Electric Flux

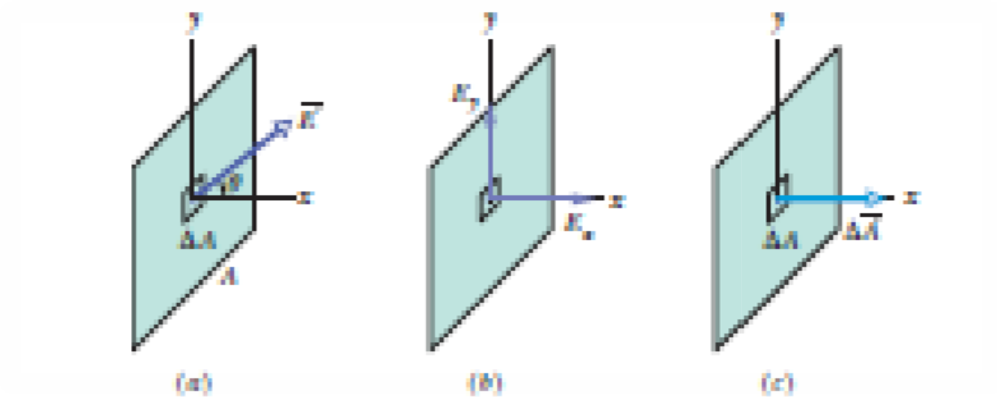


Figure: 23.4

Open Surface (Piece of paper, handkerchief)

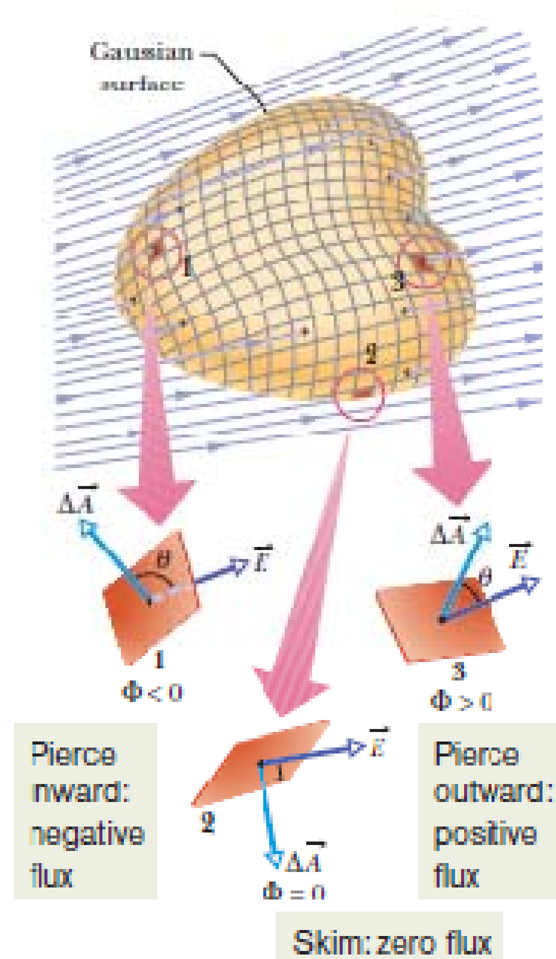


Figure 23.5

$$\Delta\Phi = \vec{E}\,.\,\Delta\vec{A} = (E\cos\theta)\Delta A$$

Total Flux = $\Phi = \sum \vec{E}\,.\,\Delta\vec{A}$ Scalar (+, -, 0)

$$d\Phi = \vec{E}\,.\,d\vec{A} = (E\cos\theta)dA$$

Total Flux = $\Phi = \int \vec{E}\,.\,d\vec{A}$

Closed Surface (Balloon, bag)

By convention, normal is always chosen from inside to outside (uniquely determined)

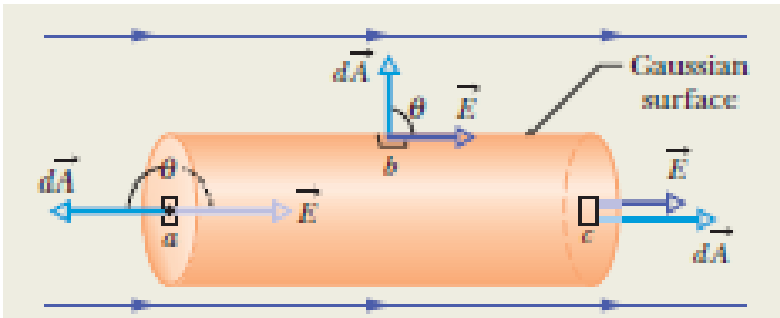


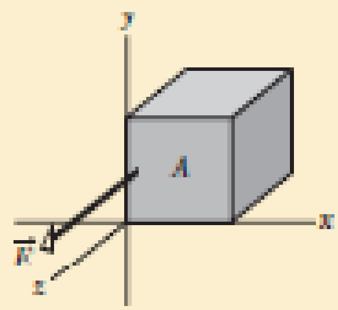
Figure 23.6

$$d\Phi = \vec{E} \,.\, d\vec{A} = (E\cos\theta)dA$$

Total Flux = $\Phi = \oint \vec{E} . d\vec{A}$

Checkpoint 1

The figure here shows a Gaussian cube of face area A immersed in a uniform electric field $\vec{E}$ that has the positive direction of the z axis. In terms of E and A, what is the flux through (a) the front face (which is in the xy plane), (b) the rear face, (c) the top face, and (d) the whole cube?



(a) $\Phi = \int_F \vec{E} \,.\, d\vec{A}$ (Open surface)

$\Phi = \int_F EdA\cos 0$ **E** and **dA** are parallel, θ=0, cos0=1

$\Phi = E \int_F dA$ E is uniform on front face; we take it out from integral

$\Phi = EA$ $\int_F dA = A$ Area of front face

(b) $\Phi = \int_F \vec{E} . d\vec{A}$ (Open surface)

$\Phi = \int_F EdAcos180$ **E** and **dA** (vector area along negative z-axis) are antiparallel, θ=180, cos180= –1

$\Phi = -E \int_F dA$ E is uniform on rear face; we take it out from integral

$\Phi = -EA$ $\int_F dA = A$ Area of rear face

(c) $\Phi = \int_F \vec{E} . d\vec{A}$ (Open surface)

$\Phi = \int_F EdAcos180$ **E** and **dA** (vector area along positive y-axis) are perpendicular, θ=90, cos90=0

$\Phi = 0$

(d) $Total\ flux\ through\ whole\ cube = \Phi = \oint \vec{E} . \overrightarrow{dA}$ (Closed surface)

$$\Phi = \oint \vec{E} . d\vec{A} = \int_F \vec{E} . d\vec{A} + \int_R \vec{E} . d\vec{A} + \int_T \vec{E} . d\vec{A} + \int_B \vec{E} . d\vec{A} + \int_R \vec{E} . d\vec{A} + \int_L \vec{E} . d\vec{A}$$

For bottom, right and left faces ***E*** and ***dA*** are perpendicular, θ=90, cos90=0

$$\Phi = EA - EA + 0 + 0 + 0 + 0$$

$\Phi = 0$ **Why?**

1 A surface has the area vector $\vec{A} = (2\hat{i} + 3\hat{j})\ m^2$. What is the flux of a uniform electric field through the area if the field is (a) $\vec{E} = 4\hat{i}$ N/C and (b) $\vec{E} = 4\hat{k}$ N/C?

Total Flux = $\Phi = \vec{E}.\vec{A}$

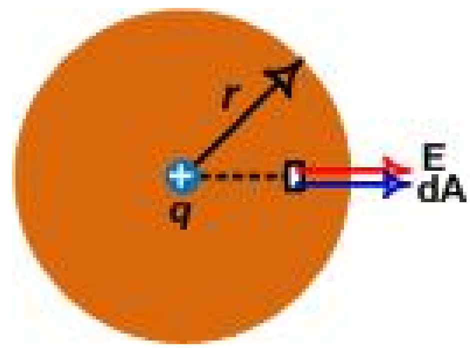
$$\Phi = 4\,\hat{\imath}.(2\,\hat{\imath} + 3\,\hat{\jmath}) = 8\,\hat{\imath}.\hat{\imath} + 12\,\hat{\imath}.\hat{\jmath} = 8\frac{Nm^2}{C}$$

$$\Phi = 4\,\hat{k}.(2\,\hat{\imath} + 3\,\hat{\jmath}) = 8\,\hat{k}.\hat{\imath} + 12\,\hat{k}.\hat{\jmath} = 0$$

Flux due to a point charge

A spherical Gaussian surface centered on a particle with charge q.

$$\Phi = \oint \vec{E}\,.d\vec{A}$$



i) E and **dA** are parallel, θ = 0, cos0 =1

$$\Phi = \oint EdAcos\theta = \oint EdA$$

ii) E at surface is same because r is same.

$\Phi = E\oint dA$ $\quad\quad$ $\Phi = E(4\pi r^2)$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

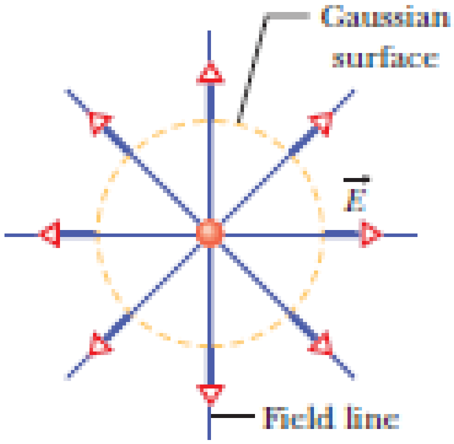
$$\Phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \times 4\pi r^2$$

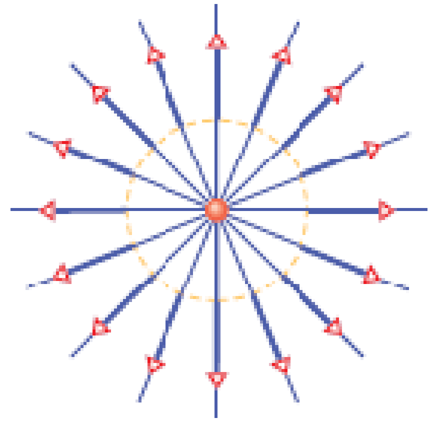
$$\Phi = \frac{q}{\varepsilon_0}$$

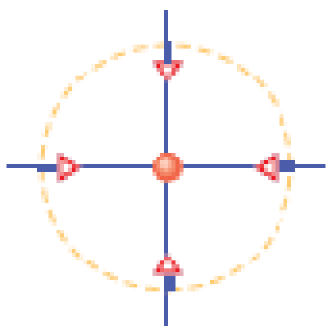
Independent of r; independent of size and shape of Gaussian surface; holds for any shape and no. of charges.

$$\Phi = \oint \vec{E}\,.d\vec{A} = \frac{\sum q_{enc}}{\varepsilon_0}$$

Holds for air. Always holds. Guass' law relates the electric field at points on a (closed) Gaussian surface to the net charge enclosed by that surface.







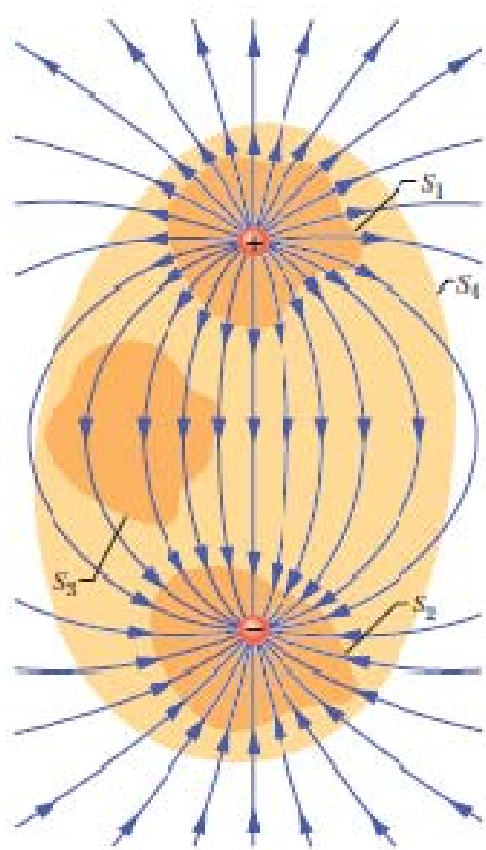
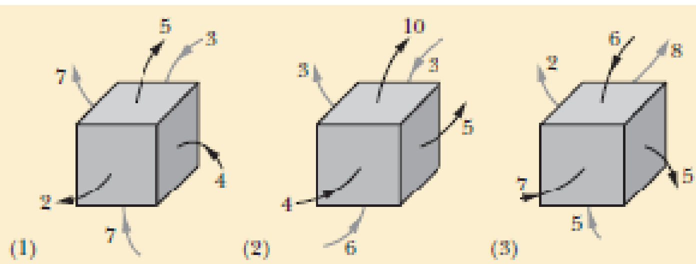


Figure 23.8

Checkpoint 2

The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicate the directions of the field lines and the magnitudes (in $N \cdot m^2/C$) of the flux through the six sides of each cube. (The lighter arrows are for the hidden faces.) In which situation does the cube enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?



Checkpoint 3

There is a certain net flux Φ_i through a Gaussian sphere of radius r enclosing an isolated charged particle. Suppose the enclosing Gaussian surface is changed to (a) a larger Gaussian sphere, (b) a Gaussian cube with edge length equal to r, and (c) a Gaussian cube with edge length equal to $2r$. In each case, is the net flux through the new Gaussian surface greater than, less than, or equal to Φ_i?

$$\Phi = \frac{q}{\varepsilon_0}$$

Electric flux are independent of size and shape of Gaussian surface.

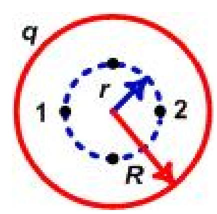
Equal to Φ_i

Spherical Symmetry

Figure shows a uniformaly charged spherical shell of radius R. Consider a Gaussian surface of radius r.

i) Spherical symmetry, charge uniformly distributed. Electric field at point 1 is equal in magnitude to electric field at point 2.

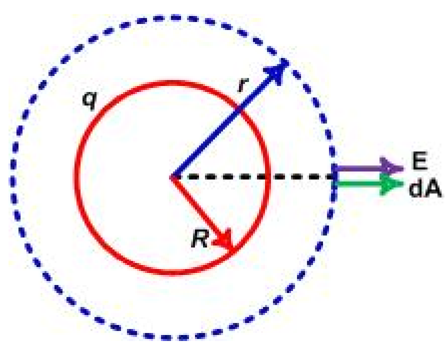
ii) Electric field radially inward or outward.



$$\Phi = \oint \vec{E}\,.\,d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{enc}}{\varepsilon_0} \qquad E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = 0 \qquad E = 0 \qquad r < R$$

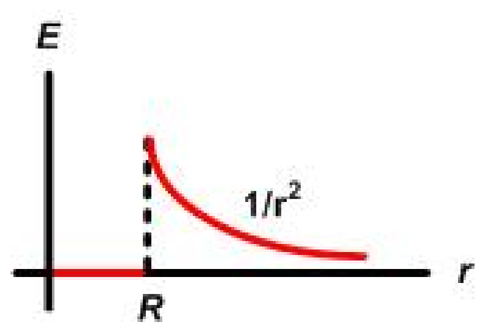


A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

$$\Phi = \oint \vec{E}\,.\,d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{enc}}{\varepsilon_0} \qquad E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad r > R$$



A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

3 Figure 23-23 shows, in cross section, a central metal ball, two spherical metal shells, and three spherical Gaussian surfaces of radii R, $2R$, and $3R$, all with the same center. The uniform charges on the three objects are: ball, Q; smaller shell, $3Q$; larger shell, $5Q$. Rank the Gaussian surfaces according to the magnitude of the electric field at any point on the surface, greatest first.

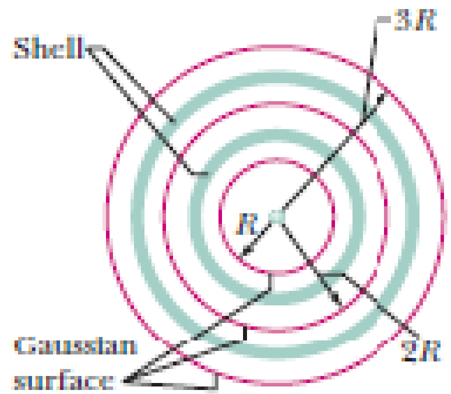


Figure 23-23 Question 3.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{4R^2} = E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{9Q}{9R^2} = E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

All tie.

4 Figure 23-24 shows, in cross section, two Gaussian spheres and two Gaussian cubes that are centered on a positively charged particle. (a) Rank the net flux through the four Gaussian surfaces, greatest first. (b) Rank the magnitudes of the electric fields on the surfaces, greatest first, and indicate whether the magnitudes are uniform or variable along each surface.

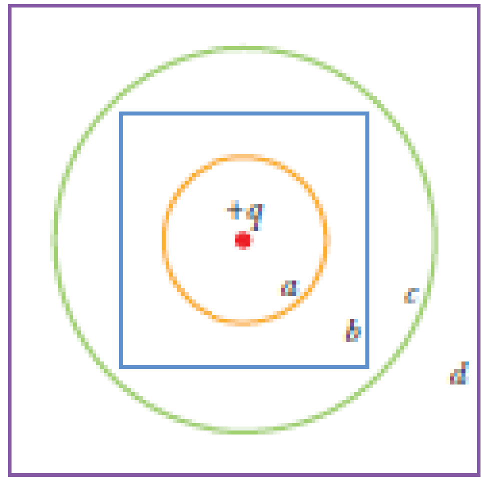


Figure 23-24 Question 4.

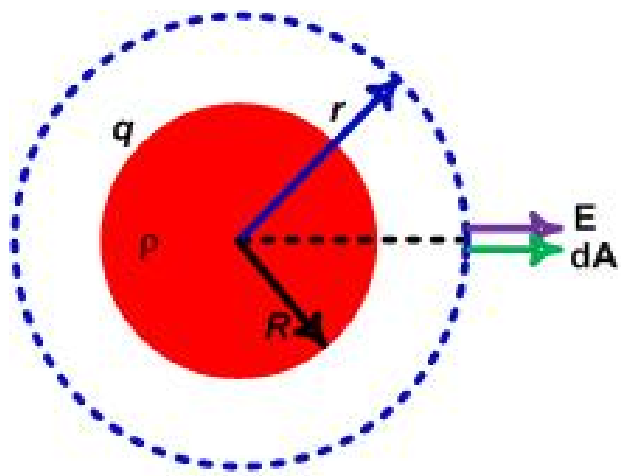
(a) All tie. Electric flux is independent of size and shape of Gaussian surface.

(b) a, b, c and d.

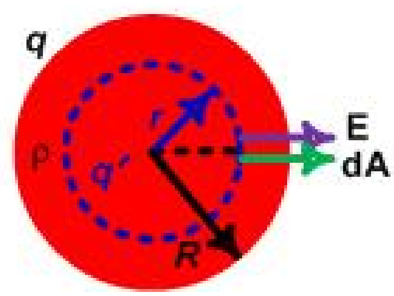
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

The magnitude of electric field is uniform along surfaces a and c and is variable along b and d.

Spherical charge distribution



Any spherically symmetric charge distribution, such as that of Fig. can be constructed with a nest of concentric spherical shells. For purposes of applying the two shell theorems, the volume charge density ρ should have a single value for each shell but need not be the same from shell to shell. Thus, for the charge distribution as a whole ρ can vary, but only with *r*, the radial distance from the center. We can then examine the effect of the charge distribution "shell by shell." In Fig. the entire charge lies within a Gaussian surface with $r > R$. The charge produces an electric field on the Gaussian surface as if the charge were that of a particle located at the center.



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad r > R$$

Figure shows a Gaussian surface with $r < R$. To find the electric field at points on this Gaussian surface, we separately consider the charge inside it and the charge outside it. The outside charge does not set up a field on the Gaussian surface. The inside charge sets up a field as though it is concentrated at the center. Letting $q^/$ represent that enclosed charge, we can then write′

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q^/}{r^2}$$

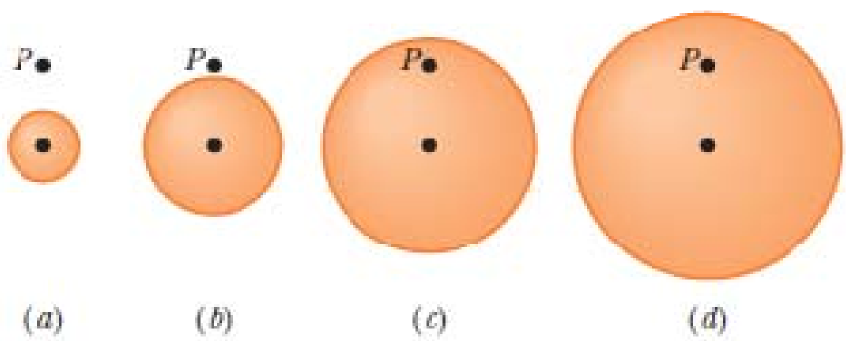
If the full charge q enclosed within radius R is uniform, then $q^/$ enclosed within radius r in Fig. is proportional to q:

$$\frac{q^/}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3} \qquad q^/ = q\frac{r^3}{R^3}$$

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r \qquad r \leq R$$

Direction of E?

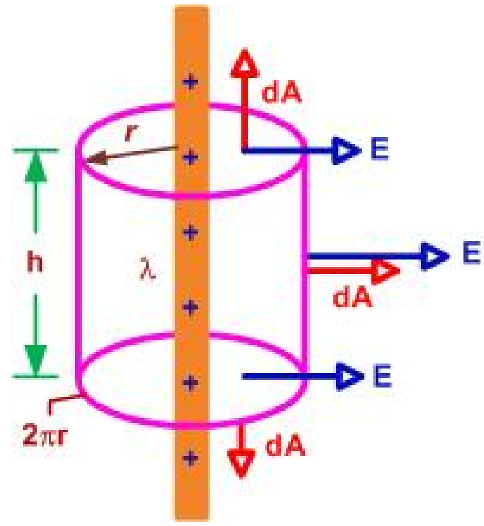
8 Figure 23 27 shows four solid spheres, each with charge Q uniformly distributed through its volume. (a) Rank the spheres according to their volume charge density, greatest first. The figure also shows a point P for each sphere, all at the same distance from the center of the sphere. (b) Rank the spheres according to the magnitude of the electric field they produce at point P, greatest first.



a) a, b, c and d. **b)** a and b tie, c and d.

Cylindrical Symmetry

Figure shows a section of an infinitely long cylindrical plastic rod with a uniform charge density λ.We want to find an expression for the electric field magnitude *E* at radius *r* from the central axis of the rod, outside the rod. The charge distribution and the field have cylindrical symmetry. To find the field at radius *r*, we enclose a section of the rod with a concentric Gaussian cylinder of radius *r* and height *h*.



$$\Phi = \oint \vec{E}\,.\,d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

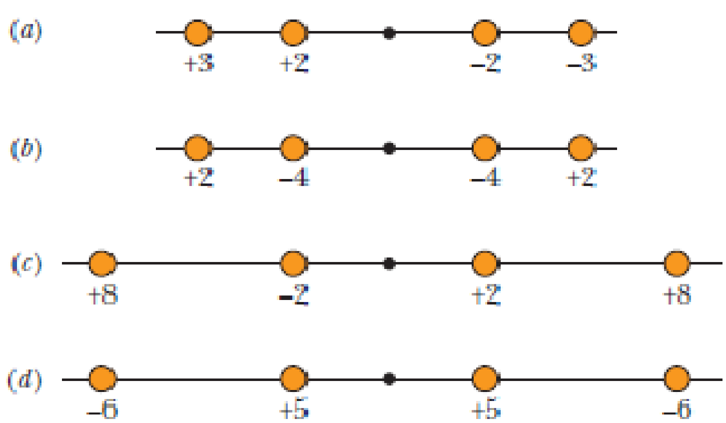
$$\int_T \vec{E}\,.\,d\vec{A} + \int_B \vec{E}\,.\,d\vec{A} + \int_C \vec{E}\,.\,d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_T EdAcos90 + \int_B EdAcos90 + \int_C EdAcos0 = \frac{q_{enc}}{\varepsilon_0}$$

$E\int_C dA = \frac{q_{enc}}{\varepsilon_0}$ $E(2\pi rh) = \frac{\lambda h}{\varepsilon_0}$ $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

What is direction of *E*?

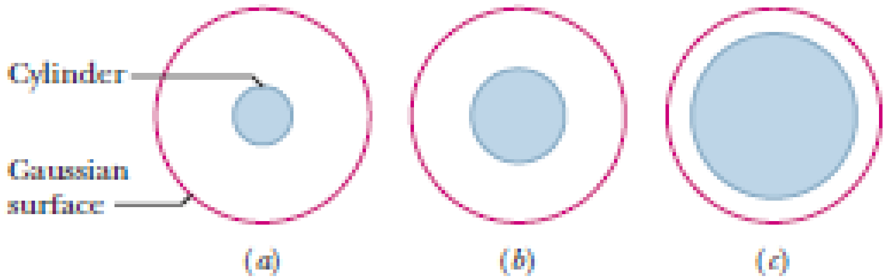
7 Figure 23-26 shows four situations in which four very long rods extend into and out of the page (we see only their cross sections). The value below each cross section gives that particular rod's uniform charge density in microcoulombs per meter. The rods are separated by either *d* or 2*d* as drawn, and a central point is shown midway between the inner rods. Rank the situations according to the magnitude of the net electric field at that central point, greatest first.



(a) $E = \frac{1}{2\pi\varepsilon_0}\left[\frac{3}{2d} + \frac{2}{d} + \frac{2}{d} + \frac{3}{2d}\right] = \frac{1}{2\pi\varepsilon_0 d}\left[\frac{6}{2} + 4\right] = \frac{7}{2\pi\varepsilon_0 d}$

a, c then b and d tie

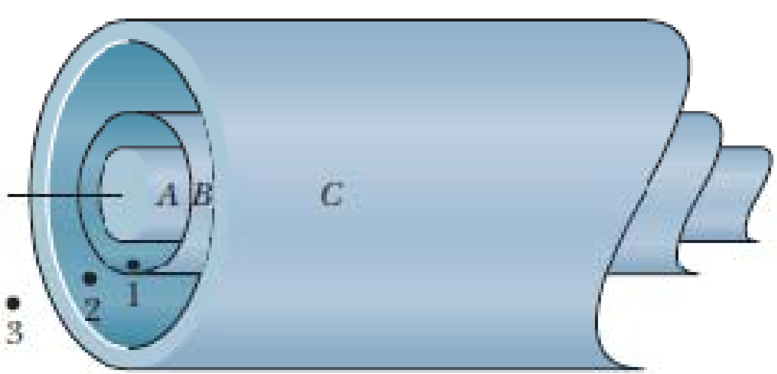
2 Figure 23-22 shows, in cross section, three solid cylinders, each of length L and uniform charge Q. Concentric with each cylinder is a cylindrical Gaussian surface, with all three surfaces having the same radius. Rank the Gaussian surfaces according to the electric field at any point on the surface, greatest first.



$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{1}{2\pi\varepsilon_0 r}\frac{Q}{L}$$

Enclosed charge and length is same. Gausssian surfaces have same radius so electric field is same. All tie.

11 Figure 23-28 shows a section of three long charged cylinders centered on the same axis. Central cylinder A has a uniform charge $q_A = +3q_0$. What uniform charges q_B and q_C should be on cylinders B and C so that (if possible) the net electric field is zero at (a) point 1, (b) point 2, and (c) point 3?



$$\Phi = \oint \vec{E} \, . d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

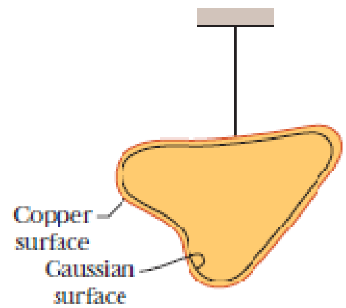
(a) E=0, therefore $q_{enc} = 0$ but $q_{enc} = q_A = +3q_0$, impossible.

(b) E=0, therefore $q_{enc} = q_A + q_B = 0$ implies $q_B = -q_A = -3q_0$

(c) E=0, therefore $q_{enc} = q_A + q_B + q_C = 0$ implies $q_C = -q_A - q_B = -3q_0 + 3q_0 = 0$ impossible.

A Charged Isolated Conductor

If an excess charge is placed on an isolated conductor that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.



Charges with the same sign repel.

Gauss' law

Figure *shows*, in cross section, an isolated lump of copper hanging from an insulating thread and having an excess charge *q*. We place a Gaussian surface just inside the actual surface of the conductor.

The electric field inside this conductor must be zero. Why?

There is no perpetual current in an isolated conductor.

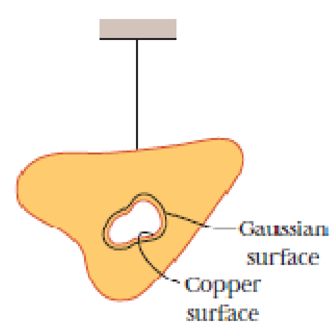
Since E = 0 everywhere inside copper conductor then ($\Phi = \oint \vec{E} \, . d\vec{A}$) shows flux through Gaussian surface must be zero (Φ = 0) and ($\Phi = \frac{q}{\varepsilon_0}$) shows charge inside the Gaussian surface must also be zero.

Then because the excess charge is not inside the Gaussian surface, it must be outside that surface, which means it must lie on the actual surface of the conductor.

An Isolated Conductor with a Cavity

Figure shows the same hanging conductor, but now with a cavity that is totally within the conductor. We draw a Gaussian surface surrounding the cavity, close to its surface but inside the conducting body. Because $E = 0$ inside the conductor.

From Gauss' law ($\Phi = \oint \vec{E} \, . d\vec{A} = \frac{q}{\varepsilon_0}$) that surface can enclose no net charge. Where is the excess charge?



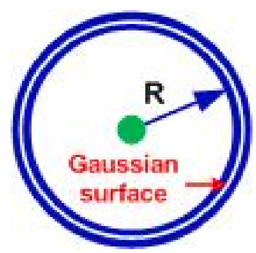
The Conductor Removed

Suppose that, by some magic, the excess charges could be "frozen" into position on the conductor's surface, perhaps by embedding them in a thin plastic coating, and suppose that then the conductor could be removed completely. This is equivalent to enlarging the cavity of Fig. until it consumes the entire conductor, leaving only the charges. The electric field would not change at all; it would remain zero inside the thin shell of charge and would remain unchanged for all external points. This shows us that the electric field is set up by the charges and not by the conductor. The conductor simply provides an initial pathway for the charges to take up their positions.

9 A small charged ball lies within the hollow of a metallic spherical shell of radius R. For three situations, the net charges on the ball and shell, respectively, are (1) $+4q$, 0; (2) $-6q$, $+10q$; (3) $+16q$, $-12q$. Rank the situations according to the charge on (a) the inner surface of the shell and (b) the outer surface, most positive first.

$\Phi = \oint \vec{E} \, . d\overrightarrow{A} = \frac{q_{enc}}{\varepsilon_0}$

E = 0 inside shell. Flux through Gaussian surface is zero and net charge enclosed in Gaussian surface is zero.



(a) $\boldsymbol{q_{net} = q_{ball} + q_{inner\ surface}}$ $\boldsymbol{0 = 4q + q_{inner\ surface}}$

$\boldsymbol{q_{inner\ surface} = -4q}$

(2, 1, 3)

(b) $q_{shell} = q_{inner\ surface} + q_{outter\ surface}$ $0 = -4q + q_{outter\ surface}$

$q_{outter\ surface} = 4q$

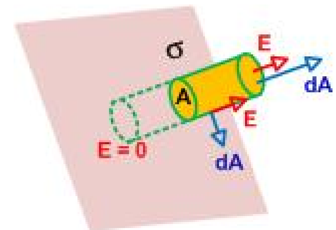
(All tie)

10 Rank the situations of Question 9 according to the magnitude of the electric field (a) halfway through the shell and (b) at a point $2R$ from the center of the shell, greatest first.

(a) All tie; E = 0 inside metal shell

(b) All tie; enclosed charge same and distance of point same.

The External Electric Field



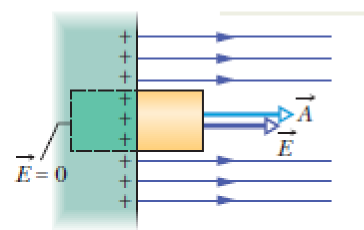
The surface charge density varies over the surface of any nonspherical conductor that makes the determination of the electric field set up by the surface charges very difficult.

The find electric field just outside the surface of a conductor, we consider a section of the surface that is small enough to permit us to neglect any curvature and thus to take the section to be flat. We then imagine a tiny cylindrical Gaussian surface to be partially embedded in the section.

The electric field at and just outside the conductor's surface must also be perpendicular to that surface. Why?

Since electric field within the conductor is zero therefore no flux through the internal (in the conductor) portion of Gaussian surface.

Flux through the external portion of Gaussian surface are:



$$\int_{curved\ surface} \vec{E} \cdot d\vec{A} + \int_{end\ cap} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{curved\ surface} EdAcos90 + \int_{end\ cap} EdAcos0 = \frac{q_{enc}}{\varepsilon_0}$$

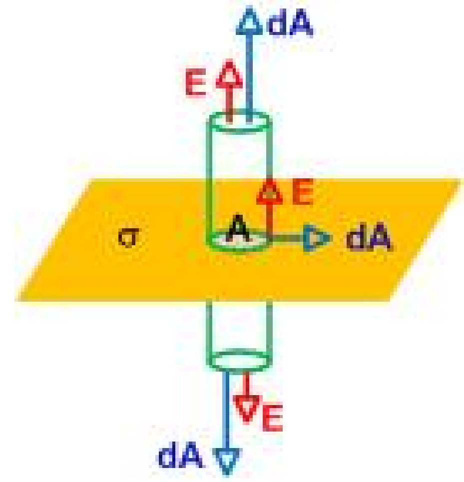
$$E \int_{end\ cap} dA = \frac{q_{enc}}{\varepsilon_0} \qquad EA = \frac{\sigma A}{\varepsilon_0} \qquad E = \frac{\sigma}{\varepsilon_0}$$

Thus, the magnitude of the electric field just outside a conductor is proportional to the surface charge density on the conductor.
What is direction of electric field?

Planar Symmetry

Figure shows a portion of a thin, infinite, non conducting sheet with a uniform (positive) surface charge density σ. Let us find electric field a distance r in front of the sheet.

Cylindrical Gaussian surface (why not spherical, tapered or conical)



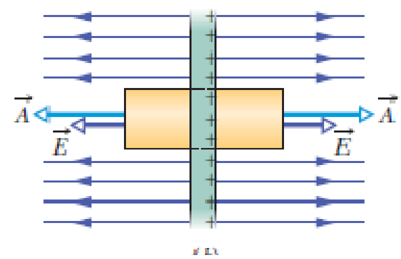
i) End caps should be flat (not convex, concave or conical) and parallel to sheet (no tilt)

ii) Curved surface should be perpendicular to sheet

iii) Distance of end caps from sheet should be same.

$$\Phi = \oint \vec{E} \, . d\vec{A} = \frac{\sum q_{enc}}{\varepsilon_0}$$

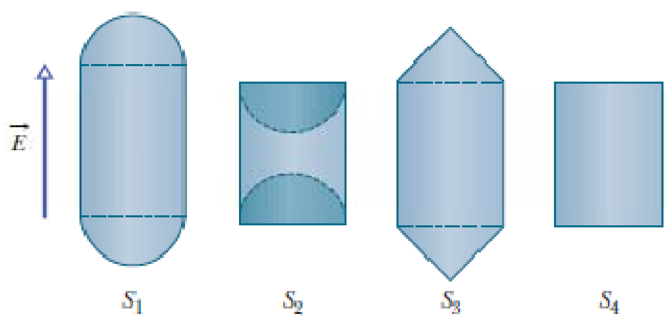
$$\int_T \vec{E} \, . d\vec{A} + \int_B \vec{E} \, . d\vec{A} + \int_C \vec{E} \, . d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

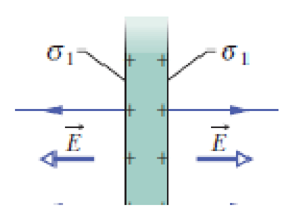


$$\int_T EdAcos0 + \int_B EdAcos0 + \int_C EdAcos90 = \frac{q_{enc}}{\varepsilon_0}$$

$$E \int_T dA + E \int_B dA = \frac{q_{enc}}{\varepsilon_0} \qquad EA + EA = 2EA = \frac{\sigma A}{\varepsilon_0} \qquad E = \frac{\sigma}{2\varepsilon_0}$$

12 Figure 23-29 shows four Gaussian surfaces consisting of identical cylindrical midsections but different end caps. The surfaces are in a uniform electric field $\vec{E}$ that is directed parallel to the central axis of each cylindrical midsection. The end caps have these shapes: S_1, convex hemispheres; S_2, concave hemispheres; S_3, cones; S_4, flat disks. Rank the surfaces according to (a) the net electric flux through them and (b) the electric flux through the top end caps, greatest first.





(a) All tie; zero (b) all tie; same

Two Conducting Plates

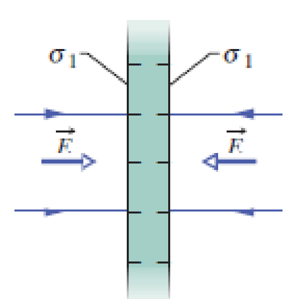
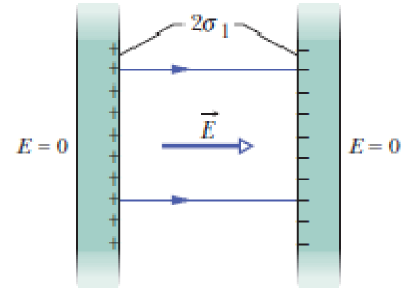


Figure shows a cross section of a thin, infinite conducting plate with excess positive charge. All the excess charge lies on the two large faces of the plate. If there is no external electric field to force the positive charge into some particular distribution, it will spread out on the two faces with a uniform surface charge density of magnitude σ_1. Just outside the plate this charge sets up an electric field of magnitude

$$E = \frac{\sigma_1}{\varepsilon_0}$$

Figure shows an identical plate with excess negative charge having the same magnitude of surface charge density σ_1.

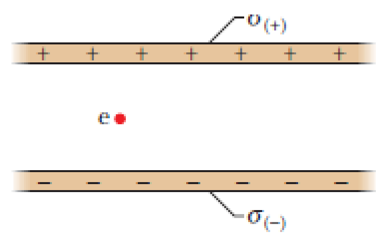
The two plates arranged so they are parallel and close. Since the plates are conductors, when we bring them into this arrangement, the excess charge on one plate attracts the excess charge on the other plate, and all the excess charge moves onto the inner faces of the plates. With twice as much charge now on each inner face, the new surface charge density (call it σ) on each inner face is twice σ_1. Thus, the electric field at any point between the plates has the magnitude



$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

5 In Fig. 23-25, an electron is released between two infinite nonconducting sheets that are horizontal and have uniform surface charge densities $\sigma_{(+)}$ and $\sigma_{(-)}$, as indicated. The electron is subjected to the following three situations involving surface charge densities and sheet separations. Rank the magnitudes of the electron's acceleration, greatest first.

Situation	$\sigma_{(+)}$	$\sigma_{(-)}$	Separation
1	$+4\sigma$	-4σ	d
2	$+7\sigma$	$-\sigma$	$4d$
3	$+3\sigma$	-5σ	$9d$



$$E = \frac{4\sigma}{2\varepsilon_0} + \frac{4\sigma}{2\varepsilon_0} = \frac{4\sigma}{2\varepsilon_0}$$

$$E = \frac{7\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{4\sigma}{2\varepsilon_0}$$

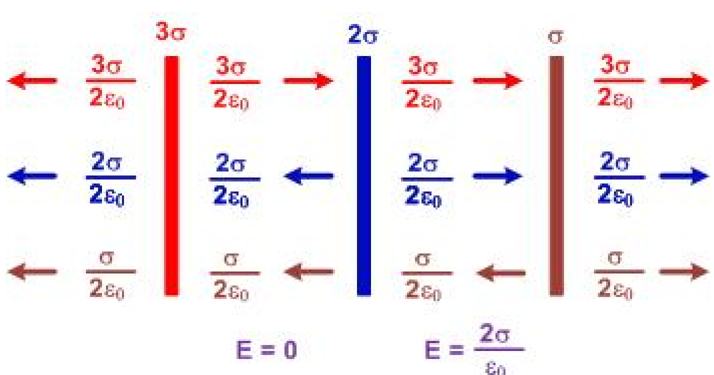
$$E = \frac{3\sigma}{2\varepsilon_0} + \frac{5\sigma}{2\varepsilon_0} = \frac{4\sigma}{2\varepsilon_0}$$

All tie; Electric field same.

Electrostatic force same.

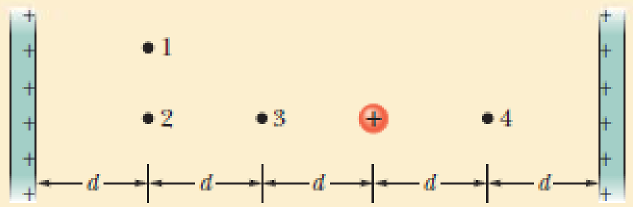
Acceleration same.

6 Three infinite nonconducting sheets, with uniform positive surface charge densities σ, 2σ, and 3σ, are arranged to be parallel like the two sheets in Fig. 23-19*a*. What is their order, from left to right, if the electric field $\vec{E}$ produced by the arrangement has magnitude $E = 0$ in one region and $E = 2\sigma/\varepsilon_0$ in another region?



Checkpoint 4

The figure shows two large, parallel, nonconducting sheets with identical (positive) uniform surface charge densities, and a sphere with a uniform (positive) volume charge density. Rank the four numbered points according to the magnitude of the net electric field there, greatest first.



Problems: 4, 5, 12, 15, 21, 29, 33, 43, 49, 54.

3 & 4 tie then 2, 1