## EXPERIMENT NO. 13

## AIM: The Relationship Between Linear and Angular Quantities

## Objectives

- To measure the radius of a cylinder using the relationship between linear and angular displacement
- To measure the radius of a cylinder using the relationship between linear and angular acceleration.
- To investigate the relationship between linear and angular velocity


## Apparatus

The experimental apparatus is shown in Figure 1 below. A light string (negligible mass) is wound around a central pulley of radius $R$, it passes over a side pulley and is attached to a mass M .

Compressed air is supplied to the apparatus making the rotating central pulley and the disk to which it is attached (shown in green in figure 1) rotate with very little frictional losses. The side pulley also is supplied with compressed air to minimize frictional losses.

The rotating disk (shown in green in Figure 1) has alternating black and white bars on its circumference which are detected by a photo-diode detector as they sweep past it.


Figure 1

## Theory:

Consider a point P on the surface of a cylinder having a radius of R shown in Figure 2 above.

If the cylinder is rotated about an axis perpendicular to the plane of the page and through the center of this circular cross-section, the arc length (S) swept out by P is given by:


Figure 2

$$
\begin{equation*}
S=\theta R \tag{1}
\end{equation*}
$$

$S$ and $R$ have units of length and the angle $\theta$ is measured in radians. When the arc length $S$ is equal to the radius R , the angle $\theta$ is 1 radian. For a complete $360^{\circ} \mathrm{circle}$, the angle, in radians is $2 \pi$ radians. If a thin string is wrapped around a cylinder N complete times, then the length of the string wrapped around the cylinder is:

$$
\begin{equation*}
\Lambda y=N(2 \Pi R) N=(2 \pi) R N \tag{2}
\end{equation*}
$$

The linear velocity (v) of a point on the cylinder's surface is related to the angular velocity of the cylinder (typically denotedby the Greek letter omega: $\omega$ ) by:

$$
\begin{equation*}
v=\omega R \tag{3}
\end{equation*}
$$

Similarly, the linear tangential acceleration of a point on the cylinder's surface (a) is related to the angular acceleration of the cylinder by:

$$
\begin{equation*}
a=\alpha R \tag{4}
\end{equation*}
$$

If the hanging mass M in Figure 1 is released from rest and allowed to fall a distance h to the ground, it will have an acceleration equal in magnitude to the tangential linear acceleration of the string that is wound around the central pulley of the apparatus. This acceleration is given by:

$$
\begin{equation*}
a=2 h / t^{\wedge} 2 \tag{5}
\end{equation*}
$$

t is the time the mass took to fall from rest $(\mathrm{v}=0)$ to the ground.

## Procedure

The apparatus is delicate. Do not rotate the rotating table or side pulley unless air is flowing through the bearings.

## Displacement

1.Turn on the air supply to the rotational apparatus and make sure the string passes over the side pulley as shown in Figure 1 and that the string is taught with the hanging mass M just barely touching the floor. The 0 cm end of the meter stick should be paced on the floor ( $\mathrm{yi}=0.0 \mathrm{~cm}$ ).
2. Turn the rotational apparatus at least $\mathrm{N}=8$ complete revolutions Record the total number of revolutions and the final position of the hanging mass in the Displacement Data Table.
3. Use equation (2) and Excel to calculate the radius of the central pulley R.
4. Repeat steps 1 through 3 three additional times. Have Excel calculate the mean value of $R$ and the standard deviation (s) and the standard deviation of the mean value (sm).

The Excel formula for the standard deviation (s) is "=STDEV(CELL1:CELL4)" and
The Excel formula for the standard deviation of the mean (sm) is " $=\mathrm{s} / \mathrm{SQRT}(4)$.

You will need the mean value of the radius and the standard deviation of this mean value for the next experiment.

## Acceleration:

1.With the air supply on, attach the hanging mass $(\mathrm{M})$ to one end of a string and wind the other end around the central pulley. The string should also pass over the side pulley such that the hanging mass is just below the side pulley. Measure and record the position of the hanging mass above the floor - each trial MUST start with the mass at this same elevation.
2. Simultaneously release the hanging mass and start the desk top timer. The START button is the white arrow at the far-left hand edge of the toolbar at the top of the screen. When the mass hits the floor stop the desk top timer.
3) We can calculate angular velocity versus time. In addition, it will construct a data table with the values of the angular velocity and time from its plot.

