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vector space

Definition:

A space consisting of vectors, together with the associative and commutative operation of addition of vectors, and the associative and distributive operation of multiplication of vectors by scalars.

Introduction

To define a vector space, first we need a few basic definitions. A **set** is a collection of distinct objects called elements. The elements are usually real or complex numbers when we use them in mathematics, but the elements of a set can also be a list of things. We notate a set by encasing the elements within curly braces. Note that to be distinct, an element cannot be repeated within the same set.

- {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} is the set of single-digit numbers that we use in mathematics.
- {a, b, c, d, . . ., y, z} is the set of letters in the alphabet.

Let's now take a closer look at elements in vector spaces. First, it's important to note that a space in mathematics is a set in which the list of elements are defined by a collection of guidelines or axioms for how each element relates to another within the set.

A **vector space** is a space in which the elements are sets of numbers themselves. Each element in a vector space is a list of objects that has a specific length, which we call **vectors**. We usually refer to the elements of a vector space as n-tuples, with n as the specific length of each of the elements in the set.

Each element of a vector space of length n can be represented as a matrix, which you may recall is a collection of numbers within parentheses. Matrix representations require multiple other lessons in matrix multiplication and addition, so we will use the parentheses notation for this assignment

.

Here's an example: In the 4-dimensional vector space of the real numbers, notated as R^4, one element is (0, 1, 2, 3). This vector has four parts and is a single element within the vector space R^4.

Now let's take a closer look at fields. We refer to any vector space as a vector space defined over a given field *F*. A **field** is a space of individual numbers, usually real or complex numbers. **A field is a set F of numbers with the property that if a, b ∈ F, then a + b, a − b, ab and a/b are also in F (assuming, of course, that b 6= 0 in the expression a/b).**

The specific axioms to define a field are similar to those of a vector space, so for the purposes of this assignment, we'll define a field as a vector space.

Technically speaking in term of math's

A *vector space* is a set V on which two operations + and · are defined, called *vector addition* and *scalar multiplication.*

. A vector space consists of a set of V (elements of V are called vectors), a field F (elements of F are scalars) and the two operations

Elements of V are mostly called vectors and the elements of F are mostly scalars. There are different types of vectors. To qualify the vector space V, the addition and multiplication operation must stick to the number of requirements called axioms. The axioms generalize the properties of vectors introduced in the field F.

If it is over the real numbers R is called a real vector space $\mathbb{R}^n$

If it is over the complex numbers, C is called the complex vector space $\mathbb{C}^n$.

In the assignment I will discuss above examples in detail.

Euclidean vectors are an example of a vector space

Difference between Vector and Vector Space

A vector is a part of a vector space whereas vector space is a group of objects which is multiplied by scalars and combined by the vector space axioms.

Where both the operations must satisfy the following condition

PROPERTIES: -

Vector addition

Vector addition is an operation that takes two vectors u, v ∈ V, and it produces the third vector u + v ∈ V

Conditions for Vector Addition

❖ The operation + (vector addition) must satisfy the following conditions:

- *Closure:*

If **u** and **v** are any vectors in V, then the sum **u** + **v** belongs to V.

- *Commutative law:*

For all vectors **u** and **v** in V, **u** + **v** = **v** + **u**

- *Associative law:*

For all vectors **u**, **v**, **w** in V, **u** + (**v** + **w**) = (**u** + **v**) + **w**

- *Additive identity:*

The set V contains an *additive identity* element, denoted by **0**, such that for any vector **v** in V, **0** + **v** = **v** and **v** + **0** = **v**

- *Additive inverses:*

For each vector **v** in V, the equations **v** + **x** = **0** and **x** + **v** = **0** have a solution **x** in V, called an *additive inverse* of **v**, and denoted by **- v**.

SACLAR MULTIPLICATION

Scalar Multiplication is an operation that takes a scalar c ∈ F and a vector v ∈ V and it produces a new vector uv∈ V.

Condition for Scalar Multiplication

❖ The operation · (scalar multiplication) is defined between real numbers (or scalars) and vectors, and must satisfy the following conditions:

- *Closure:*

If **v** in any vector in V, and c is any real number, then the product c · **v** belongs to V.

Distributive law:

For all real numbers c and all vectors **u**, **v** in V, c · (**u** + **v**) = c · **u** + c · **v**

Distributive law:

For all real numbers c, d and all vectors **v** in V, (c+d) · **v** = c · **v** + d · **v**

Associative law:

For all real numbers c,d and all vectors **v** in V,   c · (d · **v**) = (cd) · **v**

Unitary law:

For all vectors **v** in V,   1 · **v** = **v**

RESULTS DRAWN FROM PROPERTIES OF VECTOR SPACE

Here are some basic properties/results that are derived from the axioms are

- The addition operation of a finite list of vectors v_1 v_2, . . ., v_k can be calculated in any order, then the solution of the addition process will be the same.
- If x + y = 0, then the value should be y = −x.
- The negation of 0 is 0. This means that the value of −0 = 0.
- The negation or the negative value of the negation of a vector is the vector itself: −(−v) = v.
- If x + y = x, if and only if y = 0. Therefore, 0 is the only vector that behaves like 0.
- The product of any vector with zero times gives the zero vector. 0 x y = 0 for every vector in y.
- For every real number c, any scalar times of the zero vector is the zero vector. c0 = 0
- If the value cx= 0, then either c = 0 or x = 0. The product of a scalar and a vector is equal to when either scalar is 0 or a vector is 0.

- The scalar value −1 times a vector is the negation of the vector: $(-1)x = -x$. We define subtraction in terms of addition by defining $x - y$ as an abbreviation for $x + (-y)$.

$$x - y = x + (-y)$$

All the normal properties of subtraction follow:

- $x + y = z$ then the value $x = z - y$.
- $c(x - y) = cx - cy$.
- $(c - d)x = cx - dx$

Subspaces

Definition:

Let V be a vector space, and let W be a subset of V. If W is a vector space with respect to the operations in V, then W is called a *subspace* of V.

Theorem:

Let V be a vector space, with operations + and ·, and let W be a subset of V. Then W is a subspace of V if and only if the following conditions hold.

W is nonempty:

The zero vector belongs to W.

Closure under (+) addition:

If **u** and **v** are any vectors in W, then **u + v** is in W.

*Closure under (·) **dot***:

If **v** is any vector in W, and c is any real number, then c · **v** is in W.

LET V BE THE SET n BY ONE ROW AND n COLUMN MATRICE OF REAL NUMBER

R^n is the set of all n-tuple of real numbers

$\vec{v}=(x_1, x_2, x_3 \cdots x_n)$ $\vec{v}$ ε R^n

$\vec{w}=(y_1, y_2, y_3 \cdots y_n)$ $\vec{w}$ ε R^n

$\vec{u}=(z_1, z_2, z_3 \cdots z_n)$ $\vec{u}$ ε R^n

$R^n = \{((x_1, x_2, x_3 \cdots x_n): x_i \epsilon R$

Addition: -

$(x_1, x_2, x_3 \cdots x_n) + (y_1, y_2, y_3 \cdots y_n) = (x_1 + y_1, x_2 + y_2, x_3 + y_3 \cdots x_n + y_n)$

Multiplication: -

$C(x_1, x_2, x_3 \cdots x_n) = (Cx_1, Cx_2, Cx_3 \cdots Cx_n)$

PROPERTIES: -

UNDER ADDITION

Closure:

v + **w** belongs to V.

$(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n}) + (\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n}) = (\boldsymbol{x_1} + \boldsymbol{y_1}, \boldsymbol{x_2} + \boldsymbol{y_2}, \boldsymbol{x_3} + \boldsymbol{y_3} \cdots \boldsymbol{x_n} + \boldsymbol{y_n})$

PROPERTY HOLDS

Commutative law:

v+ **w**= **w**+ **v**

$(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n}) + (\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n}) = (\boldsymbol{x_1} + \boldsymbol{y_1}, \boldsymbol{x_2} + \boldsymbol{y_2}, \boldsymbol{x_3} + \boldsymbol{y_3} \cdots \boldsymbol{x_n} + \boldsymbol{y_n})$

$= (\boldsymbol{y_1} + \boldsymbol{x_1}, \boldsymbol{y_2}\, \boldsymbol{x_2} +, \boldsymbol{y_3} + \boldsymbol{x_3} \cdots \boldsymbol{y_n} + \boldsymbol{x_n})$

$= (\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n}) + (\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n})$

PROPERTY HOLDS

Associative law:

u + (**v** + **w**) = (**u** + **v**) + **w**

$(\boldsymbol{z_1}, \boldsymbol{z_2}, \boldsymbol{z_3} \cdots \boldsymbol{z_n}) + [(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n}) + (\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n})] =$

$[(\boldsymbol{z_1}, \boldsymbol{z_2}, \boldsymbol{z_3} \cdots \boldsymbol{z_n}) + (\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n})] + (\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n})$

PROPERTY HOLDS

Distributive law:

a (v+w) =av+aw

$a[(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n}) + (\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n})]) = (\boldsymbol{ax_1} + \boldsymbol{y_1}, \boldsymbol{ax_2} + \boldsymbol{y_2}, \boldsymbol{ax_3} +$
$\boldsymbol{y_3} \cdots \boldsymbol{ax_n} + \boldsymbol{y_n})$

$= (\boldsymbol{ax_1}, \boldsymbol{ax_2}, \boldsymbol{ax_3} \cdots \boldsymbol{ax_n}) + (\boldsymbol{ay_1}, \boldsymbol{ay_2}, \boldsymbol{ay_3} \cdots \boldsymbol{ay_n})$

$= a(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n}) + \mathbf{a}(\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n})$

PROPERTY HOLDS

Additive identity:

0 + **v** = **v** and **v** + **0** = **v**

$\mathbf{(, x_2, x_3 \cdots x_n) + (0,0,0 \cdots 0) = (x_1 + 0, x_2 + 0, x_3 + 0 \cdots x_n + 0)}$

PROPERTY HOLDS

Additive inverses:

v + **-v**= $\mathbf{0}$ and **-v** + **v** = $\mathbf{0}$

$(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n}) + (-\boldsymbol{x_1}, -\boldsymbol{x_2}, -\boldsymbol{x_3} \cdots - \boldsymbol{x_n})$=**0**

$=(-\boldsymbol{x_1}, -\boldsymbol{x_2}, -\boldsymbol{x_3} \cdots - \boldsymbol{x_n}) + (\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n})$

=0

PROPERTY HOLDS

UNDER MULTIPLICATION

Closure:

c ⋅ **v** belongs to V.

C$(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n}) = (\boldsymbol{Cx_1}, \boldsymbol{Cx_2}, \boldsymbol{Cx_3} \cdots \boldsymbol{Cx_n})$

PROPERTY HOLDS

Distributive law:

For all real numbers c and all vectors **v**, **w** in V, c ⋅ (**v** + **w**) = c ⋅ **v**+ c ⋅ **w**

C. $[(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n}) + (\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n})]$=

C. $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3} \cdots \boldsymbol{x_n})$ +**C.** $(\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} \cdots \boldsymbol{y_n})$

PROPERTY HOLDS

Distributive law:

For all real numbers v,w and all vectors **v** in V, (v+w) · **c**= v· **c** + w · **c**

$[(x_1, x_2, x_3 \cdots x_n) + (y_1, y_2, y_3 \cdots y_n)]$.**C**=

C. $(x_1, x_2, x_3 \cdots x_n)$+ $(y_1, y_2, y_3 \cdots y_n)$.**C**

PROPERTY HOLDS

Associative law:

v · (w · **c**) = (v, w) · **c**

$(x_1, x_2, x_3 \cdots x_n).[(y_1, y_2, y_3 \cdots y_n).C]$=$(x_1 y_1, x_2 y_2, x_3 y_3 \cdots x_n y_n)$.**C**

PROPERTY HOLDS

Unitary law:

For all vectors **v** in V, 1 · **v** = **v**

1.$(x_1, x_2, x_3 \cdots x_n) = (x_1, x_2, x_3 \cdots x_n)$

PROPERTY HOLDS

As all the properties are hold so it is a vector space and it is commonly known as real vector space $\mathbb{R}^n$

Vector Space of Column Vectors

The vector space $\mathbf{C^n}$

Vectors

The vector space C^n is the set of all column/row vectors of size n with entries from the set of complex numbers,

(A ***column vector*** of ***size*** n is an ordered list of n numbers, which is written in order vertically, starting at the top and proceeding to the bottom. At times, we will refer to a column vector as simply a ***vector***)

$\vec{u} = (z_1, z_2, z_3 \cdots z_n)$ $\vec{u}\ \varepsilon\ \mathbf{C^n}$

$\vec{v} = (w_1, w_2, w_3 \cdots w_n)$ $\vec{v}\ \varepsilon\ \mathbf{C^n}$

$\mathbf{C^n} = \{((z_1, z_2, z_3 \cdots z_n): x_i \epsilon C$

Addition: -

$(z_1, z_2, z_3 \cdots z_n) + (w_1, w_2, w_3 \cdots w_n) = (z_1 + w_1, z_2 + w_2, z_3 + w_3 \cdots z_n + w_n)$

Multiplication: -

$C(z_1, z_2, z_3 \cdots z_n) = (Cz_1, Cz_2\, Cz_3 \cdots Cz_n)$

VECTOR SPACE PROPERTIES OF COLUMN VECTORS

Suppose that C^n is the set of column vectors of size n with addition and scalar multiplication

Then

- Additive Closure, Column Vectors

If u, v ∈ C^n then u+v ∈ C^n

- Scalar Closure, Column Vectors

If α ∈ C and u ∈ C^n then α u ∈ C^n

- Commutativity, Column Vectors

If u, v ∈ C^n, then u+v=v+ u

- Additive Associativity, Column Vectors

If u, v, w ∈ C^n then u+(v+w) =(u+v) +w

- Zero Vector, Column Vectors

There is a vector, 0, called the zero vector, such that u+0=u for all u ∈ C^n

- Additive Inverses, Column Vectors

If u ∈ C^n, then there exists a vector −u ∈ C^n so that u+(−u) =0

- Scalar Multiplication Associativity, Column Vectors

If α, β∈ C and u C^n then α(βu) =(αβ)u

- Distributivity across Vector Addition, Column Vectors

If α∈Cα∈C and u, v ∈ C^n, then α(u+v) =αu+αv

- Distributivity across Scalar Addition, Column Vectors

If α, β∈C and u ∈ C^n then (α+β) u=αu+βu

One, Column Vectors

If u ∈ C^n then 1u=u

As all the above properties are hold by C^n just as by the R^n the only difference is of real and complex numbers and that of row and column notation commonly used, so it is an also a vector space.

MATRICES

The set of all m-by-n matrices with real entries, denoted by $R^{m\times n}$,is a vector space.

Let

v= $\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$, $w = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$, $u = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$,

Addition: -

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} x_{11} + y_{11} & x_{12} + y_{12} \\ x_{21} + y_{21} & x_{22} + y_{22} \end{bmatrix}$$

Multiplication: -

$$= c \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} cx_{11} & cx_{12} \\ cx_{21} & cx_{22} \end{bmatrix},$$

I will check out just some properties as I know by definition that it is a vector space.

Closure under addition

$$\begin{bmatrix} \boldsymbol{x_{11}} & \boldsymbol{x_{12}} \\ \boldsymbol{x_{21}} & \boldsymbol{x_{22}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{y_{11}} & \boldsymbol{y_{12}} \\ \boldsymbol{y_{21}} & \boldsymbol{y_{22}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x_{11}} + \boldsymbol{y_{11}} & \boldsymbol{x_{12}} + \boldsymbol{y_{12}} \\ \boldsymbol{x_{21}} + \boldsymbol{y_{21}} & \boldsymbol{x_{22}} + \boldsymbol{y_{22}} \end{bmatrix}$$

PROPERTY HOLDS

Additive identity:

$$\begin{bmatrix} \boldsymbol{x_{11}} & \boldsymbol{x_{12}} \\ \boldsymbol{x_{21}} & \boldsymbol{x_{22}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x_{11}} & \boldsymbol{x_{12}} \\ \boldsymbol{x_{21}} & \boldsymbol{x_{22}} \end{bmatrix}$$

PROPERTY HOLDS

Additive inverses:

$$\begin{bmatrix} \boldsymbol{x_{11}} & \boldsymbol{x_{12}} \\ \boldsymbol{x_{21}} & \boldsymbol{x_{22}} \end{bmatrix} + \begin{bmatrix} -\boldsymbol{x_{11}} & -\boldsymbol{x_{12}} \\ -\boldsymbol{x_{21}} & -\boldsymbol{x_{22}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

PROPERTY HOLDS

Distributive law under addition:

(a+b) v=av+bv

$$(a+b)\begin{bmatrix} \boldsymbol{x_{11}} & \boldsymbol{x_{12}} \\ \boldsymbol{x_{21}} & \boldsymbol{x_{22}} \end{bmatrix} = \begin{bmatrix} (a+b)\,\boldsymbol{x_{11}} & (a+b)\,\boldsymbol{x_{12}} \\ (a+b)\,\boldsymbol{x_{21}} & (a+b)\,\boldsymbol{x_{22}} \end{bmatrix}$$

$$= \begin{bmatrix} \boldsymbol{ax_{11}} + \boldsymbol{bx_{11}} & \boldsymbol{ax_{12}} + \boldsymbol{bx_{12}} \\ \boldsymbol{ax_{21}} + \boldsymbol{bx_{21}} & \boldsymbol{ax_{22}} + \boldsymbol{bx_{22}} \end{bmatrix}$$

$$=a\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} + b\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$$

PROPERTY HOLDS

Closure under multiplication:

$$=c\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} cx_{11} & cx_{12} \\ cx_{21} & cx_{22} \end{bmatrix},$$

PROPERTY HOLDS

Unitary law:

$$1\times\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$$

PROPERTY HOLDS

AS THE SET OF m-by-n MATRICES FORM A VECTOR SPACE WE CAN REFER SOMETIMESTHE ELEMENTS AS "VECTORS"

THE END