## Introduction

What is Conservative force?
Action at distance

$$
\vec{F}=q_{1} \vec{E}
$$

Potential energy


Figura 24-1 Particle 1 is located at point $P$ in the electric field of particle 2.

Our goals in this chapter are to

1) Define electric potential,
2) Discuss how to calculate it for various arrangements of charged particles and objects, and
3) Discuss how electric potential $V$ is related to electric potential energy $U$.


Figure 24-2 (a) A test charge has been brought in from infinity to point $P$ in the electric field of the rod. (b) We define an electric potential $V$ at $P$ based on the potential energy of the configuration in (a).

How to measure electric potential energy

$$
\begin{equation*}
U=-W \quad \text { (potential energy) } \tag{24.1}
\end{equation*}
$$

## Electric Potential and Electric Potential Energy

How to measure gravitational potential energy?

1) Assigning $U=0$ for a reference configuration (such as the object at table level) and
2) then calculating the work $W$ the gravitational force does if the object is moved up or down from that level.
We then defined the potential energy as being
3) Reference configuration for which $U=0$
4) Calculating the work the electric force does

The electric potential energy of the final configuration is then given by

$$
U=-W_{\infty}
$$

The work and thus the potential energy can be positive or negative depending on the sign of the rod’s charge.

We define the electric potential $V$ at $P$ in terms of the work done by the electric force and the resulting potential energy:

$$
\begin{equation*}
V=\frac{-W_{\propto}}{q_{0}}=\frac{U}{q_{0}} \quad \quad \text { (electric potential) } \tag{24.2}
\end{equation*}
$$

We see that $V$ is a scalar quantity and can be positive or negative (because potential energy and charge have signs).

We find that an electric potential is set up at every point in the rod's electric field.

$$
\begin{gathered}
(\text { electric potential energy })=(\text { particle's charge })\left(\frac{\text { electric potential energy }}{\text { unit charge }}\right) \\
U=q V
\end{gathered}
$$

where $q$ can be positive or negative.

## Cautions

1) The potential and potential energy are related (that is the point here) but they are very different and not interchangeable.
2) Electric potential is a scalar, not a vector.
3) A potential energy is a property of a system (or configuration) of objects, but sometimes we can get away with assigning it to a single object.
4) If a charged particle is placed in an electric field and has no noticeable effect on the field (or the charged object that sets up the field), we usually assign the electric potential energy to the particle alone.

Units

$$
\begin{aligned}
& V=\frac{U}{q_{0}} \quad(1 \text { volt }=1 \text { joule per coulomb }) \\
& 1 \frac{N}{C}=\left(1 \frac{N}{C}\right)\left(\frac{1 V}{1 \frac{J}{C}}\right)\left(1 \frac{J}{N \cdot m}\right)=1 \frac{V}{m}
\end{aligned}
$$

$1 J=1 \mathrm{~N} . \mathrm{m}$
$1 \frac{J}{N \cdot m}=1$
$1 V=1 \frac{J}{C}$

$$
\frac{1 V}{1 \frac{J}{C}}=1
$$

## Motion Through an Electric Field

## Change in Electric Potential.

If we move from an initial point $i$ to a second point $f$ in the electric field of a charged object, the electric potential changes by

$$
\Delta V=V_{f}-V_{i}
$$

If we move a particle with charge $q$ from $i$ to $f$, then, from Eq. 24-3, the potential energy of the system changes by

$$
\begin{equation*}
\Delta U=q \Delta V=q\left(V_{f}-V_{i}\right) \tag{24-4}
\end{equation*}
$$

1) The change can be positive or negative, depending on the signs of $q$ and $\Delta V$
2) The change can be zero, if there is no change in potential from $i$ to $f$ (the points have the same value of potential).
3) Because the electric force is conservative, the change in potential energy $\Delta V$ between $i$ and $f$ is the same for all paths between those points (it is path independent).

## Work by the Field.

We can relate the potential energy change $\Delta U$ to the work $W$ done by the electric force as the particle moves from $i$ to $f$ by applying the general relation for a conservative force (Eq. 8-1):

$$
\begin{equation*}
W=-\Delta U \quad \text { (work, conservative force). } \tag{24-5}
\end{equation*}
$$

Next, we can relate that work to the change in the potential by substituting from Eq. 24-4:

$$
\begin{equation*}
W=-\Delta U=-q \Delta V=-q\left(V_{f}-V_{i}\right) \tag{24-6}
\end{equation*}
$$

1) $\quad W$ is the work done on the particle by the electric field (because it, of course, produces the force). The work can be positive, negative, or zero.
2) Because $\Delta U$ between any two points is path independent, so is the work $W$ done by the field.
3) If you need to calculate work for a difficult path, switch to an easier path—you get the same result.

## Conservation of Energy

If a charged particle moves through an electric field with no force acting on it other than the electric force due to the field, then the mechanical energy is conserved. Let's assume that we can assign the electric potential energy to the particle alone. Then we can write the conservation of mechanical energy of the particle that moves from point $i$ to point $f$ as

$$
\begin{align*}
& U_{i}+K_{i}=U_{f}+K_{f}  \tag{24-7}\\
& \Delta K=-\Delta U \tag{24-8}
\end{align*}
$$

Substituting Eq. 24-4, we find a very useful equation for the change in the particle's kinetic energy as a result of the particle moving through a potential difference:

$$
\begin{equation*}
\Delta K=-q \Delta V=-q\left(V_{f}-V_{i}\right) \tag{24-9}
\end{equation*}
$$

## Work by an Applied Force

If some force in addition to the electric force acts on the particle, we say that the additional force is an applied force or external force, which is often attributed to an external agent. Such an applied force can do work on the particle, but the force may not be conservative and thus, in general, we cannot associate a potential energy with it. We account for that work $W_{\text {app }}$ by modifying Eq. 24-7:
(initial energy) $+($ work by applied force $)=($ final energy $)$

$$
\begin{equation*}
U_{i}+K_{i}+W_{a p p}=U_{f}+K_{f} \tag{24-10}
\end{equation*}
$$

Rearranging and substituting from Eq. 24-4, we can also write this as

$$
\begin{equation*}
\Delta K=-\Delta U+W_{a p p}=-q \Delta V+W_{a p p} \tag{24-11}
\end{equation*}
$$

The work by the applied force can be positive, negative, or zero, and thus the energy of the system can increase, decrease, or remain the same.

In the special case where the particle is stationary before and after the move, the kinetic energy terms in Eqs. 24-10 and 24-11 are zero and we have

$$
\begin{equation*}
W_{a p p}=q \Delta V \quad\left(\text { for } K_{i}=K_{f}\right) \tag{24-12}
\end{equation*}
$$

In this special case, the work $W_{\text {app }}$ involves the motion of the particle through the potential difference $\Delta V$ and not a change in the particle's kinetic energy.

By comparing Eqs. 24-6 and 24-12, we see that in this special case, the work by the applied force is the negative of the work by the field:

$$
\begin{array}{ll}
W=-\Delta U=-q \Delta V=-q\left(V_{f}-V_{i}\right) \\
W_{a p p}=q \Delta V & \left(\text { for } K_{i}=K_{f}\right) \\
W_{a p p}=-W & \left(\text { for } K_{i}=K_{f}\right) \tag{24-13}
\end{array}
$$

## Electron-volt

In atomic and subatomic physics, energy measures in the SI unit of joules often require awkward powers of ten. A more convenient (but non-SI unit) is the electron-volt (eV), which is defined to be equal to the work required to move a single elementary charge $e$ (such as that of an electron or proton) through a potential difference $\Delta V$ of exactly one volt. From Eq. 24-6, we see that the magnitude of this work is $q \Delta V$.Thus,

$$
\begin{gather*}
1 \mathrm{eV}=e(1 \mathrm{~V}) \\
=\left(1.602 \times 10^{-19} \mathrm{C}\right)\left(1 \frac{J}{C}\right)=1.602 \times 10^{-19} \mathrm{~J} \tag{24-14}
\end{gather*}
$$

## Checkpoint 1

In the figure, we move a proton from point $i$ to point $f$ in a uniform electric field. Is positive or negative work done by (a) the electric field and (b) our force? (c) Does the electric potential energy increase or decrease? (d) Does the proton move to a point of higher or lower electric potential?

a) $\quad W_{e}=\vec{F}_{e} \cdot \vec{d}=F_{e} d \cos 180=-F_{e} d$
b) $\quad W_{\text {app }}=\vec{F}_{\text {app }} \cdot \vec{d}=F_{\text {app }} d \cos 0=F_{\text {app }} d$
c) $\quad \Delta U=U_{f}-U_{i}=-W=-\left(-F_{e} d\right)=F_{e} d$

$$
\Delta U=U_{f}-U_{i}=W_{a p p}=F_{a p p} d
$$

As $U_{f}>U_{i} \quad$ Electric potential energy increase

d) $\quad \Delta U=q \Delta V=q\left(V_{f}-V_{i}\right)$

As $\quad \Delta U>0$ and $q>0$ therefore $\Delta V>0$ and $V_{f}>V_{i}$

## Proton moves to a point of higher potential

## Equipotential Surfaces

1) Adjacent points that have the same electric potential form an equipotential surface, which can be either an imaginary surface or a real, physical surface.
2) No net work $W$ is done on a charged particle by an electric field when the particle moves between two points $i$ and $f$ on the same equipotential surface.

$$
\begin{equation*}
W=-\Delta U=-q \Delta V=-q\left(V_{f}-V_{i}\right) \tag{24-6}
\end{equation*}
$$

If $V_{f}=V_{i}$ then $W=0$
3) Because of the path independence of work (and thus of potential energy and potential), $W=0$ for any path connecting points $i$ and $f$ on a given equipotential surface regardless of whether that path lies entirely on that surface.


Figure $24-5$ shows electric field lines and cross sections of the equipotential surfaces for a uniform electric field and for the field associated with a charged particle and with an electric dipole.


Equipotential surfaces are always perpendicular to electric field. Why?
$W=-\Delta U=-q \Delta V=-q\left(V_{f}-V_{i}\right)$

If $V_{f}=V_{i}$ then $W=0$


## Calculating the Potential from the Field

We can calculate the potential difference between any two points $i$ and $f$ in an electric field if we know the electric field vector $\vec{E}$ all along any path connecting those points.

To make the calculation, we find the work done on a positive test charge by the field as the charge moves from $i$ to $f$.

Consider an arbitrary electric field, represented by the field lines in Figure, and a positive test charge $q_{0}$ that moves along the path shown from point $i$ to point $f$. At any point on the path, an electric force $q_{0} \vec{E}$ acts on the charge as it moves through a differential displacement $d \vec{s}$.


The differential work $d W$ done on a particle by a force $\vec{F}$ during a displacement $d \vec{s}$ is given by the dot product of the force and the displacement:
$d W=\vec{F} \cdot d \vec{s}=q_{0} \vec{E} \cdot d \vec{s}$
$W=q_{0} \int_{i}^{f} \vec{E} \cdot d \vec{s}$

$$
\begin{equation*}
W=-\Delta U=-q \Delta V=-q\left(V_{f}-V_{i}\right) \tag{24-17}
\end{equation*}
$$

$V_{f}-V_{i}=-\int_{i}^{f} \vec{E} \cdot d \vec{s}$
If we set potential $V_{i}=0$
$V=-\int_{i}^{f} \vec{E} \cdot d \vec{s}$

Equation 24-19 gives us the potential $V$ at any point $f$ in the electric field relative to the zero potential at point $i$. If we let point $i$ be at infinity, then Eq. 24-19 gives us the potential $V$ at any point $f$ relative to the zero potential at infinity.

## Uniform Field

Let's apply Eq. 24-18 for a uniform field as shown in Figure. We start at point $i$ on an equipotential line with potential $V_{i}$ and move to point $f$ on an equipotential line with a lower potential $V_{f}$. The separation between the two equipotential lines is $\Delta x$. Let's also move along a path that is parallel to the electric field $\vec{E}$ (and thus perpendicular to the equipotential lines). The angle
 between $\vec{E}$ and $d \vec{s}$ in Eq. 24-18 is zero, and the dot product gives us

$$
\vec{E} \cdot d \vec{s}=E d s \cos 0=E d s
$$

Because $E$ is constant for a uniform field, Eq. 24-18 becomes

$$
V_{f}-V_{i}=-E \int_{i}^{f} d s
$$

The integral gives the sum of all the displacement elements $d$ s from $i$ to $f$, but we already know that the sum is length $\Delta x$. Thus we can write the change in potential $V_{f}-V_{i}$ in this uniform field as

$$
\Delta V=-E \Delta x \quad \text { (uniform field) }
$$

This is the change in voltage $\Delta V$ between two equipotential lines in a uniform field of magnitude $E$, separated by distance $\Delta x$. If we move in the direction of the field by distance $\Delta x$, the potential decreases. In the opposite direction, it increases.

The electric field vector points from higher potential toward lower potential.

## Checkpoint 2

The figure here shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an electron from one surface to another. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.

a) Rightward; because the electric field vector points from higher potential toward lower potential.

b) The work we do for paths $1,2,3$ and 5 is positive because $\vec{F}_{a p p}$ and $\vec{d}$ are parallel.

The work we do for path 4 is negative because $\vec{F}_{\text {app }}$ and $\vec{d}$
 are antiparallel.
c) $\quad 3$ then 1,2 and 5 tie then 4 .

## Potential Due to a Charged Particle

We now use Eq. 24-18 to derive, for the space around a charged particle, an expression for the electric potential $V$ relative to the zero potential at infinity. Consider a point $P$ at distance $R$ from a fixed particle of positive charge $q$ Figure. To use Eq. 24-18, we imagine that we move a positive test charge $q_{0}$ from point $P$ to infinity. Because the path we take does not matter, let us choose the simplest one- a line that extends radially from the fixed particle through $P$ to infinity. To use Eq. 24-18, we must evaluate the dot product

$$
\vec{E} \cdot d \vec{s}=E d s \cos 0=E d s
$$

The electric field $\vec{E}$ is directed radially outward from the fixed particle. Thus, the differential displacement $d \vec{s}$ of the test particle along its path has the same direction as $\vec{E}$.That means that, angle $\theta=0$ and $\cos 0=1$. Because the path is radial, let us write $d s$ as $d r$. Then, substituting the limits $R$ and $\propto$, we can write as

$$
V_{f}-V_{i}=-\int_{R}^{\infty} E d r
$$

Next, we set $V_{f}=0($ at $\propto)$ and $V_{i}=V($ at $R)$. Then, for the magnitude of the electric field at the site of the test charge, we substitute

$$
E=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r^{2}}
$$

With these changes, above eq. then gives us

$$
\begin{aligned}
& 0-V=-\frac{q}{4 \pi \varepsilon_{0}} \int_{R}^{\infty} \frac{d r}{r^{2}}=\frac{q}{4 \pi \varepsilon_{0}}\left[\frac{1}{r}\right] \propto \\
& R \\
&-V=-\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{R}
\end{aligned}
$$

Solving for $V$ and switching $R$ to $r$, we then have
$V=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r}$
as the electric potential $V$ due to a particle of charge $q$ at any radial distance $r$ from the particle. Although we have derived Eq. 24-26 for a positively charged particle, the derivation holds also for a negatively charged particle, in which case, $q$ is a negative quantity. Note that the sign of $V$ is the same as the sign of $q$ :

A positively charged particle produces a positive electric potential. A negatively charged particle
 produces a negative electric potential.

Equation 24-26 also gives the electric potential either outside or on the external surface of a spherically symmetric charge distribution.

## Potential Due to a Group of Charged Particles

We can find the net electric potential at a point due to a group of charged particles with the help of the superposition principle.

We calculate separately the potential resulting from each charge at the given point. Then we sum the potentials. Thus, for $n$ charges, the net potential is

$$
V=\sum_{i=1}^{n} V_{i}=\frac{1}{4 \pi \varepsilon_{0}} \sum_{i=1}^{n} \frac{q_{i}}{r_{i}}
$$

The sum in above Eq. is an algebraic sum, not a vector sum like the sum that would be used to calculate the electric field resulting from a group of charged particles. Here in lies an important computational advantage of potential over electric field: It is a lot easier to sum several scalar quantities than to sum several vector quantities whose directions and components must be considered.

## Checkpoint 3

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point $P$ by the protons, greatest first.

(a)

(b)

(c)

$$
V=\frac{q}{4 \pi \varepsilon_{0}}\left[\frac{1}{d}+\frac{1}{D}\right]
$$

All tie

## Potential Due to an Electric Dipole

Now let us apply Eq. 24-27 to an electric dipole to find the potential at an arbitrary point $P$ in Fig. 24-13a. At $P$, the positively charged particle (at distance
$r\left(\_\right)$) sets up potential $V\left({ }_{-}\right)$and the negatively charged particle (at distance $\left.r\left(\_\right)\right)$
sets up potential $V\left(\_\right)$.Then the net potential at $P$ is given by Eq. 24-27 as

$$
\begin{aligned}
V=\sum_{i=1}^{2} V_{i} & =V_{(+)}+V_{(-)}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{q}{r_{(+)}}+\frac{-q}{r_{(-)}}\right) \\
& =\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{q}{r_{(+)}}+\frac{-q}{r_{(-)}}\right) \\
& =\frac{q}{4 \pi \varepsilon_{0}}\left(\frac{r_{(-)}-r_{(+)}}{r_{(+)} r_{(-)}}\right)
\end{aligned}
$$

Naturally occurring dipoles-such as those possessed by many molecules-are quite small; so we are usually interested only in points that are relatively far from the dipole, such that $r \gg d$, where $d$ is the distance between the charges and $r$ is the distance from the dipole's


$$
(r)^{2}=\left(r_{(+)}+\frac{\Delta r}{2}\right)^{2}=r_{(+)}^{2}+\left(\frac{\Delta r}{2}\right)^{2}+r_{(+)} \Delta r
$$

$r^{2}=r_{(+)}^{2}+r_{(+)}\left(r_{(-)}-r_{(+)}\right)=r_{(-)} r_{(+)}$

$$
\begin{gathered}
=\frac{q}{4 \pi \varepsilon_{0}} \frac{d \cos \theta}{r^{2}} \\
=\frac{1}{4 \pi \varepsilon_{0}} \frac{p \cos \theta}{r^{2}} \quad \text { (electric dipole) }
\end{gathered}
$$

## Checkpoint 4

Suppose that three points are set at equal (large) distances $r$ from the center of the dipole in Fig. 24-13: Point $a$ is on the dipole axis above the positive charge, point $b$ is on the axis below the negative charge, and point $c$ is on a perpendicular bisector through the line connecting the two charges. Rank the points according to the electric potential of the dipole there, greatest (most positive) first.
a, c and b

## Induced Dipole Moment

Many molecules, such as water, have permanent electric dipole moments. In other molecules (called nonpolar molecules) and in every isolated atom, the centers of the positive and negative charges coincide (Fig. 24-14a) and thus no dipole moment is set

The electric field shifts the positive and negative charges, creating a dipole.

$\vec{p}$ up. However, if we place an atom or a nonpolar molecule in an external electric
field, the field distorts the electron orbits and separates the centers of positive and negative charge (Fig. 24-14b). Because the electrons are negatively charged, they tend to be shifted in a direction opposite the field. This shift sets up a dipole moment that points in the direction of the field. This dipole moment is said to be induced by the field, and the atom or molecule is then said to be polarized by the field (that is, it has a positive side and a negative side). When the field is removed, the induced dipole moment and the polarization disappear.

## Potential Due to a Continuous Charge Distribution

## Line of Charge

A thin nonconducting rod of length $L$ has a positive charge of uniform linear density $\lambda$. Let us determine the electric potential $V$ due to the rod at point $P$, a perpendicular distance $d$ from the left end of the rod. We consider a differential element $d x$ of the rod. This element of the rod has a differential charge of

(a)

(d)

(b)

(e)

$$
d q=\lambda d x
$$

This element produces an electric potential $d V$ at point $P$, which is a distance

$$
r=\sqrt{x^{2}+d^{2}}
$$

from the element (Fig. 24-15c). Treating the element as a point charge, we can write the potential $d V$ as
$d V=\frac{1}{4 \pi \varepsilon_{0}} \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda d x}{\left(x^{2}+d^{2}\right)^{\frac{1}{2}}}$

Since the charge on the rod is positive and we have taken $V=0$ at infinity. We now find the total potential $V$ produced by the rod at point $P$ by integrating Eq. 24-34 along the length of the $\operatorname{rod}$, from $x=0$ to $x=L$

$$
\begin{gathered}
V=\int d V=\int_{0}^{L} \frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda d x}{\left(x^{2}+d^{2}\right)^{\frac{1}{2}}} \\
=\frac{\lambda}{4 \pi \varepsilon_{0}} \int_{0}^{L} \frac{d x}{\left(x^{2}+d^{2}\right)^{\frac{1}{2}}} \\
=\frac{\lambda}{4 \pi \varepsilon_{0}}\left[\ln \left(x+\left(x^{2}+d^{2}\right)^{\frac{1}{2}}\right)\right]_{0}^{L} \\
=\frac{\lambda}{4 \pi \varepsilon_{0}}\left[\ln \left(L+\left(L^{2}+d^{2}\right)^{\frac{1}{2}}\right)-\ln d\right] \\
V=\frac{\lambda}{4 \pi \varepsilon_{0}} \ln \left[\frac{L+\left(L^{2}+d^{2}\right)^{\frac{1}{2}}}{d}\right]
\end{gathered}
$$

Because $V$ is the sum of positive values of $d V$, it too is positive, consistent with the logarithm being positive for an argument greater than 1 .

## Charged Disk

Here we derive an expression for $V(z)$, the electric potential at any point on the central axis. Because we have a circular distribution of charge on the disk, we could start with a differential element that occupies angle $d \theta$ and radial distance $d r$. We consider a differential element consisting of a flat ring of radius $R^{\prime}$ and radial width $d R^{\prime}$. Its charge has magnitude


$$
\begin{aligned}
& d q=\sigma\left(2 \pi R^{\prime}\right)\left(d R^{\prime}\right) \\
& \quad d V=\frac{1}{4 \pi \varepsilon_{0}} \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\sigma\left(2 \pi R^{\prime}\right)\left(d R^{\prime}\right)}{\left(z^{2}+R^{/ 2}\right)^{\frac{1}{2}}}
\end{aligned}
$$

We find the net potential at $P$ by adding (via integration) the contributions of all the rings from $R^{\prime}=0$ to $R^{\prime}=R$ :

$$
\begin{aligned}
& V=\int d V=\frac{\sigma}{2 \varepsilon_{0}} \int_{0}^{R} \frac{R^{/} d R^{\prime}}{\left(z^{2}+R^{\prime 2}\right)^{\frac{1}{2}}}=\frac{\sigma}{4 \varepsilon_{0}} \int_{0}^{R}\left(z^{2}+R^{/ 2}\right)^{\frac{-1}{2}}\left(2 R^{\prime} d R^{\prime}\right) \\
& =\frac{\sigma}{2 \varepsilon_{0}}\left(\sqrt{z^{2}+R^{2}}-z\right)
\end{aligned}
$$

## Calculating the Field from the Potential

In Module 24-2, you saw how to find the potential at a point $f$ if you know the electric field along a path from a reference point to point $f$. In this module, we propose to go the other way-that is, to find the electric field when we know the potential. As Fig. 24-5 shows, solving this problem graphically is easy: If we know the potential $V$ at all points near an assembly of charges, we can draw in a family of equipotential surfaces. The electric field lines, sketched perpendicular to
those surfaces, reveal the variation of $\vec{E}$. What we are seeking here is the mathematical equivalent of this graphical procedure.

Figure $24-17$ shows cross sections of a family of closely spaced equipotential surfaces, the potential difference between each pair of adjacent surfaces being $d V$. As the figure suggests, the field at any point $P$ is perpendicular to the equipotential surface through $P$.

Suppose that a positive test charge $q_{0}$ moves through a displacement from one equipotential surface to the adjacent surface. From eq.

$$
\begin{equation*}
W=-\Delta U=-q_{0} \Delta V=-q_{0}\left(V_{f}-V_{i}\right) \tag{24-6}
\end{equation*}
$$

we see that the work the electric field does on the test charge during the move is $-q \Delta V$. From Eq.


$$
d W=\vec{F} \cdot d \vec{s}=q_{0} \vec{E} \cdot d \vec{s}
$$

we see that the work done by the electric field may also be written as the scalar product $q_{0} \vec{E} \cdot d \vec{S}$ or $q_{0} E(\cos \theta) d s$. Equating these two expressions for the work yields

$$
\begin{gathered}
-q_{0} \Delta V=q_{0} E(\cos \theta) d s \\
E \cos \theta=-\frac{d V}{d s}
\end{gathered}
$$

Since $E \cos \theta$ is the component of in the direction of $d \vec{s}$. Above Eq. becomes
$E_{S}=-\frac{\partial V}{\partial s}$

We have added a subscript to $E$ and switched to the partial derivative symbols to emphasize that Eq. 24-40 involves only the variation of $V$ along a specified axis (here called the $s$ axis) and only the component of $\vec{E}$ along that axis. In words, Eq. 24-40 (which is essentially the reverse operation of Eq. 24-18) states:

The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

If we take the $s$ axis to be, in turn, the $x, y$, and $z$ axes, we find that the $x, y$, and $z$ components of $\vec{E}$ at any point are
$E_{x}=-\frac{\partial V}{\partial x} ; \quad E_{y}=-\frac{\partial V}{\partial y} ; \quad E_{z}=-\frac{\partial V}{\partial z}$
Thus, if we know $V$ for all points in the region around a charge distribution-that is, if we know the function $V(x, y, z)$-we can find the components of $\vec{E}$, and thus $\vec{E}$ itself, at any point by taking partial derivatives.

For the simple situation in which the electric field $\vec{E}$ is uniform, Eq. 24-40 becomes
$E=-\frac{\Delta V}{\Delta s}$
where $s$ is perpendicular to the equipotential surfaces. The component of the electric field is zero in any direction parallel to the equipotential surfaces because there is no change in potential along the surfaces.

## Checkpoint 5

The figure shows three pairs of parallel plates with the same separation, and the electric potential of each plate. The electric field between the

(1)

(2)

(3) plates is uniform and perpendicular to the plates. (a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first. (b) For which pair is the electric field pointing rightward? (c) If an electron is released midway between the third pair of plates, does it remain there, move rightward at constant speed, move leftward at constant speed, accelerate rightward, or accelerate leftward?
a) Uniform electric field and same separation. $\quad E=-\frac{\Delta V}{\Delta s} ; \quad \Delta V$ is $200 \mathrm{~V}, 220 \mathrm{~V}$ and -200 V in 1, 2 and 3 pair respectively.
2, then 1 and 3 tie.
b) Electric field vector points from higher potential toward lower potential. 3
c) In third pair electric field points towards right and force on electron is towards left. So electron accelerates towards leftward.

