$i = D_0 = -i_{sc}$, where i_{sc} is the short-circuit current flowing out of terminal a, which is the same as the Norton current I_N, i.e.,

$$D_0 = -I_N \tag{4.19}$$

When all the internal independent sources are turned off, $D_0 = 0$ and the circuit can be replaced by an equivalent resistance R_{eq} (or an equivalent conductance $G_{eq} = 1/R_{eq}$), which is the same as R_{Th} or R_N. Thus Eq. (4.19) becomes

$$i = \frac{v}{R_{Th}} - I_N \tag{4.20}$$

This expresses the voltage-current relation at terminals a-b of the circuit in Fig. 4.47(b), confirming that the two circuits in Fig. 4.47(a) and 4.47(b) are equivalent.

4.8 Maximum Power Transfer

In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses. It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig. 4.48, the power delivered to the load is

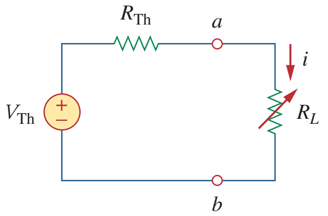

Figure 4.48
The circuit used for maximum power transfer.

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \tag{4.21}$$

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched in Fig. 4.49. We notice from Fig. 4.49 that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and ∞. We now want to show that this maximum power occurs when R_L is equal to R_{Th}. This is known as the *maximum power theorem.*

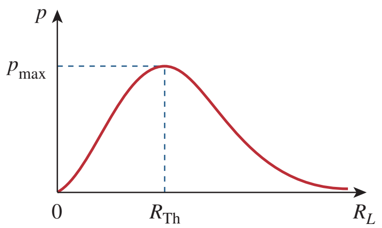

Figure 4.49
Power delivered to the load as a function of R_L.

> **Maximum power** is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate p in Eq. (4.21) with respect to R_L and set the result equal to zero. We obtain

$$\begin{aligned}\frac{dp}{dR_L} &= V_{Th}^2\left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] \\ &= V_{Th}^2\left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0\end{aligned}$$

This implies that

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L) \tag{4.22}$$

which yields

$$\boxed{R_L = R_{\text{Th}}} \tag{4.23}$$

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}. We can readily confirm that Eq. (4.23) gives the maximum power by showing that $d^2p/dR_L^2 < 0$.

The source and load are said to be *matched* when $R_L = R_{\text{Th}}$.

The maximum power transferred is obtained by substituting Eq. (4.23) into Eq. (4.21), for

$$\boxed{p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}} \tag{4.24}$$

Equation (4.24) applies only when $R_L = R_{\text{Th}}$. When $R_L \neq R_{\text{Th}}$, we compute the power delivered to the load using Eq. (4.21).

Example 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

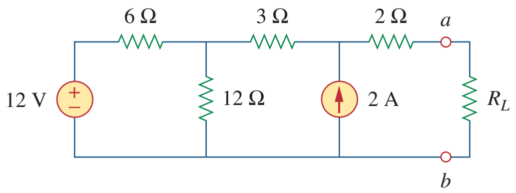

Figure 4.50
For Example 4.13.

Solution:
We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals *a-b*. To get R_{Th}, we use the circuit in Fig. 4.51(a) and obtain

$$R_{\text{Th}} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

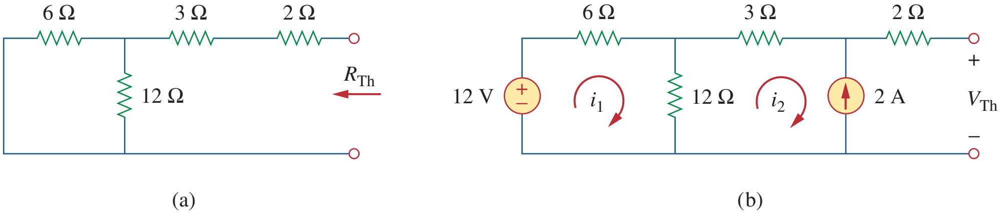

Figure 4.51
For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2 \text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals *a-b*, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

Practice Problem 4.13

Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

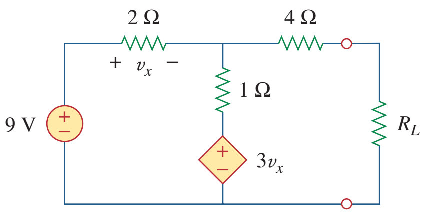

Figure 4.52
For Practice Prob. 4.13.

Answer: 4.222 Ω, 2.901 W.

4.9 Verifying Circuit Theorems with *PSpice*

In this section, we learn how to use *PSpice* to verify the theorems covered in this chapter. Specifically, we will consider using DC Sweep analysis to find the Thevenin or Norton equivalent at any pair of nodes in a circuit and the maximum power transfer to a load. The reader is advised to read Section D.3 of Appendix D in preparation for this section.

To find the Thevenin equivalent of a circuit at a pair of open terminals using *PSpice*, we use the schematic editor to draw the circuit and insert an independent probing current source, say, Ip, at the terminals. The probing current source must have a part name ISRC. We then perform a DC Sweep on Ip, as discussed in Section D.3. Typically, we may let the current through Ip vary from 0 to 1 A in 0.1-A increments. After saving and simulating the circuit, we use Probe to display a plot of the voltage across Ip versus the current through Ip. The zero intercept of the plot gives us the Thevenin equivalent voltage, while the slope of the plot is equal to the Thevenin resistance.

To find the Norton equivalent involves similar steps except that we insert a probing independent voltage source (with a part name VSRC), say, Vp, at the terminals. We perform a DC Sweep on Vp and let Vp vary from 0 to 1 V in 0.1-V increments. A plot of the current through Vp versus the voltage across Vp is obtained using the Probe menu after simulation. The zero intercept is equal to the Norton current, while the slope of the plot is equal to the Norton conductance.

To find the maximum power transfer to a load using *PSpice* involves performing a DC parametric Sweep on the component value of R_L in Fig. 4.48 and plotting the power delivered to the load as a function of R_L. According to Fig. 4.49, the maximum power occurs

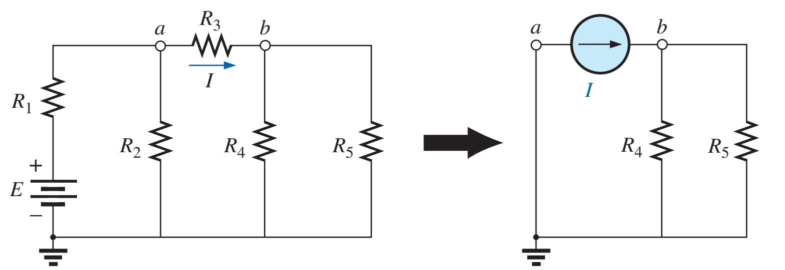

FIG. 9.105
Demonstrating the effect of knowing a current at some point in a complex network.

9.8 RECIPROCITY THEOREM

The **reciprocity theorem** is applicable only to single-source networks. It is, therefore, not a theorem used in the analysis of multisource networks described thus far. The theorem states the following:

The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current. The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.

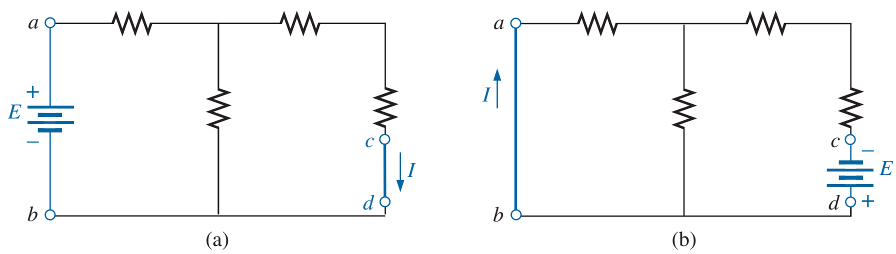

FIG. 9.106
Demonstrating the impact of the reciprocity theorem.

In the representative network in Fig. 9.106(a), the current I due to the voltage source E was determined. If the position of each is interchanged as shown in Fig. 9.106(b), the current I will be the same value as indicated. To demonstrate the validity of this statement and the theorem, consider the network in Fig. 9.107, in which values for the elements of Fig. 9.106(a) have been assigned.

The total resistance is

$$R_T = R_1 + R_2 \| (R_3 + R_4) = 12\ \Omega + 6\ \Omega \| (2\ \Omega + 4\ \Omega)$$
$$= 12\ \Omega + 6\ \Omega \| 6\ \Omega = 12\ \Omega + 3\ \Omega = 15\ \Omega$$

and $$I_s = \frac{E}{R_T} = \frac{45\ \text{V}}{15\ \Omega} = 3\ \text{A}$$

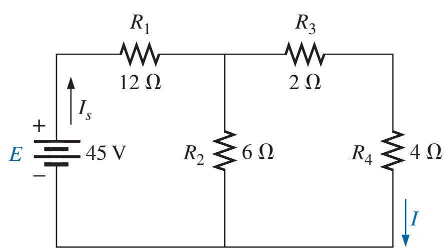

FIG. 9.107
Finding the current I due to a source E.

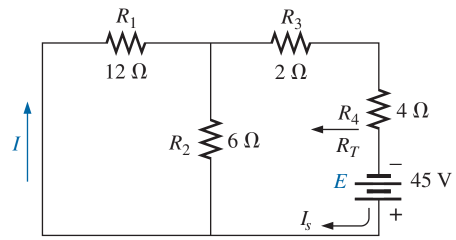

FIG. 9.108
Interchanging the location of E and I of Fig. 9.107 to demonstrate the validity of the reciprocity theorem.

with
$$I = \frac{3\ \text{A}}{2} = \mathbf{1.5\ A}$$

For the network in Fig. 9.108, which corresponds to that in Fig. 9.106(b), we find

$$R_T = R_4 + R_3 + R_1 \parallel R_2$$
$$= 4\ \Omega + 2\ \Omega + 12\ \Omega \parallel 6\ \Omega = 10\ \Omega$$

and
$$I_s = \frac{E}{R_T} = \frac{45\ \text{V}}{10\ \Omega} = 4.5\ \text{A}$$

so that
$$I = \frac{(6\ \Omega)(4.5\ \text{A})}{12\ \Omega + 6\ \Omega} = \frac{4.5\ \text{A}}{3} = \mathbf{1.5\ A}$$

which agrees with the above.

The uniqueness and power of this theorem can best be demonstrated by considering a complex, single-source network such as the one shown in Fig. 9.109.

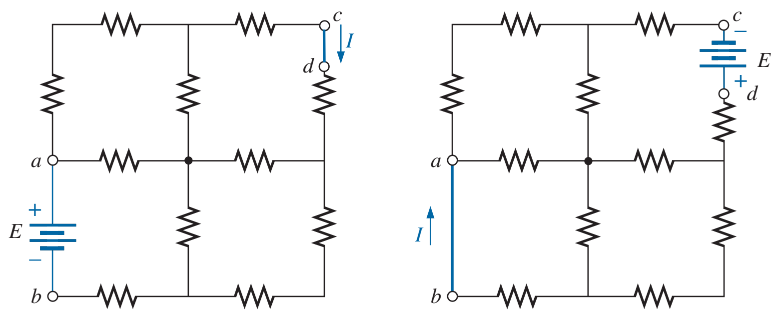

FIG. 9.109
Demonstrating the power and uniqueness of the reciprocity theorem.

9.9 COMPUTER ANALYSIS

Once you understand the mechanics of applying a software package or language, the opportunity to be creative and innovative presents itself. Through years of exposure and trial-and-error experiences, professional programmers develop a catalog of innovative techniques that are not only functional but very interesting and truly artistic in nature. Now that some of the basic operations associated with PSpice have been introduced, a few innovative maneuvers will be made in the examples to follow.

PSpice

Thévenin's Theorem The application of Thévenin's theorem requires an interesting maneuver to determine the Thévenin resistance. It is a maneuver, however, that has application beyond Thévenin's theorem whenever a resistance level is required. The network to be analyzed appears in Fig. 9.110 and is the same one analyzed in Example 9.10 (Fig. 9.48).

Since PSpice is not set up to measure resistance levels directly, a 1 A current source can be applied as shown in Fig. 9.111, and Ohm's law can be used to determine the magnitude of the Thévenin resistance in the following manner:

$$\boxed{|R_{Th}| = \left|\frac{V_s}{I_s}\right| = \left|\frac{V_s}{1\ \text{A}}\right| = |V_s|} \qquad \textbf{(9.14)}$$

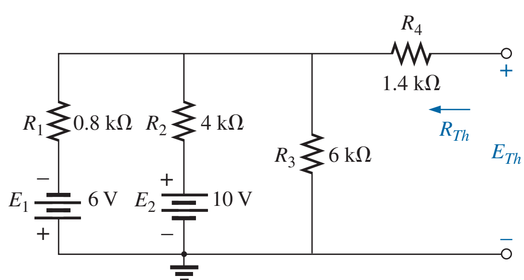

FIG. 9.110
Network to which PSpice is to be applied to determine E_{Th} and R_{Th}.