2}\left[1 - \frac{\pi}{4}\right]$$

$$\boxed{u_1(x) = \frac{1}{2} - \frac{\pi}{8}}$$

Put $n=1$

$$u_2(x) = \frac{1}{2}\int_0^{\pi/2} u_1(t)\,dt$$

$$u_2(x) = \frac{1}{2}\int_0^{\pi/2}\left(\frac{1}{2} - \frac{\pi}{8}\right)dt$$

$$u_2(x) = \frac{1}{2}\left[\frac{1}{2}t\Big|_0^{\pi/2} - \frac{\pi}{8}t\Big|_0^{\pi/2}\right]$$

Put n=2

$$u_3(x) = \frac{1}{2}\int_0^x u_2(t)\,dt$$

$$u_3(x) = \frac{1}{2}\int_0^x \left(\frac{x}{2} + \frac{x^2}{4}\right) dt$$

$$u_3(x) = \frac{1}{2}\left[-\frac{x}{2}\, t\Big|_0^x + \frac{x^2}{4}\, t\Big|_0^x\right]$$

$$u_3(x) = \frac{1}{2}\left[-\frac{x^2}{2} + \frac{x^3}{4}\right]$$

$$\boxed{u_3(x) = -\frac{x^2}{4} + \frac{x^3}{8}}$$

⋮

From Eq.(1) ⇒

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots$$

$$u(x) = x\cos x + \cancel{x} - \cancel{x} + \cancel{\frac{x}{2}} - \cancel{\frac{x^2}{2}} + \cancel{\frac{x^2}{4}} - \cancel{\frac{x^2}{4}} + \cancel{\frac{x^3}{8}} - \cdots$$

$$\boxed{u(x) = x\cos x}$$ Ans.

(12) $u(x) = \sin x + \int_0^{\pi/2} \sin x \cos t \; u(t)\,dt$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \quad -(1)$$

$$u_0(x) = f(x)$$

$\because \boxed{f(x) = \sin x}$

$$\boxed{u_0(x) = \sin x}$$

$$u_{n+1}(x) = \lambda \int_0^{\pi/2} K(x,t)\, u_n(t)\, dt$$

where $\lambda = 1$, $K(x,t) = \sin x \cos t$

$$u_{n+1}(x) = \int_0^{\pi/2} \sin x \cos t\, u_n(t)\, dt$$

Put $n=0$

$$u_1(x) = \sin x \int_0^{\pi/2} \cos t\, u_0(t)\, dt$$

$$u_1(x) = \sin x \int_0^{\pi/2} \cos t \sin t\, dt$$

$$u_1(x) = \sin x \int_0^{\pi/2} (\sin t \cos t)\, dt$$

$$u_1(x) = \sin x \left[\left| \frac{\sin^2 t}{2} \right|_0^{\pi/2} \right]$$

$$u_1(x) = \sin x \left[\frac{1}{2} - 0 \right]$$

$$\boxed{u_1(x) = \frac{\sin x}{2}}$$

Put $n=1$

$$u_2(x) = \sin x \int_0^{\pi/2} \cos t \frac{\sin t}{2}\, dt$$

$$u_2(x) = \frac{\sin x}{2} \int_0^{\pi/2} \cos t \sin t\, dt$$

$$u_2(x) = \frac{\sin x}{2} \left[\left| \frac{\sin^2 t}{2} \right|_0^{\pi/2} \right]$$

$$u_2(x) = \frac{\sin x}{4} \left[+\sin\left(\frac{\pi}{2}\right)^2 - \sin(0) \right]$$

$$u_2(x) = \frac{\sin x}{4} \left[1 \right]$$

$$\boxed{u_2(x) = \frac{\sin x}{4}}$$

$$u(x) = 1 + \frac{1}{2}\sin^2 x \int_0^{\pi/2} u(t)\,dt$$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \quad (1)$$

$$u_0(x) = f(x)$$

$$\boxed{u_0(x) = 1}$$

$$u_{n+1}(x) = \lambda \int_0^{\pi/2} k(x,t)\,u_n(t)\,dt$$

where $\lambda = \frac{1}{2}\sin^2 x$, $k(x,t) = 1$

$$u_{n+1}(x) = \frac{1}{2}\sin^2 x \int_0^{\pi/2} u_n(t)\,dt$$

Put $n=0$

$$u_1(x) = \frac{1}{2}\sin^2 x \int_0^{\pi/2} u_0(t)\,dt$$

$$u_1(x) = \frac{1}{2}\sin^2 x \int_0^{\pi/2} 1\,dt$$

$$u_1(x) = \frac{1}{2}\sin^2 x \; t\Big|_0^{\pi/2}$$

$$u_1(x) = \frac{1}{2}\sin^2 x \left(\frac{\pi}{2}\right)$$

$$\boxed{u_1(x) = \frac{\pi}{4}\sin^2 x}$$

Put $n=1$

$$u_2(x) = \frac{1}{2}\sin^2 x \int_0^{\pi/2} u_1(t)\,dt$$

$$u_2(x) = \frac{1}{2}\sin^2 x \int_0^{\pi/2} \frac{\pi}{4}\sin^2 t\,dt$$

$$u_2(x) = \frac{\pi}{8}\sin^2 x \int_0^{\pi/2} \left(\frac{1-\cos 2t}{2}\right) dt$$

$$u_2(x) = \frac{\pi}{8}\sin^2 x \left[\frac{1}{2}\int_0^{\pi/2} (1-\cos 2t)\,dt\right]$$

$$u_2(x) = \frac{\pi}{8}\sin^2 x \left[\frac{1}{2}\left(t\Big|_0^{\pi/2} - \frac{\sin 2t}{2}\Big|_0^{\pi/2}\right)\right]$$

$$u_2(x) = \frac{\pi}{8}\sin^2 x\left[\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{2}\left\{\sin(\pi) - \sin(0)\right\}\right]$$

$$u_2(x) = \frac{\pi}{8}\sin^2 x\left[\frac{\pi}{4} - 0\right]$$

$$\boxed{u_2(x) = \frac{\pi^2}{32}\sin^2 x}$$

Put $n=2$

$$u_3(x) = \frac{1}{2}\sin^2 x\int_0^{\pi/2} u_2(t)\,dt$$

$$u_3(x) = \frac{1}{2}\sin^2 x\int_0^{\pi/2}\frac{\pi^2}{32}\sin^2 t\,dt$$

$$u_3(x) = \frac{\pi^2}{64}\sin^2 x\int_0^{\pi/2}\left(\frac{1-\cos 2t}{2}\right)dt$$

$$u_3(x) = \frac{\pi^2}{64}\sin^2 x\left[\frac{1}{2}\int_0^{\pi/2}(1-\cos 2t)\,dt\right]$$

$$u_3(x) = \frac{\pi^2}{64}\sin^2 x\left[\frac{1}{2}\left(t\Big|_0^{\pi/2} - \frac{\sin 2t}{2}\Big|_0^{\pi/2}\right)\right]$$

$$u_3(x) = \frac{\pi^2}{64}\sin^2 x\left[\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{4}\left(\sin(\pi) - \sin(0)\right)\right]$$

$$u_3(x) = \frac{\pi^2}{64}\sin^2 x\left[\left(\frac{\pi}{4}\right) - 0\right]$$

$$\boxed{u_3(x) = \frac{\pi^3}{256}\sin^2 x}$$

From ① Eq $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \dots$$

$$u(x) = 1 + \frac{\pi}{4}\sin^2 x + \frac{\pi^2}{32}\sin^2 x + \frac{\pi^3}{256}\sin^2 x + \dots$$

$$u(x) = 1 + \frac{\pi}{4}\sin^2 x\left[1 + \frac{\pi}{8} + \frac{\pi^2}{64} + \dots\right]$$

Consider

$\left(1 + \frac{\pi}{8} + \frac{\pi^2}{64} + \dots\right)$ is the sum of infinite series

So $a_1 = 1$

$r = \frac{\pi}{8}$

$$S = \frac{1}{a_1 - r} \Rightarrow \frac{1}{1-\frac{\pi}{8}} \Rightarrow \frac{8}{8-\pi}$$

$$u(x) = 1 + \frac{\pi}{4}\sin^2 x\left(\frac{8\,2}{8-\pi}\right)$$

$$\boxed{u(x) = 1 + \left(\frac{2\pi}{8-\pi}\right)\sin^2 x}$$ Ans.

(16) $u(x) = \frac{\pi}{4} - \sec^2 x - \int_0^{\pi/4} u(t)\,dt$

Sol: By using ADM

$$u(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots$$

$$u_0(x) = f(x)$$

$$\boxed{u_0(x) = \frac{\pi}{4} - \sec^2 x}$$ $\because \left[f(x) = \frac{\pi}{4} - \sec^2 x\right]$

$$u_{n+1}(x) = \lambda \int_0^{\pi/4} K(x,t)\,u_n(t)\,dt$$

where $\lambda = -1$, $K(x,t) = 1$

$$u_{n+1}(x) = -\int_0^{\pi/4} u_n(t)\,dt$$

Put $n=0$

$$u_1(x) = -\int_0^{\pi/4} u_0(t)\,dt$$

$$u_1(x) = -\int_0^{\pi/4}\left(\frac{\pi}{4} - \sec^2 t\right)dt$$

$$u_1(x) = -\left[\frac{\pi}{4}t\Big|_0^{\pi/4} - \tan t\Big|_0^{\pi/4}\right]$$

$$u(x) = -\left[\frac{\pi}{4}\left(\frac{\pi}{4}\right) - 1\right]$$

$$\boxed{u_1(x) = -\frac{\pi^2}{16} + 1}$$

Put $n=1$

$$u_2(x) = -\int_0^{\pi/4} u_1(t)\,dt$$

$$u_2(x) = -\int_0^{\pi/4} \left(-\frac{\pi^2}{16}+1\right) dt$$

$$u_2(x) = -\int_0^{\pi/4} \left(-\frac{\pi^2}{16}+1\right) dt$$

$$u_2(x) = -\left[\frac{\pi^2}{16} t\Big|_0^{\pi/4} + t\Big|_0^{\pi/4}\right]$$

$$u_2(x) = -\left[-\frac{\pi^2}{16}\left(\frac{\pi}{4}\right) + \frac{\pi}{4}\right]$$

$$\boxed{u_2(x) = \frac{\pi^3}{64} - \frac{\pi}{4}}$$

Put $n=2$

$$u_3(x) = -\int_0^{\pi/4} u_2(t)\,dt$$

$$u_3(x) = -\int_0^{\pi/4} \left(\frac{\pi^3}{64} - \frac{\pi}{4}\right) dt$$

$$u_3(x) = -\left[\frac{\pi^3}{64} t\Big|_0^{\pi/4} - \frac{\pi}{4} t\Big|_0^{\pi/4}\right]$$

$$u_3(x) = -\left[\frac{\pi^3}{64}\left(\frac{\pi}{4}\right) - \frac{\pi}{4}\left(\frac{\pi}{4}\right)\right]$$

$$\boxed{u_3(x) = -\frac{\pi^4}{256} + \frac{\pi^2}{16}}$$

Put $n=3$

$$u_4(x) = -\int_0^{\pi/4} u_3(t)\,dt$$

$$u_4(x) = -\int_0^{\pi/4} \left(-\frac{\pi^4}{256} + \frac{\pi^2}{16}\right) dt$$

$$u_4(x) = -\left[-\frac{\pi^4}{256} t\Big|_0^{\pi/4} + \frac{\pi^2}{16} t\Big|_0^{\pi/4}\right]$$

$$u_4(x) = -\left[-\frac{\pi^4}{256}\left(\frac{\pi}{4}\right) + \frac{\pi^2}{16}\left(\frac{\pi}{4}\right)\right]$$

$$\boxed{u_4(x) = \frac{\pi^5}{1024} - \frac{\pi^3}{64}}$$

$\vdots$

From ① Eq $\Rightarrow$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + u_4(x) + \dots$$

$$u(x) = \frac{\pi}{4} - \sec^2 x - \frac{\pi^2}{16} + 1 + \frac{\pi^3}{64} - \frac{\pi}{4} - \frac{\pi^4}{256} + \frac{\pi^2}{16} + \frac{\pi^5}{1024} - \frac{\pi^3}{64} + \dots$$

$$u(x) = 1 - \sec^2 x$$

$$\boxed{u(x) = -\tan^2 x}$$ Ans.

$\because$ $\tan^2 x + 1 = \sec^2 x$
$1 - \sec^2 x = -\tan^2 x$