Module 23-2 Gauss' Law

•4 In Fig. 23-32, a butterfly net is in a uniform electric field of magnitude $E = 3.0$ mN/C. The rim, a circle of radius $a = 11$ cm, is aligned perpendicular to the field. The net contains no net charge. Find the electric flux through the netting.

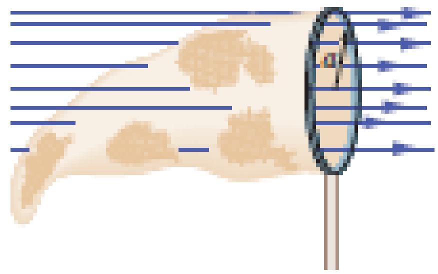


Figure 23-32 Problem 4.

$Flux\ through\ opening\ (rim) = \int \vec{E}\ .d\vec{A}$ (Open surface)

$Flux\ through\ opening\ (rim) = \int EdAcos0$ $\vec{E}$ and $d\vec{A}$ are parallel

$Flux\ through\ opening\ (rim) = E \int dA$ E is uniform on opening

$Flux\ through\ opening\ (rim) = E(\pi a^2)$

$\int dA = \pi a^2$ Area of opening

As electric field is uniform there are no free charges ~~inside~~ inside the net. The net flux is zero.
The flux through the netting must be equal to, but opposite in sign, from the flux through the opening. The flux through opening is $E\pi a^2$
Flux through netting $= -E\pi a^2$

•5 In Fig. 23-33, a proton is a distance $d/2$ directly above the center of a square of side d. What is the magnitude of the electric flux through the square? (*Hint:* Think of the square as one face of a cube with edge d.)

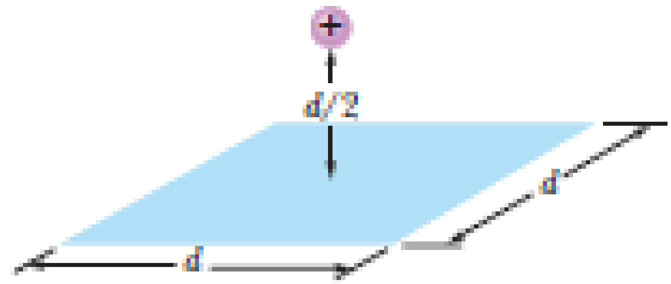
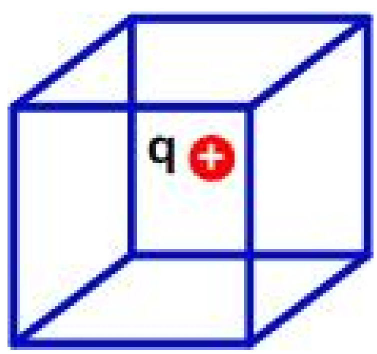


Figure 23-33 Problem 5.



The total flux through cube would be q/ϵ_0. Since charge is in the centre of the cube, the flux through any side would be the same, at $1/6$ of the total flux. Hence flux through square side is $q/6\epsilon_0$

12 Figure 23-36 shows two nonconducting spherical shells fixed in place. Shell 1 has uniform surface charge density $+6.0\ \mu C/m^2$ on its outer surface and radius 3.0 cm; shell 2 has uniform surface charge density $+4.0\ \mu C/m^2$ on its outer surface and radius 2.0 cm; the shell centers are separated by $L = 10$ cm. In unit-vector notation, what is the net electric field at $x = 2.0$ cm?

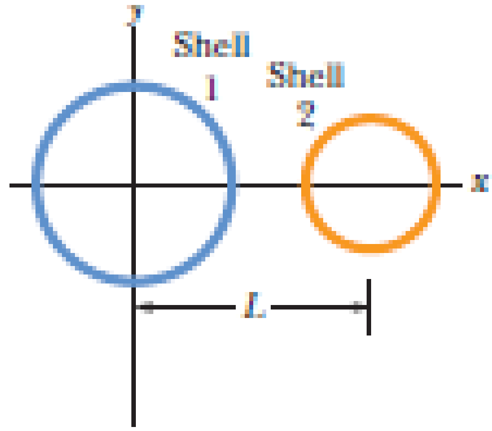
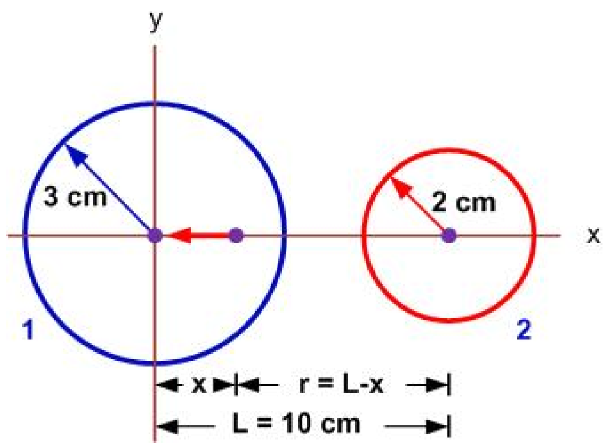
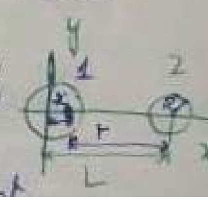


Figure 23-36 Problem 12.



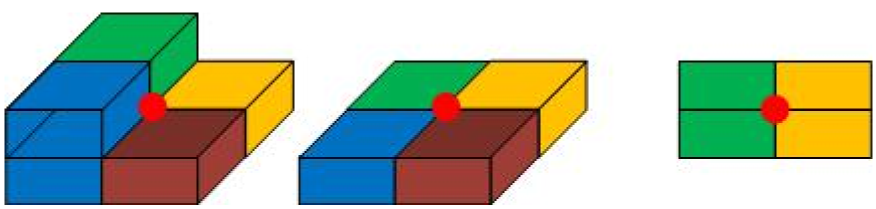
Only smaller shell contributes a (non-zero) field at designated point, since the point is inside the larger shell. The field points towards the $-x$ direction. Thus

$$\vec{E} = -E\hat{i} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{i} = -\frac{1}{4\pi\varepsilon_0}\frac{\sigma_2 4\pi R^2}{(L-x)^2}\hat{i}$$



The field lines are radial, so at each of the three cube

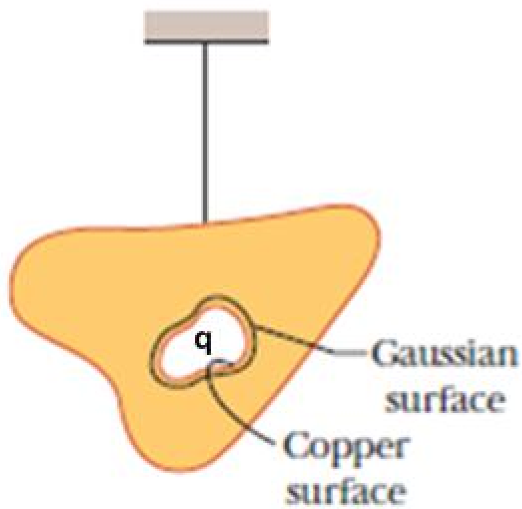
••15 A particle of charge $+q$ is placed at one corner of a Gaussian cube. What multiple of q/ε_0 gives the flux through (a) each cube face forming that corner and (b) each of the other cube faces?



) The field lines are radial, so at each of the three cube faces that meet at the charge, the lines are $\parallel$ to the face and the flux through the face is zero.

The flux through each of the other faces are the same, so flux through each of them is one third of the total. The flux through each face is $\frac{1}{3}\left(q/8\varepsilon_0\right) = \frac{q}{24\varepsilon_0}$ { Since one-eighth of field lines emanating from pt. charge Pass through a cube, Total flux through surface of a cube $q/8\varepsilon_0$

21 An isolated conductor has net charge $+10 \times 10^{-6}$ C and a cavity with a particle of charge $q = +3.0 \times 10^{-6}$ C. What is the charge on (a) the cavity wall and (b) the outer surface?



(21) Consider Gaussian surface within conductor surrounding [the cavity]. Electric field is zero on the surface, net charge it encloses is zero.

a) charge in cavity; charge on cavity wall

$q + q_w = 0 \Rightarrow q_w = -q = -3.0 \times 10^{-6}$ C

b) $q_{\text{net charge on conductor}} = q_{\text{charge on inner surface of cavity}} + q_{\text{charge on outer surface}}$

••29 SSM WWW Figure 23-42 is a section of a conducting rod of radius $R_1 = 1.30$ mm and length $L = 11.00$ m inside a thin-walled coaxial conducting cylindrical shell of radius $R_2 = 10.0R_1$ and the (same) length L. The net charge on the rod is $Q_1 = +3.40 \times 10^{-12}$ C; that on the shell is $Q_2 = -2.00Q_1$. What are the (a) magnitude E and (b) direction (radially inward or outward) of the electric field at radial distance $r = 2.00R_2$? What are (c) E and (d) the direction at $r = 5.00R_1$? What is the charge on the (e) interior and (f) exterior surface of the shell?

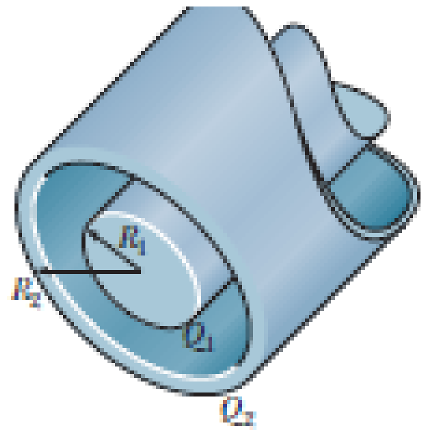
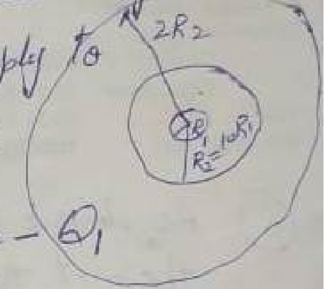


Figure 23-42 Problem 29.

(27) Take a cylindrical Gaussian surface of radius r and length L. Ignore flux through ends. Gauss' law will simplify to



a) $$q_{enc}/\epsilon_0 = \oint \vec{E}\cdot d\vec{A} = E\oint dA = E\int dA = 2\pi r L E$$

$$E = q_{enc}/2\pi\epsilon_0 r L \quad ; \quad q_{enc} = +Q_1 - 2Q_1 = -Q_1$$

$$E = -Q_1/2\pi\epsilon_0 r L$$

b) $-$ve sign indicates field inward towards axis of cylinder

c) $q_{enc} = Q_1 \; ; \; 2\pi r L E = Q_1/\epsilon_0 \Rightarrow E = Q_1/2\pi\epsilon_0 r L$

d) +ve sign indicates that field points outward.

e) Consider Gaussian surface ~~inside~~ within the shell. Electric field is zero at Gaussian surface. So ~~Q~~, q_{enc} is zero.

~~So~~ $q_{enc} = Q_1 + Q_{in} = 0 \Rightarrow Q_{in} = -Q_1$

Since shell has $Q_2 = -2Q_1$ charge. $-Q_1$ lies on inner surface. So charge on outer surface is

$$-2Q_1 = Q_{in} + Q_{out} \Rightarrow Q_{out} = -2Q_1 - Q_{in} = -2Q_1 + Q_1 = -Q_1$$

or

$$Q_{out} = Q_2 - Q_{in} = -2Q_1 - (-Q_1) = -2Q_1 + Q_1 = -Q_1$$

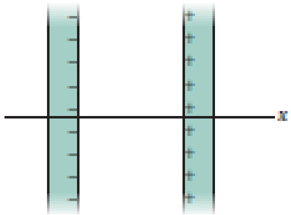


Figure 23-44 Problem 33.

•33 In Fig. 23-44, two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have excess surface charge densities of opposite signs and magnitude 7.00×10^{-22} C/m^2. In unit-vector notation, what is the electric field at points (a) to the left of the plates, (b) to the right of them, and (c) between them?

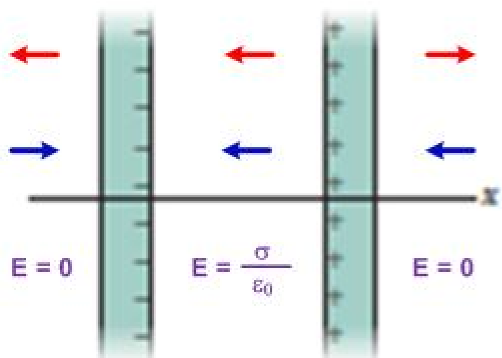


Figure 23-44 Problem 33.

$$\vec{E}_L = \frac{\sigma}{\varepsilon_0}\hat{\imath} - \frac{\sigma}{\varepsilon_0}\hat{\imath} = 0$$

$$\vec{E}_R = -\frac{\sigma}{\varepsilon_0}\hat{\imath} + \frac{\sigma}{\varepsilon_0}\hat{\imath} = 0$$

$$\vec{E}_B = -\frac{\sigma}{\varepsilon_0}\hat{\imath}$$

•••43 Figure 23-51 shows a cross section through a very large nonconducting slab of thickness $d = 9.40$ mm and uniform volume charge density $\rho = 5.80$ fC/m^3. The origin of an x axis is at the slab's center. What is the magnitude of the slab's electric field at an x coordinate of (a) 0, (b) 2.00 mm, (c) 4.70 mm, and (d) 26.0 mm?

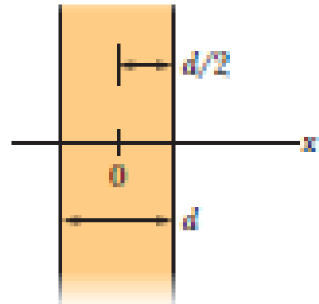
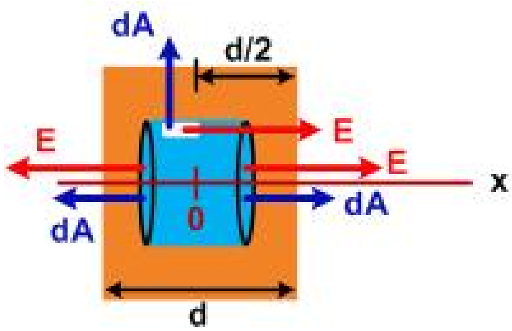
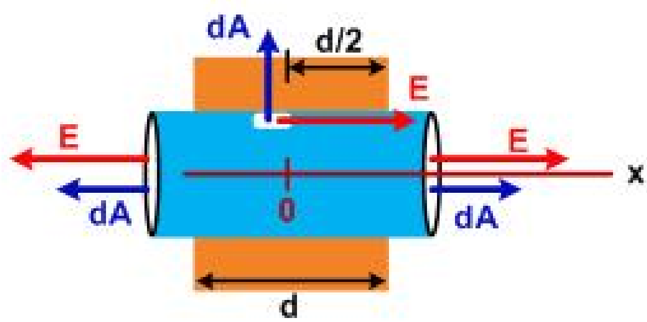


Figure 23-51 Problem 43.





(43) $\Phi = \oint \vec{E}\cdot d\vec{A} = q_{enc}/\varepsilon_0$; $\rho(A)(2x)$

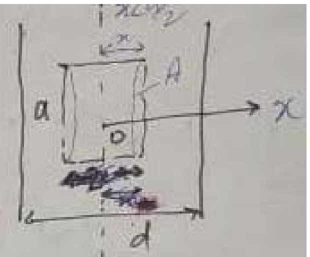
a) $0 + EA + EA = 2\rho A x/\varepsilon_0 \Rightarrow 2EA = 2\rho A x/\varepsilon_0$

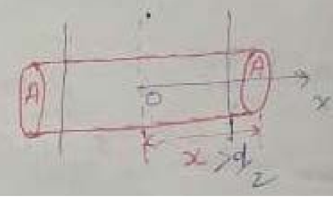
$E = \dfrac{\rho x}{\varepsilon_0}$; for $x < d/2$

$\rho(A)(d)$

b) $EA + EA = q_{enc}/\varepsilon_0 = \rho A d/\varepsilon_0$; $x > d/2$

$2EA = \rho A d/\varepsilon_0 \Rightarrow E = \dfrac{\rho d}{2\varepsilon_0}$





••49 In Fig. 23-54, a solid sphere of radius $a = 2.00$ cm is concentric with a spherical conducting shell of inner radius $b = 2.00a$ and outer radius $c = 2.40a$. The sphere has a net uniform charge $q_1 = +5.00$ fC; the shell has a net charge $q_2 = -q_1$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = a/2.00$, (c) $r = a$, (d) $r = 1.50a$, (e) $r = 2.30a$, and (f) $r = 3.50a$? What is the net charge on the (g) inner and (h) outer surface of the shell?

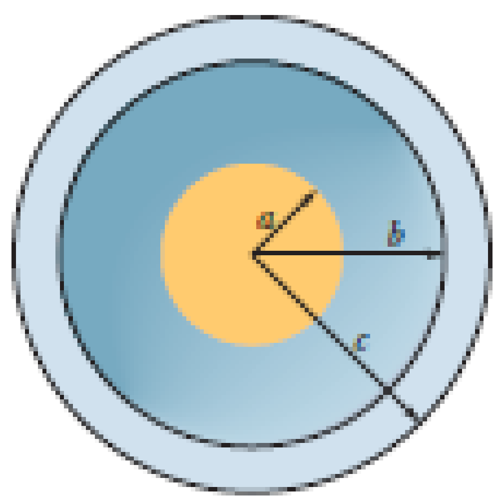


Figure 23-54 Problem 49.

(51) Consider Spherical Gaussian surface of radius ar.

for $r < a$; $q_{enc} = q_1\left(\frac{r}{a}\right)^3 = \frac{q_1 r^3}{a^3}$

$$4\pi r^2 E = \frac{q_1 r^3}{\varepsilon_0 a^3} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 r}{a^3}$$

a) $r = 0$; $E = 0$

b) $r = a/2$; $E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 a}{2a^3} = \frac{q_1}{8\pi\varepsilon_0 a^2}$

c) $r = a$; $E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a^2}$

$E(4\pi a^2) = \frac{q_1}{\varepsilon_0}$

$E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a^2}$

$E(4\pi r^2) = \frac{q_1}{\varepsilon_0}$

$E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$

d) $r = 1.5a$; $a < r < b$

$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$$

e) $b < r < c$; within shell, since shell is conducting, the electric field is zero.

$r = 2.3a$; $E = 0$

f) $r = 3.50a$; $c < r$, the charge enclosed by Gaussian surface is zero. Gauss' Law yields

$$4\pi r^2 E = 0 \Rightarrow E = 0$$

g) Consider Gaussian surface within shell. $E = 0$, $q_{enc} = 0$

$$q_1 + Q_{in} = 0 \Rightarrow Q_{in} = -q_1$$

h) $Q_{in} + Q_{out} = q_2 = -q_1$

$$Q_{out} = -q_1 - Q_{in} = -q_1 - (-q_1) = q_1 + q_1 = 0$$

Graph E versus r.

••54 Figure 23-58 shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centers of the spheres, at radial distance $R/2.00$ from the center of sphere 1. If the net electric field at point P is zero, what is the ratio q_2/q_1 of the total charges?

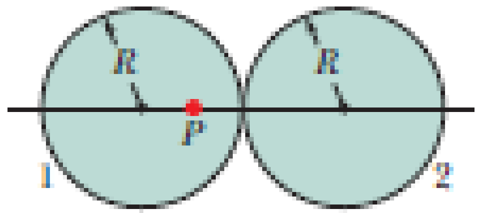
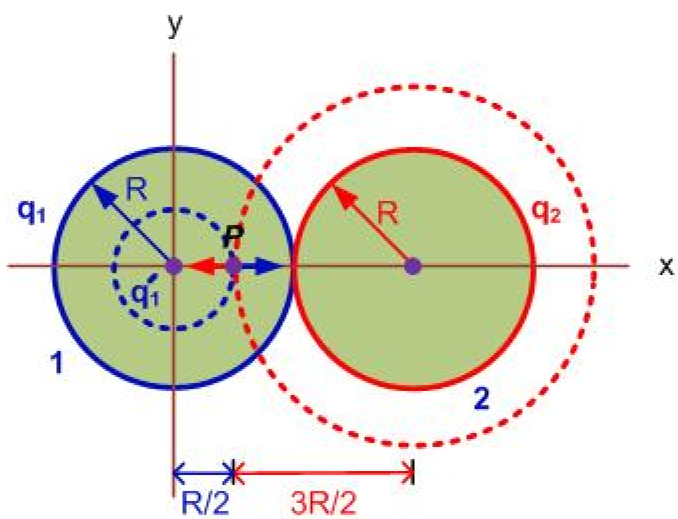


Figure 23-58 Problem 54.



$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left[\frac{3R}{2}\right]^2} = \frac{1}{9\pi\varepsilon_0}\frac{q_2}{R^2} \qquad \vec{E}_1 = \left(\frac{q_1}{4\pi\varepsilon_0 R^3}\right)\frac{R}{2} = \frac{1}{8\pi\varepsilon_0}\frac{q_1}{R^2}$$

$E_1 = E_2$

$$\frac{1}{9\pi\varepsilon_0}\frac{q_2}{R^2} = \frac{1}{8\pi\varepsilon_0}\frac{q_1}{R^2}$$

$$\frac{q_2}{q_1} = \frac{9}{8}$$

$x > d/2$

$$\oint \vec{E}\cdot d\vec{A} = \int_{ends} \vec{E}\cdot d\vec{A} + \int_{sides} \vec{E}\cdot d\vec{A} = \int_{ends} 0 + \int_{sides} E(r)\, dA = E(r)\int_{sides} dA$$

$$= E(r)\,2\pi r l$$

$$\oint \vec{E}\cdot d\vec{A} = Q_{enc}/\epsilon_0 \Rightarrow E(r)\,2\pi r l = \rho \pi r^2 l/\epsilon_0$$

$$E(r) = \frac{r\rho}{2\epsilon_0} \qquad 0 \le r \le a$$

$$\oint \vec{E}\cdot d\vec{A} = Q_{enc}/\epsilon_0$$

$$E(r)\,2\pi r l = \rho \pi a^2 l/\epsilon_0$$

$$E(r) = \frac{\rho a^2}{2\epsilon_0 r} \qquad r \geq a$$