Chapter 3

Geometric Design of Highways

3.1 INTRODUCTION

With the understanding of vehicle performance provided in Chapter 2, attention can now be directed toward highway design. The design of highways necessitates the determination of specific design elements, which include the number of lanes, lane width, median type (if any) and width, length of acceleration and deceleration lanes for on- and off-ramps, need for truck climbing lanes for steep grades, curve radii required for vehicle turning, and the alignment required to provide adequate stopping and passing sight distances. Many of these design elements are influenced by the performance characteristics of vehicles. For example, vehicle acceleration and deceleration characteristics have a direct impact on the design of acceleration and deceleration lanes (the length needed to provide a safe and orderly flow of traffic) and the highway alignment needed to provide adequate passing and stopping sight distances. Furthermore, vehicle performance characteristics determine the need for truck climbing lanes on steep grades (where the poor performance of large trucks necessitates a separate lane) as well as the number of lanes required because the observed spacing between vehicles in traffic is directly related to vehicle performance characteristics (this will be discussed further in Chapter 5). In addition, the physical dimensions of vehicles affect a number of design elements, such as the radii required for low-speed turning, height of highway overpasses, and lane widths.

When one considers the diversity of vehicles' performance and physical dimensions, and the interaction of these characteristics with the many elements constituting highway design, it is clear that proper design is a complex procedure that requires numerous compromises. Moreover, it is important that design guidelines evolve over time in response to changes in vehicle performance and dimensions, and in response to evidence collected on the effectiveness of existing highway design practices, such as the relationship between crash rates and various roadway design characteristics. Current guidelines of highway design are presented in detail in *A Policy on Geometric Design of Highways and Streets*, 6th Edition, published by the American Association of State Highway and Transportation Officials [AASHTO 2011].

Because of the sheer number of geometric elements involved in highway design, a detailed discussion of each design element is beyond the scope of this book, and the reader is referred to [AASHTO 2011] for a complete discussion of current design practices. Instead, this book focuses exclusively on the key elements of highway alignment, which are arguably the most important components of geometric design. As will be shown, the alignment topic is particularly well suited for demonstrating the effect of vehicle performance (specifically braking performance) and vehicle dimensions (such as driver's eye height, headlight height, and taillight height) on the design of highways. By concentrating on the specifics of the highway alignment problem, the reader will develop an understanding of the procedures and compromises inherent in the design of all highway-related geometric elements.

3.2 PRINCIPLES OF HIGHWAY ALIGNMENT

The alignment of a highway is a three-dimensional problem measured in x, y, and z coordinates. This is illustrated, from a driver's perspective, in Fig. 3.1. However, in highway design practice, three-dimensional design computations are cumbersome, and, what is perhaps more important, the actual implementation and construction of a design based on three-dimensional coordinates has historically been prohibitively difficult. As a consequence, the three-dimensional highway alignment problem is reduced to two two-dimensional alignment problems, as illustrated in Fig. 3.2. One of the alignment problems in this figure corresponds roughly to x and z coordinates and is referred to as horizontal alignment. The other corresponds to highway length (measured along some constant elevation) and y coordinates (elevation) and is referred to as vertical alignment. Referring to Fig. 3.2, note that the horizontal alignment of a highway is referred to as the plan view, which is roughly equivalent to the perspective of an aerial photo of the highway. The vertical alignment is represented in a profile view, which gives the elevation of all points measured along the length of the highway (again, with length measured along a constant elevation reference).

Aside from considering the alignment problem as two two-dimensional problems, one further simplification is made: instead of using x and z coordinates, highway positioning and length are defined as the distance along the highway (usually measured along the centerline of the highway, on a horizontal, constant-elevation plane) from a specified point. This distance is measured in terms of stations, with each station consisting of 100 ft of highway alignment distance.

The notation for stationing distance is such that a point on a highway 4250 ft from a specified point of origin is said to be at station 42 + 50 ft, that is, 42 stations and 50 ft, with the point of origin being at station 0 + 00. This stationing concept,

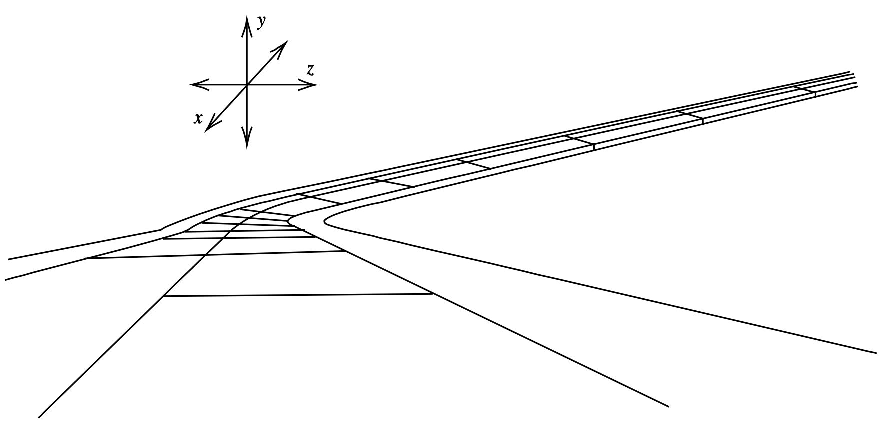

Figure 3.1 Highway alignment in three dimensions.

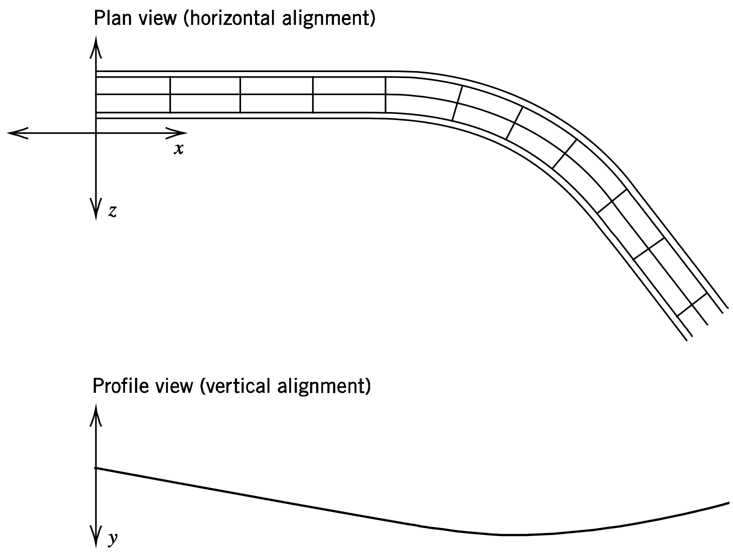

Figure 3.2 Highway alignment in two-dimensional views.

combined with the highway's alignment direction given in the plan view (horizontal alignment) and the elevation corresponding to stations given in the profile view (vertical alignment), gives a unique identification of all highway points in a manner that is virtually equivalent to using true x, y, and z coordinates.

3.3 VERTICAL ALIGNMENT

Vertical alignment specifies the elevation of points along a roadway. The elevation of these roadway points is usually determined by the need to provide an acceptable level of driver safety, driver comfort, and proper drainage (from rainfall runoff). A primary concern in vertical alignment is establishing the transition of roadway elevations between two grades. This transition is achieved by means of a vertical curve.

Vertical curves can be broadly classified into crest vertical curves and sag vertical curves, as illustrated in Fig. 3.3. Note that in Fig. 3.3, the distance from the *PVC* to the *PVI* is $L/2$. This is used in this figure because in practice the vast majority of vertical curves are arranged such that half of the curve length is positioned before the *PVI* and half after. Curves that satisfy this criterion are called equal-tangent vertical curves.

For referencing points on a vertical curve, it is important to note that the profile views presented in Fig. 3.3 correspond to all highway points even if a horizontal curve occurs concurrently with a vertical curve (as in Figs. 3.1 and 3.2). Thus, each roadway point is uniquely defined by stationing (which is measured along a horizontal plane) and elevation. This will be made clearer through forthcoming examples.

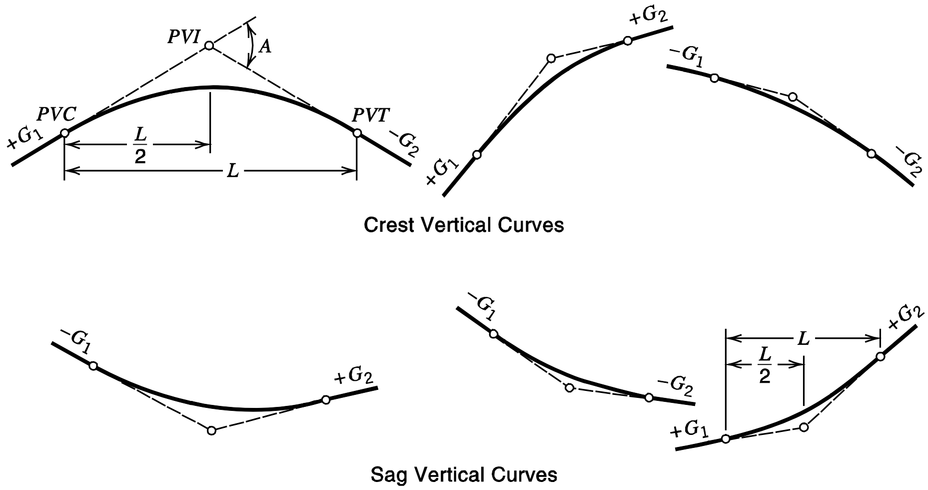

Figure 3.3 Types of vertical curves.

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G_1 = initial roadway grade in percent or ft/ft (this grade is also referred to as the initial tangent grade, viewing Fig. 3.3 from left to right),

G_2 = final roadway (tangent) grade in percent or ft/ft,

A = absolute value of the difference in grades (initial minus final, usually expressed in percent),

L = length of the curve in stations or ft measured in a constant-elevation horizontal plane.

PVC = point of the vertical curve (the initial point of the curve),

PVI = point of vertical intersection (intersection of initial and final grades), and

PVT = point of vertical tangent, which is the final point of the vertical curve (the point where the curve returns to the final grade or, equivalently, the final tangent).

3.3.1 Vertical Curve Fundamentals

In connecting roadway grades (tangents) with an appropriate vertical curve, a mathematical relationship defining elevations at all points (or equivalently, stations) along the vertical curve is needed. A parabolic function has been found suitable in this regard because, among other things, it provides a constant rate of change of slope and implies equal curve tangents. The general form of the parabolic equation, as applied to vertical curves, is

$$y = ax^2 + bx + c \tag{3.1}$$

where

y = roadway elevation at distance x from the beginning of the vertical curve (the PVC) in stations or ft,

x = distance from the beginning of the vertical curve in stations or ft,

a, b = coefficients defined below, and

c = elevation of the PVC (because $x = 0$ corresponds to the PVC) in ft.

In defining a and b, note that the first derivative of Eq. 3.1 gives the slope and is

$$\frac{dy}{dx} = 2ax + b \tag{3.2}$$

At the *PVC*, $x = 0$, so, using Eq. 3.2,

$$b = \frac{dy}{dx} = G_1 \tag{3.3}$$

where G_1 is the initial slope in ft/ft, as defined in Fig. 3.3. Also note that the second derivative of Eq. 3.1 is the rate of change of slope and is

$$\frac{d^2y}{dx^2} = 2a \tag{3.4}$$

However, the average rate of change of slope, by observation of Fig. 3.3, can also be written as

$$\frac{d^2y}{dx^2} = \frac{G_2 - G_1}{L} \tag{3.5}$$

Equating Eqs. 3.4 and 3.5 gives

$$a = \frac{G_2 - G_1}{2L} \tag{3.6}$$

with all terms as defined previously (see Fig. 3.3). Please note that the units for coefficients a and b in Eqs. 3.3 and 3.6 must be such that they provide ft when multiplied by x^2 and x, respectively. The preceding equations define all the terms in the parabolic vertical curve equation (Eq. 3.1). The following example gives a typical application of this equation.

EXAMPLE 3.1 VERTICAL CURVE STATIONS AND ELEVATIONS

A 600-ft equal-tangent sag vertical curve has the *PVC* at station 170 + 00 and elevation 1000 ft. The initial grade is −3.5% and the final grade is +0.5%. Determine the stationing and elevation of the *PVI*, the *PVT*, and the lowest point on the curve.

SOLUTION

Since the curve is equal tangent, the *PVI* will be 300 ft or three stations (measured in a horizontal plane) from the *PVC*, and the *PVT* will be 600 ft or six stations from the *PVC*. Therefore, the stationing of the *PVI* and *PVT* is 173 + 00 and 176 + 00, respectively. For the elevations of the *PVI* and *PVT*, it is known that a −3.5% grade can be equivalently written as −3.5 ft/station (a 3.5 ft drop per 100 ft of horizontal distance). Since the *PVI* is three stations from the *PVC*, which is known to be at elevation 1000 ft, the elevation of the *PVI* is

$$1000 - 3.5 \text{ ft/station} \times (3 \text{ stations}) = 989.5 \text{ ft}$$

Similarly, with the *PVI* at elevation 989.5 ft, the elevation of the *PVT* is

$$989.5 + 0.5 \text{ ft/station} \times (3 \text{ stations}) = \underline{\underline{991.0 \text{ ft}}}$$

It is clear from the values of the initial and final grades that the lowest point on the vertical curve will occur when the first derivative of the parabolic function (Eq. 3.1) is zero because the initial and final grades are opposite in sign. When initial and final grades are not opposite in sign, the low (or high) point on the curve will not be where the first derivative is zero because the slope along the curve will never be zero. For example, a sag curve with an initial grade of −2.0% and a final grade of −2.0% will have its lowest elevation at the *PVT*, and the first derivative of Eq. 3.1 will not be zero at any point along the curve. However, in our example problem the derivative will be equal to zero at some point, so the low point will occur when

$$\frac{dy}{dx} = 2ax + b = 0$$

From Eq. 3.3 we have

$$b = G_1 = -3.5$$

with G_1 in percent. From Eq. 3.6 (with L in stations and G_1 and G_2 in percent),

$$a = \frac{0.5 - (-3.5)}{2(6)} = 0.33333$$

Substituting for *a* and *b* gives

$$\frac{dy}{dx} = 2(0.33333)x + (-3.5) = 0$$
$$x = 5.25 \text{ stations}$$

This gives the stationing of the low point at $\underline{\underline{175 + 25}}$ (5 + 25 stations from the *PVC*). For the elevation of the lowest point on the vertical curve, the values of *a*, *b*, *c* (elevation of the *PVC*), and *x* are substituted into Eq. 3.1, giving

$$\begin{aligned} y &= 0.33333(5.25)^2 + (-3.5)(5.25) + 1000 \\ &= 990.81 \text{ ft} \end{aligned}$$

Note that the preceding equations can also be solved with grades expressed as the decimal equivalent of percent (for example, 0.02 ft/ft for 2%) if *x* is expressed in feet instead of stations. Care must be taken not to mix units. A dimensional analysis of Eq. 3.1 must ensure that each right-side element of the equation has resulting units of feet.

Another interesting vertical curve problem that is sometimes encountered is one in which the curve must be designed so that the elevation of a specific location is met. An example might be to have the roadway connect with another (at the same elevation) or to have the roadway at some specified elevation so as to pass under another roadway. This type of problem is referred to as a curve-through-a-point problem and is demonstrated by the following example.

EXAMPLE 3.2 ELEMENTS OF VERTICAL CURVE DESIGN

An equal-tangent vertical curve is to be constructed between grades of −2.0% (initial) and +1.0% (final). The *PVI* is at station 110 + 00 and at elevation 420 ft. Due to a street crossing the roadway, the elevation of the roadway at station 112 + 00 must be at 424.5 ft. Design the curve.

SOLUTION

The design problem is one of determining the length of the curve required to ensure that station 112 + 00 is at elevation 424.5 ft. To begin, we use Eq. 3.1:

$$y = ax^2 + bx + c$$

From Eq. 3.3,

$$b = G_1 = -2.0$$

and from Eq. 3.6,

$$a = \frac{G_2 - G_1}{2L}$$

Substituting $G_1 = -2.0$ and $G_2 = 1.0$, we have

$$a = \frac{G_2 - G_1}{2L} = \frac{1.0 - (-2.0)}{2L} = \frac{1.5}{L}$$

Now note that c (the elevation of the *PVC*) in Eq. 3.1 will be equal to the elevation of the *PVI* plus $G_1 \times 0.5L$ (this is simply using the slope of the initial grade to determine the elevation difference between the *PVI* and *PVC*). With G_1 in percent (which is ft/station) and the curve length L in stations, we have

$$c = 420 + 2.0(0.5L) = 420 + L$$

Finally, the value of x to be used in Eq. 3.1 will be $0.5L + 2$ because the point of interest (station 112 + 00) is two stations from the *PVI* (which is at station 110 + 00). Substituting $b = -2.0$, the expressions for a, c, and x, and $y = 424.5$ ft (the given elevation) into Eq. 3.1 gives

$$424.5 = (1.5/L)(0.5L + 2)^2 + (-2.0)(0.5L + 2) + (420 + L)$$
$$4.5 = 0.375L + 3 + 6/L - 4$$
$$0 = -0.375\,L^2 + 5.5L - 6$$

Solving this quadratic equation gives $L = 1.187$ stations (which is not feasible because we know that the point of interest is 2.00 stations beyond the *PVI*, so the curve must be longer than 1.187 stations) or $L = 13.466$ stations (which is the only feasible solution). This means that the curve must be 1346.6 ft long. Using this value of L,

$$\text{elevation of } PVC = c = 420 + L = 420 + 13.466 + 433.47 \text{ ft}$$
$$\text{station of } PVC = 110 + 00 - (13 + 46.6)/2 = 103 + 26.7$$

$$\text{elevation of } PVT = \text{elevation of } PVI + (0.5L)G_2 = 420 + [0.5(13.466)](1.0) = 426.73 \text{ ft}$$

$$\text{station of } PVT = 110 + 00 + (13 + 46.6)/2 = 116 + 73.3$$

and

$$x = 0.5L + 2.0 = 6.733 + 2.0 = 8.733 \text{ stations from the } PVC$$

To check the elevation of the curve at station 112 + 00, we apply Eq. 3.1 with $x = 8.733$:

$$\begin{aligned} y &= ax^2 + bx + c \\ &= \left(\frac{3}{2(13.466)}\right)(8.733)^2 + (-2.0)(8.733) + 433.47 \\ &= 424.5 \text{ ft} \end{aligned}$$

Therefore, all calculations are correct.

Some additional properties of vertical curves can now be formalized. For example, offsets, which are vertical distances from the initial tangent to the curve, as illustrated in Fig. 3.4, are extremely important in vertical curve design and construction.

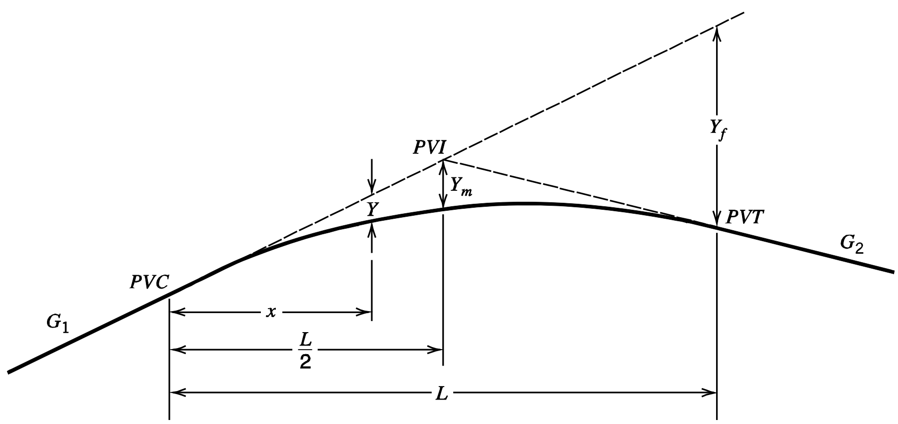

Figure 3.4 Offsets for equal-tangent vertical curves.

G_1 = initial roadway grade in percent or ft/ft (this grade is also referred to as the initial tangent grade, viewing Fig. 3.4 from left to right),

G_2 = final roadway (tangent) grade in percent or ft/ft,

Y = offset at any distance x from the PVC in ft,

Y_m = midcurve offset in ft,

Y_f = offset at the end of the vertical curve in ft,

x = distance from the PVC in ft,

L = length of the curve in stations or ft measured in a constant-elevation horizontal plane,

PVC = point of the vertical curve (the initial point of the curve),

PVI = point of vertical intersection (intersection of initial and final grades), and

PVT = point of vertical tangent, which is the final point of the vertical curve (the point where the curve returns to the final grade or, equivalently, the final tangent).

Referring to the elements shown in Fig. 3.4, the properties of an equal-tangent parabola can be used to give

$$Y = \frac{A}{200L}x^2 \qquad \textbf{(3.7)}$$

where

A = absolute value of the difference in grades ($|G_1 - G_2|$) expressed in percent, and
Other terms are as defined in Fig. 3.4.

Note that in this equation, 200 is used in the denominator instead of 2 because A is expressed in percent instead of ft/ft (this division by 100 also applies to Eqs. 3.8 and 3.9 below). It follows from Fig. 3.4 that

$$Y_m = \frac{AL}{800} \qquad \textbf{(3.8)}$$

and

$$Y_f = \frac{AL}{200} \qquad \textbf{(3.9)}$$

Another useful vertical curve property is one that gives the length of curve required to effect a 1% change in slope. Because the parabolic equation used for roadway elevations (Eq. 3.1) gives a constant rate of change of slope, it can be shown that the horizontal distance required to change the slope by 1% is

$$K = \frac{L}{A} \qquad \textbf{(3.10)}$$

where

K = value that is the horizontal distance, in ft, required to affect a 1% change in the slope of the vertical curve,
L = length of curve in ft, and
A = absolute value of the difference in grades ($|G_1 - G_2|$) expressed as a percentage.

This K-value can also be used to compute the high and low point locations of crest and sag vertical curves, respectively (provided the high or low point does not occur at the *PVC* or *PVT*). As shown in Example 3.1, setting $dy/dx = 0$ in Eq. 3.2 and solving for x gives the distance from the *PVC* to the high/low point. If Eq. 3.6 is used to substitute for a in Eq. 3.2 (with $L = KA$), it can be shown that setting $dy/dx = 0$ in Eq. 3.2 gives

$$x_{hl} = K \times |G_1| \qquad \textbf{(3.11)}$$

where

x_{hl} = distance from the *PVC* to the high/low point in ft, and
Other terms are as defined previously.

In addition to high/low point computations, *K*-values have an important application in the design of vertical curves, as will be demonstrated in Sections 3.3.3 and 3.3.4.

EXAMPLE 3.3 VERTICAL CURVE DESIGN WITH *K*-VALUES

A curve has initial and final grades of +3% and −4%, respectively, and is 700 ft long. The *PVC* is at elevation 100 ft. Graph the vertical curve elevations and the slope of the curve against the length of curve. Compute the *K*-value and use it to locate the high point of the curve (distance from the *PVC*).

SOLUTION

Recall that to find the slope at any point on the curve, we take the derivative of Eq. 3.1, which gives Eq. 3.2. To apply this equation, *a* and *b* need to be determined. From Eq. 3.6,

$$a = \frac{-4.0 - 3.0}{2(7)} = -0.5$$

and from Eq. 3.3,

$$b = G_1 = 3$$

The results of applying Eq. 3.2 and solving for the slope at all points along the curve, as well as a profile view of the curve itself (by application of Eq. 3.1), are shown graphically in Fig. 3.5 (exaggerating the vertical scale). Figure 3.5 shows the constant rate of change of the slope along the length of the curve. The circular points on the slope-of-curve line correspond to changes in grade of 1%, and these points occur at equal intervals of 100 ft.

To show that this is consistent with the *K*-value, Eq. 3.10 gives

$$K = \frac{L}{A} = \frac{700}{|3-(-4)|} = 100 \text{ ft}$$

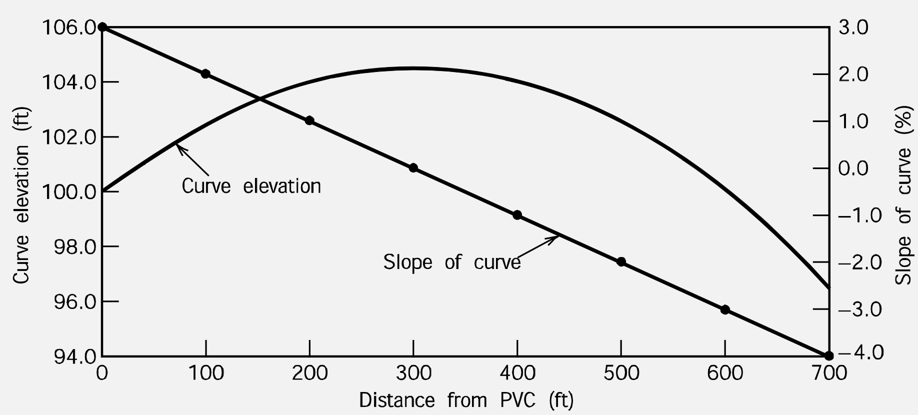

Figure 3.5 Profile view of vertical curve for Example 3.3 with the graph of the slope at all points along the curve overlaid.

This indicates that there should be a change in grade of 1% for every 100 ft of curve length (measured in the horizontal plane), and this is consistent with Fig. 3.5. Applying Eq. 3.11 with the K-value of 100 ft gives the high point at 300 ft from the beginning of the curve ($x_{hl} = 100 \times 3 = 300$ ft). This is shown in Fig. 3.5, where the slope of the curve at 300 ft is zero (the same result obtained by setting the derivative of Eq. 3.2 equal to zero and solving for x). This result can also be explained conceptually based on the definition of the K-value. The K-value gives the horizontal distance required to effect a 1% change in the slope of the curve, and for this curve that value is 100 ft. Thus, to go from an initial grade (G_1) of 3% to a grade of 0% (the high point) requires a horizontal distance equal to $K \times 3$, or 300 ft.

EXAMPLE 3.4 VERTICAL CURVE DESIGN USING OFFSETS

A vertical curve crosses a 4-ft diameter pipe at right angles. The pipe is located at station 110 + 85 and its centerline is at elevation 1091.60 ft. The *PVI* of the vertical curve is at station 110 + 00 and elevation 1098.4 ft. The vertical curve is equal tangent, 600 ft long, and connects an initial grade of +1.20% and a final grade of −1.08%. Using offsets, determine the depth, below the surface of the curve, of the top of the pipe and determine the station of the highest point on the curve.

SOLUTION

The *PVC* is at station 107 + 00 (110 + 00 minus 3 + 00, which is half of the curve length), so the pipe is 385 ft (110 + 85 minus 107 + 00) from the beginning of the curve (*PVC*). The elevation of the *PVC* will be the elevation of the *PVI* minus the drop in grade over one-half the curve length,

$$1098.4 - (3 \text{ stations} \times 1.2 \text{ ft/station}) = 1094.8 \text{ ft}$$

Using this, the elevation of the initial tangent above the pipe is

$$1094.8 + (3.85 \text{ stations} \times 1.2 \text{ ft/station}) = 1099.42 \text{ ft}$$

Using Eq. 3.7 to determine the offset above the pipe at $x = 385$ ft (the distance of the pipe from the *PVC*), we have

$$Y = \frac{A}{200L} x^2$$

$$Y = \frac{|1.2 - (-1.08)|}{200(600)} (385)^2 = 2.82 \text{ ft}$$

Thus the elevation of the curve above the pipe is 1096.6 ft (1099.42 − 2.82). The elevation of the top of the pipe is 1093.60 ft (elevation of the centerline plus one-half of the pipe's diameter), so the pipe is 3.0 ft below the surface of the curve (1096.6 − 1093.6).

To determine the location of the highest point on the curve, we find K from Eq. 3.10 as

$$K = \frac{600}{|1.2 - (-1.08)|} = 263.16$$

and the distance from the *PVC* to the highest point is (from Eq. 3.11)

$$x_{hl} = K \times |G_1| = 263.16 \times 1.2 = 315.79 \text{ ft}$$

This gives the station of the highest point at 110 + 15.79 (107 + 00 plus 3 + 15.79). Note that this example could also be solved by applying Eq. 3.1, setting Eq. 3.2 equal to zero (for determining the location of the highest point on the curve), and following the procedure used in Example 3.1.

3.3.2 Stopping Sight Distance

Construction of a vertical curve is generally a costly operation requiring the movement of significant amounts of earthen material. Thus one of the primary challenges facing highway designers is to minimize construction costs (usually by making the vertical curve as short as possible) while still providing an adequate level of safety. An appropriate level of safety is usually defined as that level of safety that gives drivers sufficient sight distance to allow them to safely stop their vehicles to avoid collisions with objects obstructing their forward motion. The provision of adequate roadway drainage is sometimes an important concern as well, but is not discussed in terms of vertical curves in this book (see [AASHTO 2011]). Referring back to the vehicle braking performance concepts discussed in Chapter 2, we can compute this necessary stopping sight distance (SSD) as the summation of vehicle practical stopping distance (Eq. 2.47) and the distance traveled during driver perception/reaction time (Eq. 2.49). That is,

$$\text{SSD} = \frac{V_1^2}{2g\left(\left(\frac{a}{g}\right) \pm G\right)} + V_1 \times t_r \qquad \textbf{(3.12)}$$

where

SSD = stopping sight distance in ft,
V_1 = initial vehicle speed in ft/s,
g = gravitational constant, 32.2 ft/s^2,
a = deceleration rate in ft/s^2,
G = roadway grade (+ for uphill and − for downhill) in percent/100, and
t_r = perception/reaction time in s.

Recall from Sections 2.9.5 and 2.9.6 that a value of 11.2 ft/s^2 for a and a value of 2.5 s for t_r were recommended for roadway design purposes. The design speed of the highway is defined as the maximum safe speed at which a highway can be negotiated assuming near-worst-case conditions (wet-weather conditions). The application of Eq. 3.12 (assuming G = 0) produces the stopping sight distances presented in Table 3.1.

Table 3.1 Stopping Sight Distance

Design speed (mi/h)	Brake reaction distance (ft)	Braking distance on level (ft)	Stopping sight distance	
			Calculated (ft)	Design (ft)
15	55.1	21.6	76.7	80
20	73.5	38.4	111.9	115
25	91.9	60.0	151.9	155
30	110.3	86.4	196.7	200
35	128.6	117.6	246.2	250
40	147.0	153.6	300.6	305
45	165.4	194.4	359.8	360
50	183.8	240.0	423.8	425
55	202.1	290.3	492.4	495
60	220.5	345.5	566.0	570
65	238.9	405.5	644.4	645
70	257.3	470.3	727.6	730
75	275.6	539.9	815.5	820
80	294.0	614.3	908.3	910

Note: Brake reaction distance is based on a time of 2.5 s; a deceleration rate of 11.2 ft/s^2 is used to determine calculated stopping sight distance.

Source: American Association of State Highway and Transportation Officials, *A Policy on Geometric Design of Highways and Streets*, 6th Edition, Washington, DC, 2011. Used by permission.

3.3.3 Stopping Sight Distance and Crest Vertical Curve Design

The length of curve (L in Fig. 3.3) is the critical element in providing sufficient SSD on a vertical curve. Longer curve lengths provide more SSD, all else being equal, but are more costly to construct. Shorter curve lengths are less expensive to construct but may not provide adequate SSD due to more rapid changes in slope. What is needed, then, is an expression for minimum curve length given a required SSD. In developing such an expression, crest and sag vertical curves are considered separately.

The case of designing a crest vertical curve for adequate stopping sight distance is illustrated in Fig. 3.6. To determine the minimum length of curve for a required sight distance, the properties of a parabola for an equal tangent curve can be used to show that

For $S < L$

$$L_m = \frac{AS^2}{200\left(\sqrt{H_1} + \sqrt{H_2}\right)^2} \tag{3.13}$$

For $S > L$

$$L_m = 2S - \frac{200\left(\sqrt{H_1} + \sqrt{H_2}\right)^2}{A} \tag{3.14}$$

where

L_m = minimum length of vertical curve in ft,

A = absolute value of the difference in grades ($|G_1 - G_2|$), expressed as a percentage, and

Other terms are as defined in Fig. 3.6.

For the sight distance required to provide adequate SSD, current AASHTO design guidelines [2011] use a driver eye height, H_1, of 3.5 ft and a roadway object height, H_2, of 2.0 ft (the height of an object to be avoided by stopping before a collision). In applying Eqs. 3.13 and 3.14 to determine the minimum length of curve required to provide adequate SSD, we set the sight distance, S, equal to the stopping sight distance, SSD (note that the relatively small distance from the driver's eye position to the front of the vehicle is ignored). Substituting AASHTO guidelines for H_1 and H_2 and letting S = SSD in Eqs. 3.13 and 3.14 gives

For SSD < L

$$L_m = \frac{A \times \text{SSD}^2}{2158} \tag{3.15}$$

For SSD > L

$$L_m = 2 \times \text{SSD} - \frac{2158}{A} \tag{3.16}$$

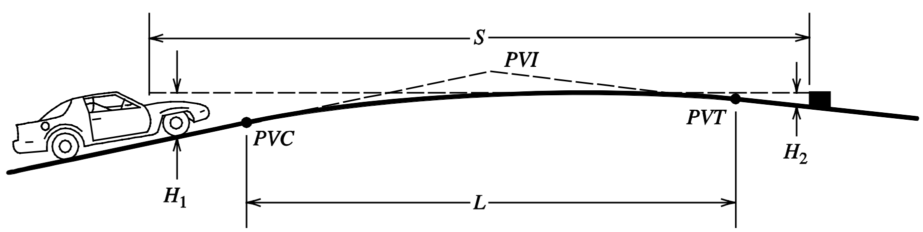

Figure 3.6 Stopping sight distance considerations for crest vertical curves.

S = sight distance in ft,

H_1 = height of driver's eye above roadway surface in ft,

H_2 = height of object above roadway surface in ft,

L = length of the curve in ft,

PVC = point of vertical curve (the initial point of the curve),

PVI = point of vertical intersection (intersection of initial and final grades), and

PVT = point of vertical tangent, which is the final point of the vertical curve (the point where the curve returns to the final grade or, equivalently, the final tangent).

EXAMPLE 3.5 DESIGN SPEED AND CREST VERTICAL CURVE DESIGN

A highway is being designed to AASHTO guidelines with a 70-mi/h design speed, and at one section, an equal-tangent vertical curve must be designed to connect grades of +1.0% and −2.0%. Determine the minimum length of curve necessary to meet SSD requirements.

SOLUTION

If we ignore the effect of grades (Gs = 0), the SSD can be read directly from Table 3.1. In this case, the SSD corresponding to a speed of 70 mi/h is 730 ft. If we assume that L > SSD (an assumption that is typically made), Eq. 3.15 gives

$$L_m = \frac{A \times SSD^2}{2158} = \frac{3 \times 730^2}{2158} = \underline{\underline{740.82 \text{ ft}}}$$

Since 740.82 > 730, the assumption that L > SSD was correct.

The assumption that $G = 0$, made at the beginning of Example 3.5, is not really correct. If $G \neq 0$, we cannot use the SSD values in Table 3.1 and instead must apply Eq. 3.12 with the appropriate G value. In this problem, if we use the initial grade in Eq. 3.12 (+1.0%), we will underestimate the stopping sight distance because the vertical curve has a slope as steeply positive as this only at the *PVC*. If we use the final grade in Eq. 3.12 (−2.0%), we will overestimate the stopping sight distance because the vertical curve has a slope as steeply negative as this only at the *PVT*. If we knew where the vehicle began to brake, we could use the first derivative of the parabolic curve function (from Eq. 3.2) to give G in Eq. 3.12 and set up the equation to solve for SSD exactly. In practice, policies vary as to how this grade issue is handled. Fortunately, because sight distance tends to be greater on downgrades (which require longer stopping distances) than on upgrades, a self-correction for the effect of grades is generally provided. As a consequence, some design agencies ignore the effect of grades completely, while others assume G is equal to zero for grades less than 3% and use simple adjustments to the SSD, depending on the initial and final grades, for grades of 3% or more. For the remainder of this chapter, we will ignore the effect of grades (G = 0 will be used in Eq. 3.12). However, it must be pointed out that the use of SSD grade corrections is very easy and straightforward, and all of the equations presented herein still apply.

The use of Eqs. 3.15 and 3.16 can be simplified if the initial assumption that L > SSD is made, in which case Eq. 3.15 is always used. The advantage of this assumption is that the relationship between A and L_m is linear, and Eq. 3.10 can be used to give

$$L_m = KA \tag{3.17}$$

where K = horizontal distance, in ft, required to effect a 1% change in the slope (as in Eq. 3.10), defined as

$$K = \frac{SSD^2}{2158} \tag{3.18}$$

With known SSD for a given design speed (assuming $G = 0$), K-values can be computed for crest vertical curves as shown in Table 3.2. Thus the minimum curve length can be obtained (as shown in Eq. 3.17) simply by multiplying A by the K-value read from Table 3.2.

Some discussion about the assumption that L > SSD is warranted. This assumption is made because there are two complications that could arise when SSD > L. First, if SSD > L the relationship between A and L_m is not linear, so K-values cannot be used in the $L = KA$ formula (Eq. 3.10). Second, at low values of A, it is possible to get negative minimum curve lengths (see Eq. 3.16). As a result of these complications, the assumption that L > SSD is almost always made in practice, and Eqs. 3.17 and 3.18 and the K-values presented in Table 3.2 are used. It is important to note that the assumption that L > SSD (upon which Eqs. 3.17 and 3.18 are based) is a good one because in many cases, L is greater than SSD, and when it is not (SSD > L), use of the L > SSD formula (Eq. 3.15 instead of Eq. 3.16) gives longer curve lengths and thus the error is on the conservative, safe side.

A final point relates to the smallest allowable length of curve. Very short vertical curves can be difficult to construct and may not be warranted for safety purposes. As a result, it is common practice to set minimum curve length limits that range from 100 to 325 ft depending on individual jurisdictional guidelines. A common alternative to these limits is to set the minimum curve length limit at three times the design speed (with speed in mi/h and length in ft) [AASHTO 2011].

Table 3.2 Design Controls for Crest Vertical Curves Based on Stopping Sight Distance

Design speed (mi/h)	Stopping sight distance (ft)	Rate of vertical curvature, K^*	
		Calculated	Design
15	80	3.0	3
20	115	6.1	7
25	155	11.1	12
30	200	18.5	19
35	250	29.0	29
40	305	43.1	44
45	360	60.1	61
50	425	83.7	84
55	495	113.5	114
60	570	150.6	151
65	645	192.8	193
70	730	246.9	247
75	820	311.6	312
80	910	383.7	384

*Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A): $K = L/A$.

Source: American Association of State Highway and Transportation Officials, *A Policy on Geometric Design of Highways and Streets*, 6th Edition, Washington, DC, 2011. Used by permission.

EXAMPLE 3.6 DESIGN SPEED AND CREST VERTICAL CURVE DESIGN WITH *K*-VALUES

Solve Example 3.5 using the *K*-values in Table 3.2.

SOLUTION

From Example 3.5, $A = 3$. For a 70-mi/h design speed, $K = 247$ (from Table 3.2). Therefore, application of Eq. 3.17 gives

$$L_m = KA = 247(3) = \underline{\underline{741.00\ \text{ft}}}$$

which is almost identical to the 740.82 ft obtained in Example 3.5. This difference is due to rounding. In this example the rounded *K* of 247 was used as opposed to the calculated *K* of 246.9. The rounded values are typically used in design for computational convenience. Note, however, that fractional calculated values are always rounded up to the nearest integer value, to be conservative.

EXAMPLE 3.7 STOPPING-SIGHT DISTANCE AND CREST VERTICAL CURVE DESIGN

If the grades in Example 3.5 intersect at station 100 + 00, determine the stationing of the *PVC*, *PVT*, and curve high point for the minimum curve length based on SSD requirements.

SOLUTION

Using the curve length from Example 3.6, $L = 741$ ft. Since the curve is equal tangent (as are virtually all curves used in practice), one-half of the curve will occur before the *PVI* and one-half after, so that

$$PVC \text{ is at } 100 + 00 - L/2 = 100 + 00 \text{ minus } 3 + 70.5 = \underline{\underline{96 + 29.5}}$$

$$PVT \text{ is at } 100 + 00 + L/2 = 100 + 00 \text{ plus } 3 + 70.5 = \underline{\underline{103 + 70.5}}$$

For the stationing of the high point, Eq. 3.11 is used:

$$x_{hl} = K \times |G_1| = 247(1) = 247\ \text{ft}$$

or

$$\text{station } 96 + 29.5 \text{ plus } 2 + 47 = \underline{\underline{98 + 76.5}}$$

3.3.4 Stopping Sight Distance and Sag Vertical Curve Design

Sag vertical curve design differs from crest vertical curve design in the sense that sight distance is governed by nighttime conditions because in daylight, sight distance on a sag vertical curve is unrestricted. Thus the critical concern for sag vertical curve design is the length of roadway illuminated by the vehicle headlights, which is a function of the height of the headlight above the roadway and the inclined angle of the headlight beam, relative to the horizontal plane of the car. The sag vertical curve sight distance design problem is illustrated in Fig. 3.7.

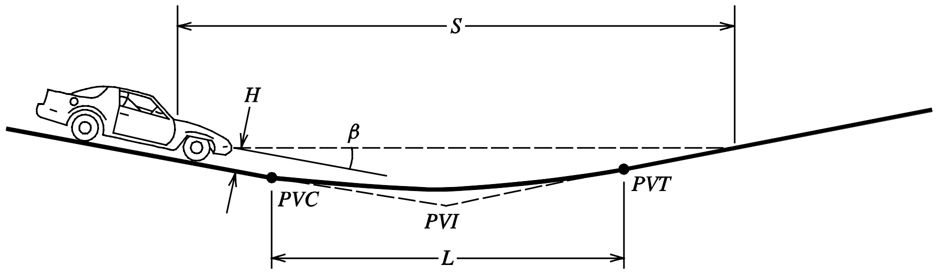

Figure 3.7 Stopping sight distance considerations for sag vertical curves.

S = sight distance in ft,
H = height of headlight in ft,
β = inclined angle of headlight beam in degrees,
L = length of the curve in ft
PVC = point of the vertical curve (the initial point of the curve)
PVI = point of vertical intersection (intersection of initial and final grades), and
PVT = point of vertical tangent, which is the final point of the vertical curve (the point where the curve returns to the final grade or, equivalently, the final tangent).

To determine the minimum length of curve for a required sight distance, the properties of a parabola for an equal-tangent curve can be used to show that

For $S < L$

$$L_m = \frac{AS^2}{200(H + S\tan\beta)} \tag{3.19}$$

For $S > L$

$$L_m = 2S - \frac{200(H + S\tan\beta)}{A} \tag{3.20}$$

where

L_m = minimum length of vertical curve in ft,
A = absolute value of the difference in grades ($|G_1 - G_2|$), expressed as a percentage, and
Other terms are as defined in Fig. 3.7.

For the sight distance required to provide adequate SSD, current AASHTO design guidelines [2011] use a headlight height of 2.0 ft and an upward angle of one degree. Substituting these design guidelines and S = SSD (as was done in the crest vertical curve case, and again ignoring the relatively small distance from the driver's eye position to the front of the vehicle) into Eqs. 3.19 and 3.20 gives

For SSD < L

$$L_m = \frac{A \times \text{SSD}^2}{400 + 3.5 \times \text{SSD}} \quad \textbf{(3.21)}$$

For SSD > L

$$L_m = 2 \times \text{SSD} - \frac{400 + 3.5 \times \text{SSD}}{A} \quad \textbf{(3.22)}$$

where

SSD = stopping sight distance in ft, and
Other terms are as defined previously.

As was the case for crest vertical curves, K-values can be computed by assuming L > SSD, which gives us the linear relationship between L_m and A as shown in Eq. 3.21. Thus for sag vertical curves (with $L_m = KA$),

$$K = \frac{\text{SSD}^2}{400 + 3.5\ \text{SSD}} \quad \textbf{(3.23)}$$

where

K = horizontal distance, in ft, required to effect a 1% change in the slope (as in Eq. 3.10), and
Other terms are as defined previously.

The K-values corresponding to design-speed-based SSDs are presented in Table 3.3. As was the case for crest vertical curves, some caution should be exercised in using this table because the assumption that G = 0 (for determining SSD) is used. Also, assume that L > SSD is a safe, conservative assumption (as was the case for crest vertical curves) and the smallest allowable curve lengths for sag curves are the same as those for crest curves (see discussion in Section 3.3.3).

EXAMPLE 3.8 SAG VERTICAL CURVE FUNDAMENTALS WITH DESIGN SPEED

An equal tangent sag vertical curve has an initial grade of –2.5%. It is known that the final grade is positive and that the low point is at elevation 270 ft and station 141+00. The PVT of the curve is at elevation 274 ft and the design speed of the curve is 35 mi/h. Determine the station and elevation of the PVC and PVI.

Table 3.3 Design Controls for Sag Vertical Curves Based on Stopping Sight Distance

Design speed (mi/h)	Stopping sight distance (ft)	Rate of vertical curvature, K*	
		Calculated	Design
15	80	9.4	10
20	115	16.5	17
25	155	25.5	26
30	200	36.4	37
35	250	49.0	49
40	305	63.4	64
45	360	78.1	79
50	425	95.7	96
55	495	114.9	115
60	570	135.7	136
65	645	156.5	157
70	730	180.3	181
75	820	205.6	206
80	910	231.0	231

*Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A): $K = L/A$.

Source: American Association of State Highway and Transportation Officials, *A Policy on Geometric Design of Highways and Streets*, 6th Edition, Washington, DC, 2011. Used by permission.

SOLUTION

From Table 3.3 it can be seen the K = 49 for a design speed of 35 mi/h. With this, equation 3.11 is used to find the distance of the low point from the *PVC*:

$$x_{hl} = K \times |G_1| = 49(2.5) = 122.5 \text{ ft}$$

Knowing the elevation of the low point (270 ft) and the distance of the low point from the *PVC* (122.5 ft), Eq. 3.1 can be applied to determine the elevation of the *PVC* (c in the Eq. 3.1):

$$y = ax^2 + bx + c$$

From Eq. 3.3,

$$b = G_1 = -2.5$$

and from Eq. 3.6,

$$a = \frac{G_2 - G_1}{2L}$$

Because the final grade is known to be positive (and with G_2 being negative),

$$G_2 - G_1 = |G_1 - G_2| = A,$$

Using $L = KA$ from Eq. 3.17, Eq. 3.6 becomes

$$a = \frac{G_2 - G_1}{2L} = \frac{A/100}{2KA} = \frac{0.01}{2K} = \frac{0.01}{2(49)} = 0.000102$$

Note that A is divided by 100 to make certain that the units are consistent with the denominator since L is in feet from the $L = KA$ equation. At the low point, $y = 270$ ft so solving for c in Eq. 3.1 with $x = 122.5$, $a = 0.000102$ and $b = -0.025$ gives

$$270 = 0.000102(122.5)^2 + (-0.025)(122.5) + c$$
$$c = \underline{\underline{271.53}} = \text{elevation of the } PVC$$

Knowing that the station of the low point is 141+00 and the distance from the PVC to the low point is 122.5 ft,

$$\text{station of } PVC = 141+00 \text{ minus } 1+22.5 = \underline{\underline{139+77.5}}$$

Next, the length of the curve is determined by applying Eq. 3.1. Because it is known that the elevation of the PVT is 274 ft, using $y = 274$ means that $x = L$ in Eq. 3.1, so (with c = 271.53)

$$y = aL^2 + bL + c$$
$$274 = 0.000102L^2 + (-0.025)L + 271.53$$
$$L = \underline{\underline{320.624 \text{ ft}}}$$

The station of the PVI is

$$\text{station of } PVI = \text{station of } PVC + L/2$$
$$= 139+77.5 \text{ plus } (3+20.624/2) = \underline{\underline{141+37.812}}$$

Finally, the elevation of the PVI is determined as

$$\text{elevation of } PVI = \text{elevation of } PVC + G_1(L/2)$$
$$= 271.53 - 0.025(320.624/2) = \underline{\underline{267.52}}$$

EXAMPLE 3.9 COMBINED SAG AND CREST VERTICAL CURVES WITHOUT A CONSTANT GRADE CONNECTION

An existing tunnel needs to be connected to a newly constructed bridge with sag and crest vertical curves. The profile view of the tunnel and bridge is shown in Fig. 3.8. Develop a vertical alignment to connect the tunnel and bridge by determining the highest possible common design speed for the sag and crest (equal-tangent) vertical curves needed. Compute the stationing and elevations of PVC, PVI, and PVT curve points.

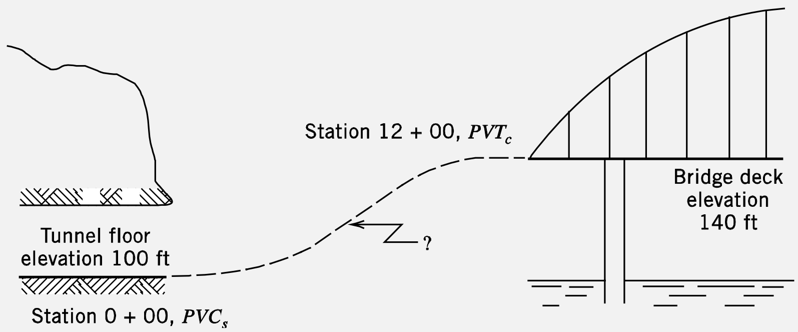

Figure 3.8 Profile view (vertical alignment diagram) for Example 3.9.

SOLUTION

From left to right (see Fig. 3.8), a sag vertical curve (with subscript *s*) and a crest vertical curve (with subscript *c*) are needed to connect the tunnel and bridge. From the given information, it is known that G_{1s} = 0% (the initial slope of the sag vertical curve) and G_{2c} = 0% (the final slope of the crest vertical curve). To obtain the highest possible design speed, we want to use all of the horizontal distance available. This means we want to connect the curve so that the *PVT* of the sag curve (PVT_s) will be the *PVC* of the crest curve (PVC_c). If this is the case, $G_{2s} = G_{1c}$ and since $G_{1s} = G_{2c} = 0$, $A_s = A_c = A$, the common algebraic difference in the grades.

Since 1200 ft separates the tunnel and bridge,

$$L_s + L_c = 1200$$

Also, the summation of the end-of-curve offset for the sag curve and the beginning-of-curve offset (relative to the final grade) for the crest curve must equal 40 ft. Using the equation for the final offset, Eq. 3.9, we have

$$\frac{AL_s}{200} + \frac{AL_c}{200} = 40$$

Rearranging,

$$\frac{A}{200}\left(L_s + L_c\right) = 40$$

and since $L_s + L_c = 1200$,

$$\frac{A}{200}\left(1200\right) = 40$$

Solving for A gives A = 6.667%. The problem now becomes one of finding K-values that allow $L_s + L_c = 1200$. Since $L = KA$ (Eq. 3.17), we can write

$$K_s A + K_c A = 1200$$

Substituting $A = 6.667$,

$$K_s + K_c = 180$$

To find the highest possible design speed, Tables 3.2 and 3.3 are used to arrive at K-values to solve $K_s + K_c = 180$. From Tables 3.2 and 3.3 it is apparent that the highest possible design speed is 50 mi/h, at which speed $K_c = 84$ and $K_s = 96$ (the summation of K's is 180).

To arrive at the stationing of curve points, we first determine curve lengths as

$$L_s = K_s A = 96(6.667) = 640.0 \text{ ft}$$

$$L_c = K_c A = 84(6.667) = 560.0 \text{ ft}$$

Since the station of the PVC_s is $\underline{\underline{0 + 00}}$ (given), it is clear that the $PVI_s = \underline{\underline{3 + 20.0}}$, $PVT_s = PVC_c = \underline{\underline{6 + 40.0}}$, $PVI_c = \underline{\underline{9 + 20.0}}$, and $PVT_c = \underline{\underline{12 + 00.0}}$. For elevations, $PVC_s = PVI_s = \underline{\underline{100 \text{ ft}}}$ and $PVI_c = PVT_c = \underline{\underline{140 \text{ ft}}}$. Finally, the elevation of PVT_s and PVC_c can be computed as

$$100 + \frac{AL_s}{200} = 100 + \frac{6.667(640.0)}{200} = 121.33 \text{ ft}$$

EXAMPLE 3.10 COMBINED SAG AND CREST VERTICAL CURVES WITH A CONSTANT GRADE CONNECTION

Consider the conditions described in Example 3.9. Suppose a design speed of only 35 mi/h is needed. Determine the lengths of curves required to connect the bridge and tunnel while keeping the connecting grade as small as possible.

SOLUTION

It is known that the 1200 ft separating the tunnel and bridge are more than enough to connect a 35-mi/h alignment because Example 3.8 showed that 50 mi/h is possible. Therefore, to connect the tunnel and bridge and keep the connecting grade as small as possible, we will place a constant-grade section between the sag and crest curves (as shown in Fig. 3.9).

The elevation change will be the final offsets of the sag and crest curves plus the change in elevation resulting from the constant-grade section connecting the two curves. Let G_{con} be the grade of the constant-grade section. This means that $G_{2s} = G_{1c} = G_{con}$, and since $G_{1s} = G_{2c} = 0$ (as in Example 3.8), $G_{con} = A_s = A_c = A$. The equation that will solve the vertical alignment for this problem is

$$\frac{AL_s}{200} + \frac{AL_c}{200} + \frac{A(1200 - L_s - L_c)}{100} = 40$$

where the third term accounts for the elevation difference attributable to the constant-grade section connecting the sag and crest curves (the 100 in the denominator of this term converts A from percent to ft/ft). Using $L = KA$, we have

$$\frac{A^2 K_s}{200} + \frac{A^2 K_c}{200} + \frac{A(1200 - K_s A - K_c A)}{100} = 40$$

From Table 3.2, $K_c = 29$, and from Table 3.3, $K_s = 49$. Putting these values in the above equation gives

$$0.39A^2 + 12A - 0.78A^2 = 40$$

$$-0.39A^2 + 12A - 40 = 0$$

Solving this gives A = 3.803 and A = 26.966; A = 3.803% is chosen because we want to minimize the grade. For this value of A, the curve lengths are

$$L_s = K_s A = 49(3.803) = \underline{\underline{186.35 \text{ ft}}}$$

$$L_c = K_c A = 29(3.803) = \underline{\underline{110.29 \text{ ft}}}$$

and the length of the constant-grade section will be 903.36 ft. This means that about 34.35 ft of the elevation difference will occur in the constant-grade section, with the remainder of the elevation difference attributable to the final curve offsets.

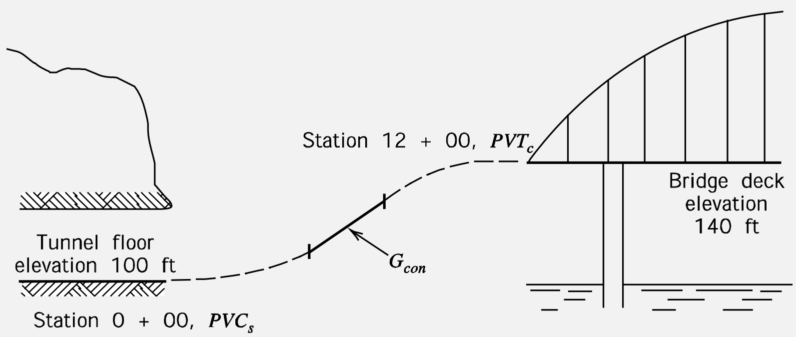

Figure 3.9 Profile view (vertical alignment diagram) for Example 3.10.

Another variation of this type of problem is the case when the initial and final grades are not equal to zero, as in the following example.

EXAMPLE 3.11 COMBINED SAG AND CREST VERTICAL CONNECTING HIGHWAY SEGMENTS WITH NON-ZERO GRADES

Two sections of highway are separated by 1800 ft, as shown in Fig. 3.10. Determine the curve lengths required for a 60-mi/h vertical alignment to connect these two highway segments while keeping the connecting grade as small as possible.

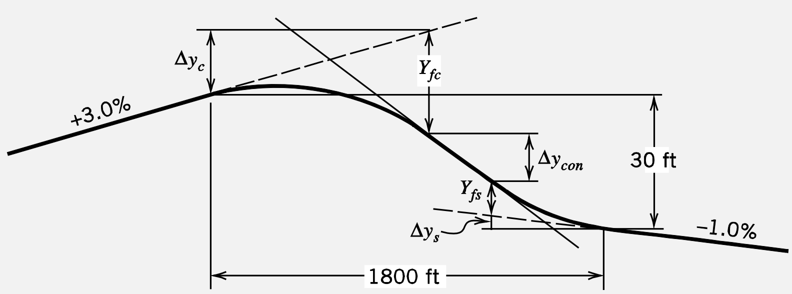

Figure 3.10 Profile view (vertical alignment diagram) for Example 3.11.

SOLUTION

Let Y_{fc} and Y_{fs} be the final offsets of the crest and sag curves, respectively. Let G_{con} be the slope of a constant-grade section connecting the crest and sag curves (we will assume that the horizontal distance is sufficient to connect the highway with a 60-mi/h alignment; if this assumption is incorrect, the following equations will produce an obviously erroneous answer and a lower design speed will have to be chosen). Finally, let Δy_{con} be the change in elevation over the constant-grade section, and let Δy_c and Δy_s be the changes in elevation due to the extended curve tangents. The elevation equation is then (see Fig. 3.10)

$$Y_{fc} + Y_{fs} + \Delta\, y_{con} + \Delta y_s = 30 + \Delta\, y_c$$

Substituting offset equations and equations for elevation changes (with subscripts *c* for crest and *s* for sag),

$$\frac{A_c L_c}{200} + \frac{A_s L_s}{200} + \frac{G_{con}\left(1800 - L_c - L_s\right)}{100} + \frac{1.0\, L_s}{100} = 30 + \frac{3.0\, L_c}{100}$$

Using $L = KA$, this equation becomes

$$\frac{A_c^2 K_c}{200} + \frac{A_s^2 K_s}{200} + \frac{G_{con}\left(1800 - K_c A_c - K_s A_s\right)}{100} + \frac{1.0\, K_s A_s}{100} = 30 + \frac{3.0\, K_c A_c}{100}$$

From Tables 3.2 and 3.3, K_c = 151 and K_s = 136. Substituting and defining *A*'s (and arranging the equation so that G_{con} will be positive, and assuming G_{con} will be greater than 1%) gives

$$\frac{\left(3+G_{con}\right)^2 151}{200} + \frac{\left(G_{con}-1\right)^2 136}{200} + \frac{G_{con}\left(1800 - 151\left(3+G_{con}\right) - 136\left(G_{con}-1\right)\right)}{100} + \frac{1.0\left(136\left(G_{con}-1\right)\right)}{100} = 30 + \frac{3.0\left(151\left(3+G_{con}\right)\right)}{100}$$

or

$$-1.435G_{con}^2 + 14.83G_{con} - 37.475 = 0$$

which gives G_{con} = 4.40 (the other possible solution is 5.93, which is rejected because we want to minimize the grade). Using $L = KA$ gives L_c = 1117.40 ft (151 × 7.40) and L_s = 462.40 ft (136 × 3.40). Accordingly, the length of the constant-grade section is 220.20 ft ($1800 - L_c - L_s$). Elevations and the locations of curve points can be readily computed with this information.

3.3.5 Passing Sight Distance and Crest Vertical Curve Design

In addition to stopping sight distance, in some instances it may be desirable to provide adequate passing sight distance, which can be an important issue in two-lane highway design (one lane in each direction). Passing sight distance is a factor only in crest vertical curve design because, for sag curves, the sight distance is unobstructed looking up or down the grade, and at night, the headlights of oncoming or opposing vehicles will be seen. In determining the sight distance required to pass on a crest vertical curve, Eqs. 3.13 and 3.14 will apply; however, whereas the driver's eye height, H_1, will remain at 3.5 ft, H_2 will now also be set to 3.5 ft. This value for H_2 is the assumed value for the portion of a vehicle's height necessary to be visible such that it can be recognized as an opposing vehicle to a driver performing a passing maneuver. Using the same height for both H_1 and H_2 provides a reciprocal design relationship; that is, if the driver of the passing vehicle can see the opposing vehicle, then the opposing vehicle driver can see the passing vehicle. Substituting these H values into Eqs. 3.13 and 3.14 and letting the sight distance S equal the passing sight distance, PSD, gives

For PSD < L

$$L_m = \frac{A \times \text{PSD}^2}{2800} \qquad \textbf{(3.24)}$$

For PSD > L

$$L_m = 2 \times \text{PSD} - \frac{2800}{A} \qquad \textbf{(3.25)}$$

where

L_m = minimum length of vertical curve in ft,

A = absolute value of the difference in grades ($|G_1 - G_2|$), expressed as a percentage, and

PSD = passing sight distance in ft.

As was the case for stopping sight distance, it is typically assumed that the length of curve is greater than the required sight distance (in this case L > PSD), so

$$K = \frac{\text{PSD}^2}{2800} \qquad \textbf{(3.26)}$$

where

K = horizontal distance, in ft, required to effect a 1% change in the slope (as in Eq. 3.10), and

PSD = passing sight distance in ft.

The passing sight distance (PSD) used for design is assumed to consist of four distances: (1) the initial maneuver distance (which includes the driver's perception/reaction time and the time it takes to bring the vehicle from its trailing speed to the point of encroachment on the left lane), (2) the distance that the passing vehicle traverses while occupying the left lane, (3) the clearance length between the passing and opposing vehicles at the end of the passing maneuver, and (4) the distance traversed by the opposing vehicle during two-thirds of the time the passing vehicle occupies the left lane. The determination of these distances is undertaken using assumptions regarding the time of the initial maneuver, average vehicle acceleration, and the speeds of passing, passed, and opposing vehicles. The sum of these four distances gives the required passing sight distance. The reader is referred to [AASHTO 2011] for a complete description of the assumptions made in determining required passing sight distances.

The minimum distances needed to pass (PSDs) at various design speeds, along with the corresponding K-values as computed from Eq. 3.26, are presented in Table 3.4. Notice that the K-values in this table are much higher than those required for stopping sight distance (as given in Table 3.2). As a result, designing a crest curve to provide adequate passing sight distance is often an expensive proposition (due to the length of curve required).

Table 3.4 Design Controls for Crest Vertical Curves Based on Passing Sight Distance.

Design speed (mi/h)	Passing sight distance (ft)	Rate of vertical curvature, K*
20	400	57
25	450	72
30	500	89
35	550	108
40	600	129
45	700	175
50	800	229
55	900	289
60	1000	357
65	1100	432
70	1200	514
75	1300	604
80	1400	700

*Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A): $K = L/A$.

Source: American Association of State Highway and Transportation Officials, *A Policy on Geometric Design of Highways and Streets*, Washington, DC, 2011. Used by permission.

EXAMPLE 3.12 VERTICAL CURVE DESIGN WITH PASSING SIGHT DISTANCE

An equal-tangent crest vertical curve is 1000 ft long and connects a +2.5% and a −1.5% grade. If the design speed of the roadway is 55 mi/h, does this curve have adequate passing sight distance?

SOLUTION

To determine the length of curve required to provide adequate passing sight distance at a design speed of 55 mi/h, we use $L = KA$ with $K = 289$ (as read from Table 3.4). This gives

$$L = 289(4.0) = 1156 \text{ ft}$$

Since the curve is only 1000 ft long, it is not long enough to provide adequate passing sight distance. Alternatively, the K-value for the existing design can be compared with that required for a PSD-based design. The K-value for the existing design is

$$K = \frac{1000}{4} = 250$$

Since the K-value of 250 for the existing curve design is less than 289, this curve does not provide adequate PSD for a 55-mi/h design speed.

3.3.6 Underpass Sight Distance and Sag Vertical Curve Design

As mentioned in Section 3.3.4, design for sag curves is based on nighttime conditions because during daytime conditions a driver can see the entire sag curve. However, in the case of a sag curve being built under an overhead structure (such as roadway or railroad crossing), a driver's line of sight may be restricted so that the entire curve length is not visible. An example of this situation is shown in Fig. 3.11.

In designing the sag curve, it is essential that the curve be long enough to provide a suitably gradual rate of curvature such that the overhead structure does not block the line of sight and allows the required stopping sight distance for the specified design speed to be maintained.

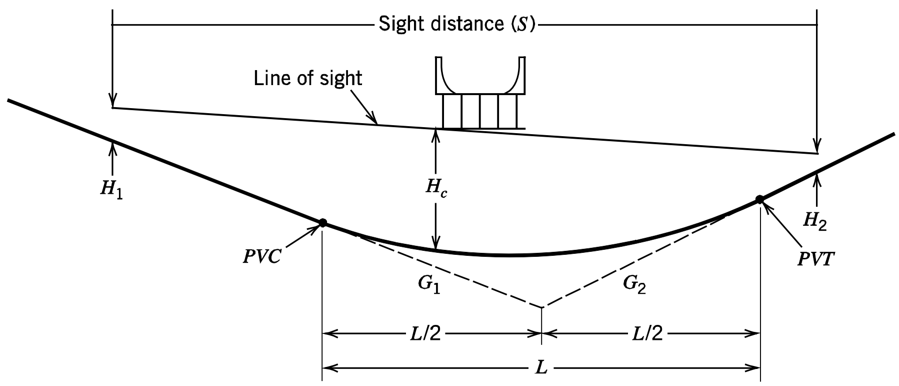

Figure 3.11 Stopping sight distance considerations for underpass sag curves.

Used by permission from American Association of State Highway and Transportation Officials, *A Policy on Geometric Design of Highways and Streets*, 6th Edition, Washington, DC, 2011.

S = sight distance in ft,

H_1 = height of driver's eye in ft,

H_2 = height of object in ft,

H_c = clearance height of overpass structure above roadway in ft,

L = length of the curve in ft

G_1 = initial roadway grade in percent or ft/ft,

G_2 = final roadway grade in percent or ft/ft,

PVC = point of the vertical curve (the initial point of the curve), and

PVT = point of vertical tangent, which is the final point of the vertical curve (the point where the curve returns to the final grade or, equivalently, the final tangent).

Again, by using the properties of a parabola for an equal-tangent vertical curve, it can be shown that the minimum length of sag curve for a required sight distance and clearance height is

For $S < L$

$$L_m = \frac{AS^2}{800\left(H_c - \left(\frac{H_1 + H_2}{2}\right)\right)} \tag{3.27}$$

For $S > L$

$$L_m = 2S - \frac{800\left(H_c - \left(\frac{H_1 + H_2}{2}\right)\right)}{A} \tag{3.28}$$

where

L_m = minimum length of vertical curve in ft,
A = absolute value of the difference in grades ($|G_1 - G_2|$), expressed as a percentage, and
Other terms are as defined in Fig. 3.11.

Current AASHTO design guidelines [2011] use a driver eye height, H_1, of 8 ft for a truck driver, and an object height, H_2, of 2 ft for the taillights of a vehicle. Substituting these values and S = SSD into Eqs. 3.27 and 3.28 gives

For SSD < L

$$L = \frac{A \times \text{SSD}^2}{800(H_c - 5)} \tag{3.29}$$

For SSD > L

$$L = 2 \times \text{SSD} - \frac{800(H_c - 5)}{A} \tag{3.30}$$

where

SSD = stopping sight distance in ft, and
Other terms are as defined previously.

In the case where there is an existing sag curve alignment and a new overpass structure is to be built over it, the above equations can be rearranged to solve for the necessary clearance height, H_c, of the overpass structure to provide for the required stopping sight distance. When the clearance height is determined in this manner, it is necessary to check this value against the minimum clearance heights based on maximum vehicle height regulations and AASHTO recommendations. Maximum vehicle heights as regulated by state laws range from 13.5 to 14.5 ft. AASHTO [2011] recommends a minimum structure clearance height of 14.5 ft and a desirable clearance height of 16.5 ft. AASHTO [2011] also recommends that clearance heights be no less than 1 ft greater than the maximum allowable vehicle height. This provides a margin for snow or ice accumulation, some over-height vehicles, and future roadway resurfacings. Thus, in building a new overpass structure over an existing sag curve alignment, the clearance height must be determined for both required stopping sight distance and maximum allowable vehicle height for that roadway, and the greater of the two values should be used.

EXAMPLE 3.13 UNDERPASS VERTICAL CURVE CLEARANCE

An equal-tangent sag curve has an initial grade of −4.0%, a final grade of +3.0%, and a length of 1270 ft. An overpass is being placed directly over the *PVI* of this curve. At what height above the roadway should the bottom of this sign be placed?

SOLUTION

For this situation, Eq. 3.29 or 3.30 must be used to solve for the necessary clearance height based on stopping sight distance. Thus, the required SSD must be determined for the given sag curve specifications, based on the design speed. The design speed for the curve can be determined from the *K*-value by applying Eq. 3.10 as follows:

$$K = \frac{L}{A} = \frac{1270}{|-4-3|} = 181.4$$

From Table 3.3, this *K*-value corresponds approximately to a design speed of 70 mi/h (K = 181). For a 70-mi/h design speed, the required stopping sight distance is 730 ft. Since the curve length is greater than the required SSD (1270 > 730), Eq. 3.29 applies:

$$L = \frac{A \times \text{SSD}^2}{800(H_c - 5)}$$

Rearranging this equation to solve for the clearance height, H_c, and substituting A = 7%, SSD = 730 ft, and L = 1270 ft gives

$$\begin{aligned} H_c &= \frac{A \times \text{SSD}^2}{800L} + 5 \\ &= \frac{7 \times 730^2}{800(1270)} + 5 \\ &= 8.67 \text{ ft} \end{aligned}$$

Although only 8.67 ft is needed for SSD requirements, AASHTO [2011] recommends a minimum clearance height of 14.5 ft to take maximum vehicle height into account. Thus, the bottom of the overpass should be placed at least 14.5 ft above the roadway surface (at the *PVI*), but desirably at a height of 16.5 ft according to AASHTO [2011].

3.4 HORIZONTAL ALIGNMENT

The critical aspect of horizontal alignment is the horizontal curve, with the focus on design of the directional transition of the roadway in a horizontal plane. Stated differently, a horizontal curve provides a transition between two straight (or tangent) sections of roadway. A key concern in this directional transition is the ability of the vehicle to negotiate a horizontal curve. (Provision of adequate drainage is also important, but is not discussed in this book; see [AASHTO 2011].) As was the case with the straight-line vehicle performance characteristics discussed at length in Chapter 2, the highway engineer must design a horizontal alignment to accommodate the cornering capabilities of a variety of vehicles, ranging from nimble sports cars to ponderous trucks. A theoretical assessment of vehicle cornering at the level of detail given to straight-line performance in Chapter 2 is beyond the scope of this book (see [Campbell 1978] and [Wong 2008]). Instead, vehicle cornering performance is viewed only at the practical design-oriented level, with equations simplified in a manner similar to that used for the stopping-distance equation discussed in Section 2.9.5.

3.4.1 Vehicle Cornering

Figure 3.12 illustrates the forces acting on a vehicle during cornering.

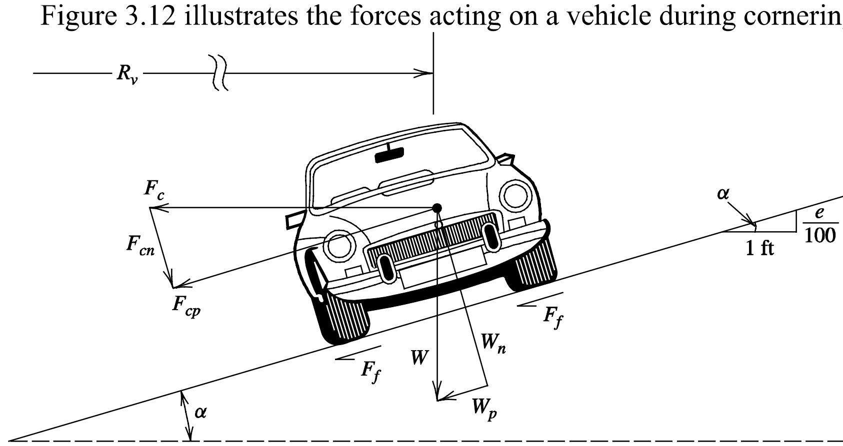

Figure 3.12 Vehicle cornering forces.

R_v = radius defined to the vehicle's traveled path in ft,

α = angle of incline in degrees,

e = number of vertical ft of rise per 100 ft of horizontal distance,

W = weight of the vehicle in lb,

W_n = vehicle weight normal to the roadway surface in lb,

W_p = vehicle weight parallel to the roadway surface in lb,

F_f = side frictional force (centripetal, in lb),

F_c = centripetal force (lateral acceleration × mass, in lb),

F_{cp} = centripetal force acting parallel to the roadway surface in lb, and

F_{cn} = centripetal force acting normal to the roadway surface in lb.

Some basic horizontal curve relationships can be derived by noting that

$$W_p + F_f = F_{cp} \tag{3.31}$$

From basic physics this equation can be written as [with $F_f = f_s(W_n + F_{cn})$]

$$W \sin \alpha + f_s \left(W \cos \alpha + \frac{WV^2}{gR_v} \sin \alpha \right) = \frac{WV^2}{gR_v} \cos \alpha \quad \textbf{(3.32)}$$

where

f_s = coefficient of side friction (unitless),
V = vehicle speed in ft/s,
g = gravitational constant, 32.2 ft/s^2, and
Other terms are as defined in Fig 3.12.

Dividing both sides of Eq. 3.32 by $W \cos \alpha$ gives

$$\tan \alpha + f_s = \frac{V^2}{gR_v}(1 - f_s \tan \alpha) \quad \textbf{(3.33)}$$

The term tan α indicates the superelevation of the curve (banking) and can be expressed in percent; it is denoted e (e = 100 tan α). In words, the superelevation is the number of vertical feet (meters) of rise per 100 feet (meters) of horizontal distance (see Fig. 3.12). The term f_s tan α in Eq. 3.33 is conservatively set equal to zero for practical applications due to the small values that f_s and α typically assume (this is equivalent to ignoring the normal component of centripetal force). With e = 100 tan α, Eq. 3.33 can be arranged as

$$R_v = \frac{V^2}{g\left(f_s + \frac{e}{100} \right)} \quad \textbf{(3.34)}$$

EXAMPLE 3.14 SUPERELEVATION ON HORIZONTAL CURVES

A roadway is being designed for a speed of 70 mi/h. At one horizontal curve, it is known that the superelevation is 8.0% and the coefficient of side friction is 0.10. Determine the minimum radius of curve (measured to the traveled path) that will provide for safe vehicle operation.

SOLUTION

The application of Eq. 3.34 gives [with 1.467 (5280/3600) converting mi/h to ft/s]

$$R_v = \frac{V^2}{g\left(f_s + \frac{e}{100} \right)} = \frac{(70 \times 1.467)^2}{32.2(0.10 + 0.08)} = \underline{\underline{1819.40 \text{ ft}}}$$

This value is the minimum radius, because radii smaller than 1819.40 ft will generate centripetal forces higher than those that can be safely supported by the superelevation and the side frictional force.

In the actual design of a horizontal curve, the engineer must select appropriate values of e and f_s. The value selected for superelevation, e, is critical because high rates of superelevation can cause vehicle steering problems on the horizontal curve, and in cold climates, ice on the roadway can reduce f_s such that vehicles traveling at less than the design speed on an excessively superelevated curve could slide inward off the curve due to gravitational forces. AASHTO provides general guidelines for the selection of e and f_s for horizontal curve design, as shown in Table 3.5. The values presented in this table are grouped by five values of maximum e. The selection of any one of these five maximum e values is dependent on the type of road (for example, higher maximum e's are permitted on freeways than on arterials and local roads) and local design practice. Limiting values of f_s are simply a function of design speed. Table 3.5 also presents calculated radii (given V, e, and f_s) by applying Eq. 3.34.

3.4.2 Horizontal Curve Fundamentals

In connecting straight (tangent) sections of roadway with a horizontal curve, several options are available. The most obvious of these is the simple circular curve, which is just a curve with a single, constant radius. Other options include reverse curves, compound curves, and spiral curves. Reverse curves generally consist of two consecutive curves that turn in opposite directions. They are used to shift the alignment of a highway laterally. The curves used are usually circular and have equal radii. Reverse curves, however, are not recommended because drivers may find it difficult to stay within their lane as a result of sudden changes to the alignment. Compound curves consist of two or more curves, usually circular, in succession. Compound curves are used to fit horizontal curves to very specific alignment needs, such as interchange ramps, intersection curves, or difficult topography. In designing compound curves, care must be taken not to have successive curves with widely different radii, as this will make it difficult for drivers to maintain their lane position as they transition from one curve to the next. Spiral curves are curves with a continuously changing radius. Spiral curves are sometimes used to transition a tangent section of roadway to a circular curve. In such a case, the radius of the spiral curve is equal to infinity where it connects to the tangent section and ends with the radius value of the connecting circular curve at the other end. Because motorists usually create their own transition paths between tangent sections and circular curves by utilizing the full lane width available, spiral curves are not often used. However, there are exceptions. Spiral curves are sometimes used on high-speed roadways with sharp horizontal curves and also to gradually introduce the superelevation of an upcoming horizontal curve. To illustrate the basic principles involved in horizontal curve design, this book will focus only on the single simple circular curve. For detailed information regarding these additional horizontal curve types, refer to standard route-surveying texts, such as Hickerson [1964]. Figure 3.13 shows the basic elements of a simple horizontal curve.

Table 3.5 Minimum Radius Using Limiting Values of *e* and f_s

Design speed (mi/h)	Maximum *e* (%)	Limiting values of f_s	Total ($e/100 + f_s$)	Calculated radius, R_v (ft)	Rounded radius, R_v (ft)
10	4.0	0.38	0.42	15.9	16
15	4.0	0.32	0.36	41.7	42
20	4.0	0.27	0.32	86.0	86
25	4.0	0.23	0.27	154.3	154
30	4.0	0.20	0.24	250.0	250
35	4.0	0.18	0.22	371.2	371
40	4.0	0.16	0.20	533.3	533
45	4.0	0.15	0.19	710.5	711
50	4.0	0.14	0.18	925.9	926
55	4.0	0.13	0.17	1186.3	1190
60	4.0	0.12	0.16	1500.0	1500
10	6.0	0.38	0.44	15.2	15
15	6.0	0.32	0.38	39.5	39
20	6.0	0.27	0.33	80.8	81
25	6.0	0.23	0.29	143.7	144
30	6.0	0.20	0.26	230.8	231
35	6.0	0.18	0.24	340.3	340
40	6.0	0.16	0.22	484.8	485
45	6.0	0.15	0.21	642.9	643
50	6.0	0.14	0.20	833.3	833
55	6.0	0.13	0.19	1061.4	1060
60	6.0	0.12	0.18	1333.3	1330
65	6.0	0.11	0.17	1656.9	1660
70	6.0	0.10	0.16	2041.7	2040
75	6.0	0.09	0.15	2500.0	2500
80	6.0	0.08	0.14	3047.6	3050
10	8.0	0.38	0.46	14.5	14
15	8.0	0.32	0.40	37.5	38
20	8.0	0.27	0.35	76.2	76
25	8.0	0.23	0.31	134.4	134
30	8.0	0.20	0.28	214.3	214
35	8.0	0.18	0.26	314.1	314
40	8.0	0.16	0.24	444.4	444
45	8.0	0.15	0.23	587.0	587
50	8.0	0.14	0.22	757.6	758
55	8.0	0.13	0.21	960.3	960
60	8.0	0.12	0.20	1200.0	1200
65	8.0	0.11	0.19	1482.5	1480
70	8.0	0.10	0.18	1814.8	1810
75	8.0	0.09	0.17	2205.9	2210
80	8.0	0.08	0.16	2666.7	2670
10	10.0	0.38	0.48	13.9	14
15	10.0	0.32	0.42	35.7	36
20	10.0	0.27	0.37	72.1	72
25	10.0	0.23	0.33	126.3	126
30	10.0	0.20	0.30	200.0	200
35	10.0	0.18	0.28	291.7	292
40	10.0	0.16	0.26	410.3	410
45	10.0	0.15	0.25	540.0	540
50	10.0	0.14	0.24	694.4	694
55	10.0	0.13	0.23	876.8	877
60	10.0	0.12	0.22	1090.9	1090
65	10.0	0.11	0.21	1341.3	1340
70	10.0	0.10	0.20	1633.3	1630
75	10.0	0.09	0.19	1973.7	1970
80	10.0	0.08	0.18	2370.4	2370
10	12.0	0.38	0.50	13.3	13
15	12.0	0.32	0.44	34.1	34
20	12.0	0.27	0.39	68.4	68
25	12.0	0.23	0.35	119.0	119
30	12.0	0.20	0.32	187.5	188
35	12.0	0.18	0.30	272.2	272
40	12.0	0.16	0.28	381.0	381
45	12.0	0.15	0.27	500.0	500
50	12.0	0.14	0.26	641.0	641
55	12.0	0.13	0.25	806.7	807
60	12.0	0.12	0.24	1000.0	1000
65	12.0	0.11	0.23	1224.6	1220
70	12.0	0.10	0.22	1484.8	1480
75	12.0	0.09	0.21	1785.7	1790
80	12.0	0.08	0.20	2133.3	2130

Note: In recognition of safety considerations, use of e_{max} = 4.0% should be limited to urban conditions.

Source: American Association of State Highway and Transportation Officials, *A Policy on Geometric Design of Highways and Streets*, 6th Edition, Washington, DC, 2011. Used by permission.

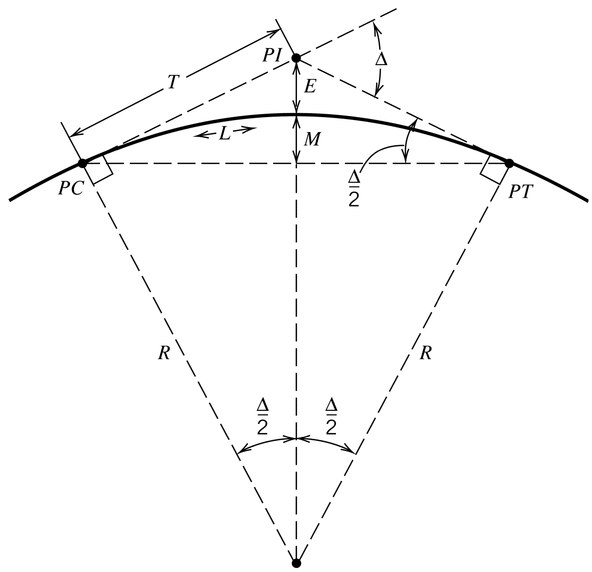

Figure 3.13 Elements of a simple circular horizontal curve.

R = radius, usually measured to the centerline of the road, in ft,
Δ = central angle of the curve in degrees,
T = tangent length in ft,
E = external distance in ft,
M = middle ordinate in ft,
PC = point of curve (the beginning point of the horizontal curve),
PI = point of tangent intersection,
PT = point of tangent (the ending point of the horizontal curve), and
L = length of curve in ft.

Another important term is the degree of curve, which is defined as the angle subtended by a 100-ft arc along the horizontal curve. It is a measure of the sharpness of the curve and is frequently used instead of the radius in the construction of the curve. The degree of curve is directly related to the radius of the horizontal curve by

$$D = \frac{100\left(\frac{180}{\pi}\right)}{R} = \frac{18000}{\pi R} \tag{3.35}$$

where

D = degree of curve [angle subtended by a 100-ft arc along the horizontal curve], and
Other terms are as defined in Fig. 3.13.

Note that the quantity $180/\pi$ converts from radians to degrees.

Geometric and trigonometric analyses of Fig. 3.13 reveal the following relationships:

$$T = R \tan \frac{\Delta}{2} \tag{3.36}$$

$$E = R\left(\frac{1}{\cos(\Delta/2)} - 1\right) \tag{3.37}$$

$$M = R\left(1 - \cos\frac{\Delta}{2}\right) \tag{3.38}$$

$$L = \frac{\pi}{180} R\Delta \tag{3.39}$$

where all terms are as defined in Fig. 3.13.

It is important to note that horizontal curve stationing, curve length, and curve radius (R) are usually measured to the centerline of the road. In contrast, the radius determined on the basis of vehicle forces (R_v in Eq. 3.34) is measured from the innermost vehicle path, which is assumed to be the midpoint of the innermost vehicle lane. Thus, a slight correction for lane width is required in equating the R_v of Eq. 3.34 with the R in Eqs. 3.35 through 3.39.

EXAMPLE 3.15 STATIONING ON HORIZONTAL CURVES

A horizontal curve is designed with a 2000-ft radius. The curve has a tangent length of 400 ft and the *PI* is at station 103 + 00. Determine the stationing of the *PT*.

SOLUTION

Equation 3.36 is applied to determine the central angle, Δ:

$$T = R\tan\frac{\Delta}{2}$$

$$400 = 2000\tan\frac{\Delta}{2}$$

$$\Delta = 22.62°$$

So, from Eq. 3.39, the length of the curve is

$$L = \frac{\pi}{180} R\Delta$$

$$L = \frac{3.1416}{180} 2000(22.62) = 789.58 \text{ ft}$$

Given that the tangent length is 400 ft,

station of *PC* = 103 + 00 minus 4 + 00 = 99 + 00

Since horizontal curve stationing is measured along the alignment of the road,

$$\text{station of } PT = \text{station of } PC + L$$
$$= 99 + 00 \text{ plus } 7 + 89.58 = \underline{\underline{106 + 89.58}}$$

3.4.3 Stopping Sight Distance and Horizontal Curve Design

As is the case for vertical curve design, adequate stopping sight distance must be provided in the design of horizontal curves. Sight distance restrictions on horizontal curves occur when obstructions are present, as shown in Fig. 3.14. Such obstructions are frequently encountered in highway design due to the cost of right-of-way acquisition or the cost of moving earthen materials, such as rock outcroppings. When such an obstruction exists, the stopping sight distance is measured along the horizontal curve from the center of the traveled lane (the assumed location of the driver's eyes). As shown in Fig. 3.14, for a specified stopping distance, some distance M_s (the middle ordinate of a curve that has an arc length equal to the stopping sight distance) must be visually cleared so that the line of sight is such that sufficient stopping sight distance is available.

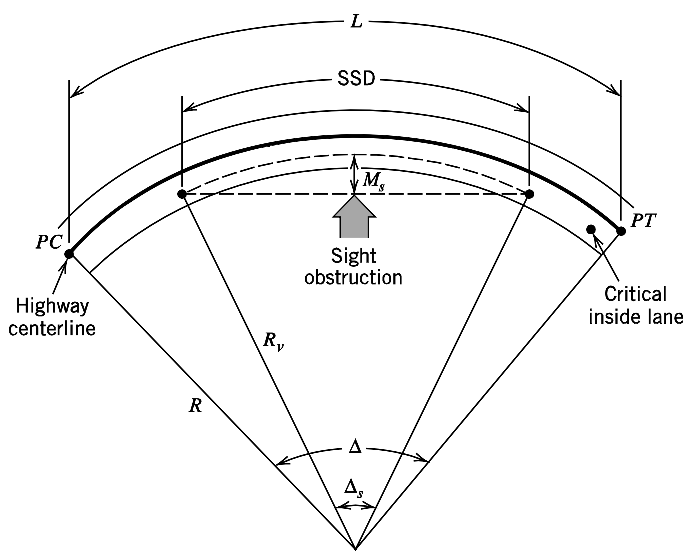

Figure 3.14 Stopping sight distance considerations for horizontal curves.

R = radius measured to the centerline of the road in ft,

R_v = radius to the vehicle's traveled path (usually measured to the center of the innermost lane of the road) in ft,

Δ = central angle of the curve in degrees,

Δ_s = angle (in degrees) subtended by an arc equal in length to the required stopping sight distance (SSD),

L = length of curve in ft,

M_s = middle ordinate necessary to provide adequate stopping sight distance (SSD) in ft.

SSD = stopping sight distance in ft,

PC = point of curve (the beginning point of the horizontal curve), and

PT = point of tangent (the ending point of the horizontal curve).

Equations for computing stopping sight distance (SSD) relationships for horizontal curves can be derived by first determining the angle, Δ_s, for an arc length equal to the required stopping sight distance (see Fig. 3.14 and note that this is not the central angle, Δ of the horizontal curve whose arc length is equal to L). Assuming that the length of the horizontal curve exceeds the required SSD (as shown in Fig. 3.14), we have (as with Eq. 3.39)

$$\text{SSD} = \frac{\pi}{180} R_v \Delta_s \quad \textbf{(3.40)}$$

Rearranging terms,

$$\Delta_s = \frac{180 \text{ SSD}}{\pi R_v} \quad \textbf{(3.41)}$$

Substituting this into the general equation for the middle ordinate of a simple horizontal curve (Eq. 3.38) to get an expression for M_s gives

$$M_s = R_v \left(1 - \cos \frac{90 \, \text{SSD}}{\pi R_v} \right) \quad \textbf{(3.42)}$$

Solving Eq. 3.42 for SSD gives

$$\text{SSD} = \frac{\pi R_v}{90} \left[\cos^{-1} \left(\frac{R_v - M_s}{R_v} \right) \right] \quad \textbf{(3.43)}$$

Note that Eqs. 3.40 to 3.43 can also be applied directly to determine sight distance requirements for passing. If these equations are to be used for passing, distance values given in Table 3.4 would apply and SSD in the equations would be replaced by PSD.

EXAMPLE 3.16 HORIZONTAL CURVE STATIONING AND DESIGN SPEED

A horizontal curve on a 4-lane highway (two lanes each direction with no median) has a superelevation of 6% and a central angle of 40 degrees. The *PT* of the curve is at station 322 + 50 and the *PI* is at 320 + 08. The road has 10-ft lanes and 8-ft shoulders on both sides with high retaining walls going up immediately next to the shoulders. What is the highest safe speed of this curve (highest in 5 mi/h increments) and what is the station of the *PC*?

SOLUTION

The tangent will be equal to the *PI* – *PC* so *T* = 320 + 08 – *PC*. The length of the curve will be equal to the *PT* – *PC* so *L* = 322 + 50 – *PC*. With the equations for the tangent and length of curve both put in terms of the *PC*, Eq. 3.36 and Eq. 3.39 can be rearranged respectively as,

$$R = \frac{T}{\tan \frac{\Delta}{2}} \quad \text{and} \quad R = \frac{L}{\frac{\pi}{180}\Delta} \quad \text{so that} \quad \frac{T}{\tan \frac{\Delta}{2}} = \frac{L}{\frac{\pi}{180}\Delta}$$

Substituting the previous tangent and length-of-curve equations (*T* = 32008 – *PC* and *L* = 32250 – *PC*),

$$\frac{32008 - PC}{\tan \frac{40}{2}} = \frac{32250 - PC}{\frac{\pi}{180} 40}$$

Which gives $\underline{\underline{PC = 317 + 44.25}}$. Using this value of *PC*, the tangent can be computed as T = 32008 – 31744.25 = 263.75 ft. This value of T can then be used to determine R (see equations above),

$$R = \frac{T}{\tan\frac{\Delta}{2}} = \frac{263.75}{\tan\frac{40}{2}} = 724.59 \text{ ft}$$

Because the curve radius is usually taken to the centerline of the roadway and there are two 10-ft lanes before the centerline (working from the inside of the curve to the outside), R_v = R – 10 – 10/2 = 724.59 – 15 = 709.59 ft. From Table 3.5 with a superelevation of 6%, at 45 mi/h a radius of 643 ft is needed; and at 50 mi/h a radius of 833 ft is needed. Therefore the highest deign speed for centripetal force is 45 mi/h (since 709.59 > 643, the design is acceptable for 45 mi/h because more than the needed radius is available, but with 833 > 709.59 the design is not acceptable for 50 mi/h since insufficient radius is available).

To check for adequate sight distance, M_s is going to be the shoulder width plus half of the inside lane width or 8 + 10/2 = 13 ft. Consider the stopping sight distance required at 40 mi/h. At 40 mi/h the required stopping sight distance (SSD) is 305 ft (from Table 3.1). Applying Eq. 3.42 gives,

$$M_s = R_v\left(1 - \cos\frac{90\ \text{SSD}}{\pi R_v}\right) = 709.59\left(1 - \cos\frac{90\ (305)}{\pi(709.59)}\right) = 16.34 \text{ ft}$$

Because 16.34 ft is greater than the 13 ft of available M_s, 40 mi/h is too fast. Consider a speed of 35 mi/h which gives SSD = 250 ft (from Table 3.1). The application of Eq. 3.42 then gives,

$$M_s = R_v\left(1 - \cos\frac{90\ \text{SSD}}{\pi R_v}\right) = 709.59\left(1 - \cos\frac{90\ (250)}{\pi(709.59)}\right) = 10.99 \text{ ft}$$

Because 10.99 ft is less than 13 ft, the highway is safe at 35 mi/h. Considering both the maximum safe speeds for centripetal force (45 mi/h) and sight distance (35 mi/h), the lower of the two speeds will govern. Thus 35 mi/h (the highest safe speed for sight distance) is the lower of the two speeds and is the highest safe speed for this curve.

3.5 COMBINED VERTICAL AND HORIZONTAL ALIGNMENT

Thus far the discussion on highway alignment has treated vertical and horizontal curves independently. The combination of vertical and horizontal curves, however, is quite common in geometric design, and often necessary. Obvious examples are highways through mountainous terrain and freeway interchange ramp roadways, which typically have to make significant changes in direction and elevation over a relatively short distance.

As previously mentioned, the design of an alignment that consists of a vertical and horizontal curve in combination usually consists of two two-dimensional alignment problems. The following examples illustrate this process.

EXAMPLE 3.17 COMBINED HORIZONTAL/VERTICAL ALIGNMENT—DESIGN ADEQUACY

A two-lane highway (two 12-ft lanes) has a posted speed limit of 50 mi/h and, on one section, has both horizontal and vertical curves, as shown in Fig. 3.15. A recent daytime crash (driver traveling eastbound and striking a stationary roadway object) resulted in a fatality and a lawsuit alleging that the 50-mi/h posted speed limit is an unsafe speed for the curves in question and was a major cause of the crash. Evaluate and comment on the roadway design.

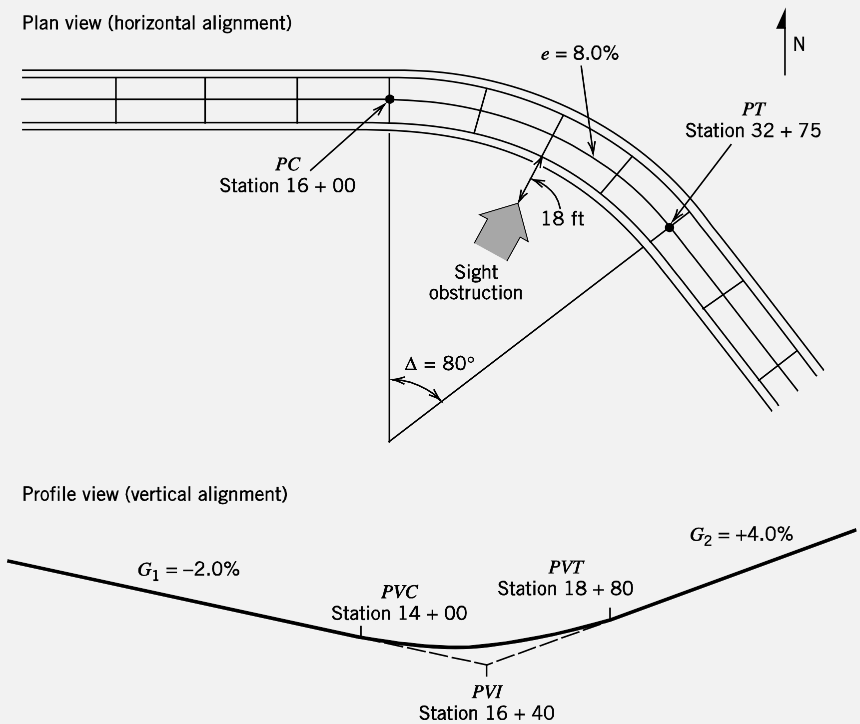

Figure 3.15 Horizontal and vertical alignment for Example 3.17.

SOLUTION

Begin with an assessment of the horizontal alignment. Two concerns must be considered: the adequacy of the curve radius and superelevation, and the adequacy of the sight distance on the eastbound (inside) lane. For the curve radius, note from Fig. 3.15 that

$$\begin{aligned} L &= \text{station of } PT - \text{station of } PC \\ &= 32+75 \text{ minus } 16+00 = 1675 \text{ ft} \end{aligned}$$

Rearranging Eq. 3.39, we get

$$R = \frac{180}{\pi\Delta}L = \frac{180}{\pi(80)}(1675) = 1198.65 \text{ ft}$$

Using the posted speed limit of 50 mi/h with e = 8.0%, we find that Eq. 3.34 can be rearranged to give (with the vehicle traveling in the middle of the inside lane, $R_v = R -$ half the lane width, or $R_v = 1199.63 - 6 = 1193.63$ ft)

$$f_s = \frac{V^2}{gR_v} - e = \frac{(50 \times 1.47)^2}{32.2(1192.65)} - 0.08 = 0.061$$

From Table 3.5, the maximum f_s for 50 mi/h is 0.14. Since 0.060 does not exceed 0.14, the radius and superelevation are sufficient for the 50-mi/h design speed. For sight distance, the available M_s is 18 ft plus the 6-ft distance to the center of the eastbound (inside) lane, or 24 ft. Application of Eq. 3.43 gives

$$\begin{aligned} \text{SSD} &= \frac{\pi R_v}{90}\left[\cos^{-1}\left(\frac{R_v - M_s}{R_v}\right)\right] \\ &= \frac{\pi(1192.65)}{90}\left[\cos^{-1}\left(\frac{1192.65-24}{1192.65}\right)\right] \\ &= 479.3 \text{ ft} \end{aligned}$$

From Table 3.1, the required SSD at 50 mi/h is 425 ft, so the 479.5 ft of SSD provided is sufficient. Turning to the sag vertical curve, the length of curve is

$$\begin{aligned} L &= \text{station of } PVT - \text{station of } PVC \\ &= 18+80 \text{ minus } 14+00 = 480 \text{ ft} \end{aligned}$$

Using $A = 6\%$ (from Fig. 3.15) and applying Eq. 3.10, we obtain

$$K = \frac{L}{A} = \frac{480}{6} = 80$$

For the 50-mi/h design speed, Table 3.3 indicates a necessary K-value of 96. Thus the K-value of 80 reveals that the curve is inadequate for the 50-mi/h speed. However, because the crash occurred in daylight and sight distances on sag vertical curves are governed by nighttime conditions, this design did not contribute to the crash.

EXAMPLE 3.18 DESIGN OF A COMBINED HORIZONTAL/VERTICAL ALIGNMENT

A new highway is to be constructed over an existing highway. The two highways will intersect at right angles and are to be grade-separated. Both highways are level grade (constant elevation). The new highway will run east-west and the existing highway runs north-south at elevation of 565.5 ft. The proposed bridge structure for the new highway is such that the bridge girder thickness is 6 ft (measured from the road surface to the bottom of the girder). A single-lane ramp is to be constructed to allow eastbound traffic to go southbound. A single horizontal curve, with a central angle of 90 degrees, is to be used. With a design speed of 40 mi/h and a required superelevation of 4%, determine the following: the stationing of the *PC*, *PI*, and *PT*, assuming the curve begins at station 40 + 00; the stationing and elevation of all key points along the vertical alignment; the distance that must be cleared from the inside of the horizontal curve so that the line of sight is sufficient to provide sufficient stopping sight distance. Fig. 3.16 displays the horizontal and vertical alignments.

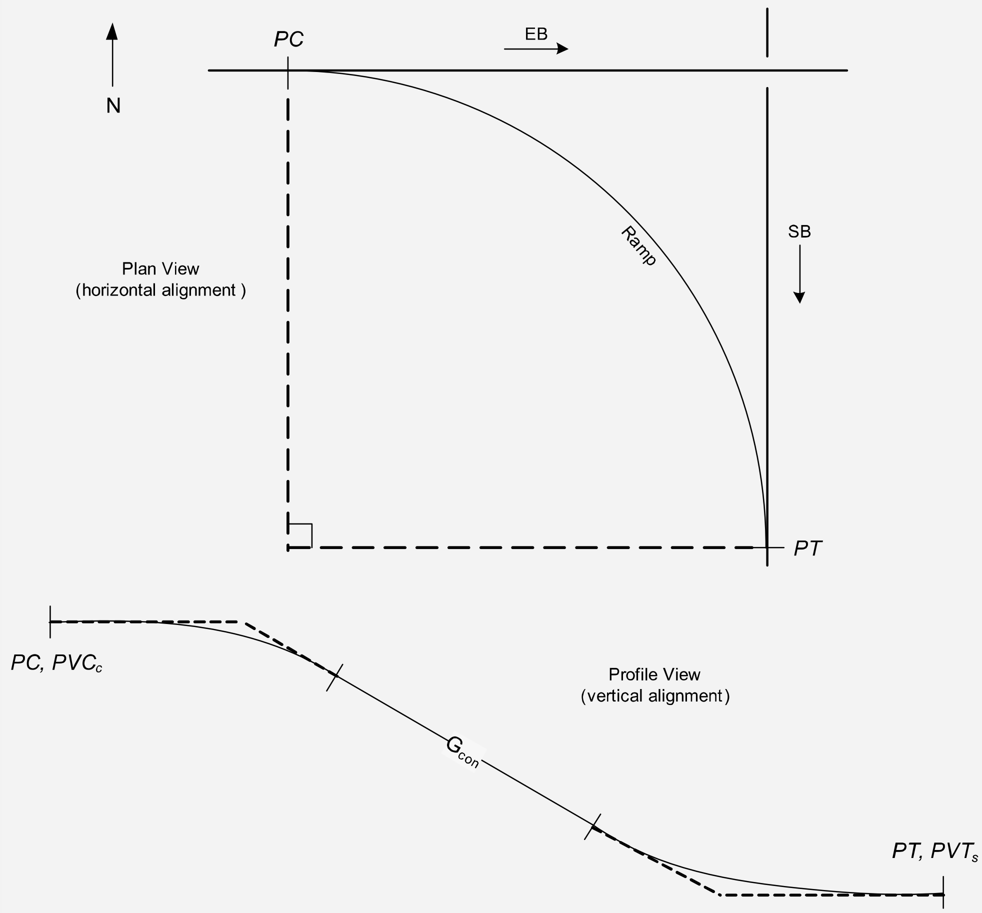

Figure 3.16 Horizontal and vertical alignment for Example 3.18.

SOLUTION

To begin, the required radius to the vehicle path (R_v) is determined to be 533 ft from Table 3.5 with a 40 mi/h design speed and 4% superelevation. Because the ramp is a single lane, the horizontal curve radius will be equal to the radius to the vehicle path ($R = R_v$). Applying Eq. 3.39 gives the length of the horizontal curve as (with R = 533 ft and Δ = 90 degrees):

$$\begin{aligned} L &= \frac{\pi}{180} R\Delta \\ &= \frac{3.1416}{180} 533(90) = 837.24 \text{ ft} \end{aligned}$$

Also, by inspection of Fig. 3.16 (or application of Eq. 3.36), the tangent length $T = R = 533$ ft. The stationing for the horizontal curve is as follows:

$$\text{station of } PC = \underline{\underline{40+00}}$$

$$\text{station of } PI = \text{station of } PC + T$$
$$= 40+00 \text{ plus } 5+33 = \underline{\underline{45+33}}$$

$$\text{station of } PT = \text{station of } PC + L$$
$$= 40+00 \text{ plus } 8+37.24 = \underline{\underline{48+37.24}}$$

For the vertical alignment, both a sag and crest curve are necessary. From Table 3.2 for 40 mi/h, $K_c = 44$, and from Table 3.3, $K_s = 64$.

Adequate clearance must be provided over the existing highway. As shown in Section 3.3.6, AASHTO [2011] specifies a desirable clearance height of 16.5 ft. The bridge girder thickness is given as 6 ft so the total elevation difference between the two highways is 22.5 ft (16.5 + 6).

For the vertical alignment, the elevation change will be the final offsets of the sag and crest curves plus the change in elevation resulting from the constant-grade section connecting the two curves. The constant-grade section is included because the available distance of 837.24 ft (known from the length of the horizontal curve) is likely to be more than sufficient to affect a 22.5 ft elevation difference at a 40 mi/h design speed. Because both the sag and crest curves connect to a level grade at one end and have the constant grade in common at the other end, the A value will be the same for both curves and will also be the grade for the constant-grade section. That is,

$$A_s = A_c = A$$

With this information, the equation that will solve the vertical alignment for this problem is

$$\frac{AL_s}{200} + \frac{AL_c}{200} + \frac{A\left(837.24 - L_s - L_c\right)}{100} = 22.5$$

where the third term accounts for the elevation difference attributable to the constant-grade section connecting the sag and crest curves (see also Example 3.9 for comparison). Using $L = KA$, we have

$$\frac{A^2K_s}{200} + \frac{A^2K_c}{200} + \frac{A\left(837.24 - K_sA - K_cA\right)}{100} = 22.5$$

From Table 3.2 for 40 mi/h, $K_c = 44$, and from Table 3.3 for 40 mi/h, $K_s = 64$. Putting these values in the above equation gives

$$0.54A^2 + 8.374A - 1.08\,A^2 = 22.5$$
$$-0.54A^2 + 8.374A - 22.5 = 0$$

Solving this gives A = 3.458 and A = 12.049; A = 3.458% is chosen because we want to minimize the grade. For this value of A, the curve lengths are

$$L_s = K_sA = 64\left(3.458\right) = \underline{\underline{221.31 \text{ ft}}}$$

$$L_c = K_cA = 44\left(3.458\right) = \underline{\underline{152.15 \text{ ft}}}$$

and the length of the constant-grade section (L_{con}) will be 463.78 ft (837.24 – 221.31 – 152.15). This means that about 16.04 ft of the elevation difference will occur in the constant-grade section, with the remainder of the elevation difference attributable to the final curve offsets.

The stationing and elevation of the key points along the vertical alignment can now be calculated:

$$\text{station of } PVC_c = \text{station of } PC$$
$$= \underline{\underline{40+00}}$$
$$\text{station of } PVI_c = 40+00+L_c/2 = 40+00 \text{ plus } (1+52.15/2) = \underline{\underline{41+52.15}}$$
$$\text{station of } PVT_c = \text{station of } PVI + L_c$$
$$= 40+00 \text{ plus } \left(1+52.15\right) = \underline{\underline{41+52.15}}$$
$$\text{station of } PVC_s = \text{station of } PVT_c + L_{con}$$
$$= 41+52.15 \text{ plus } 4+63.78 = \underline{\underline{46+15.93}}$$
$$\text{station of } PVI_s = \text{station of } PVC_s + L_s/2$$
$$= 46+15.93 \text{ plus } \left(2+21.31\right)/2 = \underline{\underline{47+26.59}}$$
$$\text{station of } PVT_s = \text{station of } PT$$
$$= \text{station of } PVC_s + L_s$$
$$= 46+15.93 \text{ plus } \left(2+21.31\right) = \underline{\underline{48+37.24}}$$

The elevation of the new east-west road will be 588 ft which is determined by adding 22.5 ft (the 16.5 ft clearance plus the 6 ft girder depth) above the north-south road elevation of 565.5 ft. The *PC* and PVC_c are both at station 40 + 00 and elevation 588 ft (by inspection, the PVI_c will also be at this elevation)

$$\text{elevation } PVT_c = \text{elev } PVI_c - \frac{AL_c}{200}$$
$$= 588 - \frac{3.458(152.15)}{200} = \underline{\underline{585.37 \text{ ft}}}$$

$$\text{elevation } PVC_s = \text{elev } PVT_c - \frac{AL_{con}}{100}$$
$$= 585.37 - \frac{3.458(463.78)}{100} = \underline{\underline{569.33 \text{ ft}}}$$

$$\text{elevation } PVT_s = \text{elev } PVC_s - \frac{AL_c}{200}$$
$$= 569.33 - \frac{3.458(221.31)}{200} = \underline{\underline{565.00 \text{ ft}}}$$

The *PT* and PVT_s are both at station 48 + 37.24 and will thus both be at 565 ft (by inspection, the PVI_s will also be at this elevation).

Finally, the distance that must be cleared from the inside of the horizontal curve to provide sufficient stopping sight distance is determined by applying Equation 3.42:

$$M_s = R_v\left(1 - \cos\frac{90\,\text{SSD}}{\pi R_v}\right)$$

With R_v = 533 and SSD = 305 ft (from Table 3.1 at 40 mi/h),

$$M_s = 533\left(1 - \cos\frac{90(305)}{3.1416(533)}\right)$$
$$= \underline{\underline{21.67\ \text{ft}}}$$

Thus, a distance of at least 21.67 ft must be cleared from the center of the ramp's lane to the nearest sight obstruction on the inside of the curve.

NOMENCLATURE FOR CHAPTER 3

A	absolute value of the algebraic difference in grades (in percent)
a	coefficient in the parabolic curve equation or the deceleration in the stopping distance equation
b	coefficient in the parabolic curve equation
c	elevation of the *PVC*
D	degree of curvature
e	rate of superelevation
F_f	frictional side force
F_c	centripetal force
F_{cn}	centripetal force normal to the roadway surface
F_{cp}	centripetal force parallel to the roadway surface
f_s	coefficient of side friction
G	grade
G_1	initial roadway grade
G_2	final roadway grade
g	gravitational constant
H	height of vehicle headlights
H_c	clearance height of structure above sag curve
H_1	height of driver's eye
H_2	height of roadway object for stopping, height of oncoming car for passing
K	horizontal distance required to effect a 1% change in slope
L	length of curve
L_m	minimum length of curve
M	middle ordinate
M_s	middle ordinate for stopping sight distance
PC	initial point of horizontal curve
PI	point of tangent intersection (horizontal curve)
PSD	passing sight distance
PT	final point of horizontal curve
PVC	initial point of vertical curve
PVI	point of tangent intersection (vertical curve)
PVT	final point of vertical curve
R	radius of curve measured to the roadway centerline
R_v	radius of curve to the vehicle's traveled path
S	sight distance
SSD	stopping sight distance
T	tangent length
V	vehicle speed
V_1	initial vehicle speed
W	vehicle weight
W_n	vehicle weight normal to the roadway surface
W_p	vehicle weight parallel to the roadway surface
x	distance from the beginning of the vertical curve to specified point
x_{hl}	distance from the beginning of the vertical curve to high or low point
Y	vertical curve offset
Y_f	end-of-curve offset (vertical curve)
Y_m	midcurve offset (vertical curve)
α	angle of superelevation
β	upward angle of headlight beam
Δ	central angle
Δ_s	angle subtended by the stopping sight distance (SSD) arc

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PROBLEMS

Crest Vertical Curves (Section 3.3)

3.1 A 520-ft–long equal-tangent crest vertical curve connects tangents that intersect at station 340 + 00 and elevation 1325 ft. The initial grade is +4.0% and the final grade is −2.5%. Determine the elevation and stationing of the high point, *PVC*, and *PVT*.

3.2 Consider Example 3.4. Solve this problem with the parabolic equation (Eq. 3.1) rather than by using offsets.

3.3 Again consider Example 3.4. Does this curve provide sufficient stopping sight distance for a speed of 60 mi/h?

3.4 An equal-tangent crest vertical curve is designed for 70 mi/h. The high point is at elevation 1011.4 ft. The initial grade is +2% and the final grade is −1%. What is the elevation of the *PVT*?

3.5 An equal-tangent crest curve has been designed for 70 mi/h to connect a +2% initial grade and a −1% final grade for a new vehicle that has a 3 ft driver's eye height; the curve was designed to avoid an object that is 1 ft high. Standard practical stopping distance design was used but, unlike current design standards, the vehicle was assumed to make a 0.5g stop, although driver reactions are assumed to be the same as in current highway design standards. If the *PVC* of the curve is at elevation 848 ft and station 43 + 48, what is the station and elevation of the high point of the curve?

3.6 A vertical curve is designed for 55 mi/h and has an initial grade of +2.5% and a final grade of −1.0%. The *PVT* is at station 114 + 50. It is known that a point on the curve at station 112 + 35 is at elevation 245 ft. What is the stationing and elevation of the *PVC*? What is the stationing and elevation of the high point on the curve?

3.7 An equal-tangent crest vertical curve is designed for 65 mi/h. The initial grade is +3.4% and the final grade is negative. What is the elevation difference between the *PVC* and the high point of the curve?

3.8 An equal-tangent crest vertical curve has a 50-mi/h design speed. The initial grade is +3%. The high point is at station 33 + 40.76 and the *PVT* is at station 37 + 24.66. What is the elevation difference between the high point and the *PVT*?

3.9 An equal-tangent crest curve connects a +2% initial grade with a −1% final grade, and is designed for 55 mi/h. The station of the *PVI* is 233 + 40 with elevation 1203 ft. What is the elevation of the curve at station 234 + 00?

3.10 An equal-tangent vertical curve was designed in 2012 (to 2011 AASHTO guidelines) for a design speed of 70 mi/h to connect grades G_1 = +1.2% and G_2 = −2.1%. The curve is to be redesigned for a 70-mi/h design speed in the year 2025. Vehicle braking technology has advanced so that the recommended design deceleration rate is 25% greater than the 2011 value used to develop Table 3.1, but due to the higher percentage of older persons in the driving population, design reaction times have increased by 20%. Also, vehicles have become smaller so that the driver's eye height is assumed to be 3.0 ft above the pavement and roadway objects are assumed to be 1.0 ft above the pavement surface. Compute the difference in design curve lengths for the 2012 and 2025 designs.

3.11 An equal-tangent crest vertical curve is designed with a *PVI* at station 110 + 00 (elevation 927.2 ft) and a PVC at station 107 + 43.3 (elevation 921.55 ft). If the high point is at station 110 + 75.5, what is the design speed of the curve?

3.12 An equal-tangent crest vertical curve connects a +3.2% and a −1.1% grade. The *PVI* is at station 98 + 20. Due to drainage considerations, the highest point of the curve is at station 100 + 79.35. Determine the station of the *PVC* and *PVT*, and the design speed of the curve.

3.13 A 1200-ft equal-tangent crest vertical curve is currently designed for 50 mi/h. A civil engineering student contends that 60 mi/h is safe in a van because of the higher driver's-eye height. If all other design inputs are standard, what must the driver's-eye height (in the van) be for the student's claim to be valid?

3.14 A highway reconstruction project is being undertaken to reduce crash rates. The reconstruction involves a major realignment of the highway such that a 60-mi/h design speed is attained. At one point on the highway, a 720-ft equal-tangent crest vertical curve exists. Measurements show that at 3 + 40 stations from the *PVC*, the vertical curve offset is 3.5 ft. Assess the adequacy of this existing curve in light of the reconstruction design speed of 60 mi/h and, if the existing curve is inadequate, compute a satisfactory curve length.

3.15 An equal-tangent crest curve connects a +1.0% and a −0.5% grade. The *PVC* is at station 54 + 24 and the *PVI* is at station 56 + 92. Is this curve long enough to provide passing sight distance for a 60-mi/h design speed?

Sag Vertical Curves (Section 3.3)

3.16 A 1400-ft–long sag vertical curve (equal tangent) has a *PVC* at station 115 + 00 and elevation 750 ft. The initial grade is −3.5% and the final grade is +6.5%. Determine the elevation and stationing of the low point, *PVI*, and *PVT*.

3.17 An equal-tangent sag vertical curve is designed with the *PVC* at station 109 + 00 and elevation 950 ft, the *PVI* at station 110 + 77 and elevation 947.34 ft, and the low point at station 110 + 50. Determine the design speed of the curve.

3.18 An equal tangent vertical curve connects a −2% and a +3% grade. The low point of the curve is at elevation 297.88 ft. If the *PVI* is at elevation 295 ft, what is the design speed of the curve?

3.19 An equal-tangent sag equal tangent vertical curve is designed for 45 mi/h. The low point is 237 ft from the *PVC* at station 112 + 37 and the final offset at the *PVT* is 19.355 ft. If the *PVC* is at station 110 + 00, what is the elevation difference between the *PVT* and a point on the curve at station 111 + 00?

3.20 An equal tangent vertical curve connects an initial grade of -3% and a final grade of +1% and is designed for 60 mi/h. The *PVI* is at station 250+50 and elevation 732 ft. What is the station and elevation of the lowest point on the curve?

3.21 An overpass is being built over the *PVI* of an existing equal-tangent sag curve. The sag curve has a 70-mi/h design speed and G_1 = −5%, G_2 = +3%. Determine the minimum necessary clearance height of the overpass and the resultant elevation of the bottom of the overpass over the *PVI*. (Ignore the cross-sectional width of the overpass.)

3.22 An existing highway-railway at-grade crossing is being redesigned as grade separated to improve traffic operations. The railway must remain at the same elevation. The highway is being reconstructed to travel under the railway. The underpass will be a sag curve that connects to 2.25% tangent sections on both ends, and the *PVI* will be centered under the railway (a symmetrical alignment). The sag curve design speed is 45 mi/h. How many feet below the railway should the curve *PVI* be located?

3.23 An existing equal-tangent sag vertical curve is designed for 60 mi/h. The initial grade is −3% and the elevation of the *PVT* is 754 ft. The *PVC* of the curve is at station 134 + 16 and the *PVI* is at 137 + 32. An overpass is being constructed directly above the *PVI*. The highway is for cars only (AASHTO minimum and recommended structure clearances do not apply) and the overpass design assumes the driver's eye height is set conservatively to 5 ft. What is the lowest possible elevation of the bottom of the overpass structure to ensure sufficient stopping sight distance at 60 mi/h?

3.24 An equal-tangent sag curve has its *PVI* at station 10 + 00 and elevation at 138 ft. Directly above the *PVI*, the bottom of an overpass structure is at elevation 162 ft. The *PVC* is at station 4 + 00. If the initial grade is −4%, what is the highest possible value of the final grade given that a 70-mi/h design speed is to be provided in daytime conditions? What is the highest possible final grade in nighttime conditions? (Note: Be careful of units of *A*, and ignore the cross-sectional width of the overpass.)

Combined Crest and Sag Vertical Curves (Section 3.3)

3.25 Consider the bridge-tunnel problem in Example 3.9. Suppose a 70 mi/h interstate design speed is needed. If so, what would be the minimum bridge-tunnel separation distance (something higher than the current 1200 ft separation) needed to connect the elevations of the bridge and tunnel with 70 mi/h design-speed curves?

3.26 Two level sections of an east-west highway (*G* = 0) are to be connected. Currently, the two sections of highway are separated by a 4000-ft (horizontal distance), 2% grade. The westernmost section of highway is the higher of the two and is at elevation 100 ft. If the highway has a 60-mi/h design speed, determine, for the crest and sag vertical curves required, the stationing and elevation of the *PVC*s and *PVT*s given that the *PVC* of the crest curve (on the westernmost level highway section) is at station 0 + 00 and elevation 100 ft. In solving this problem, assume that the curve *PVI*s are at the intersection of $G = 0$ and the 2% grade, that is, $A = 2$.

3.27 Consider Problem 3.26. Suppose it is necessary to keep the entire alignment within the 4000 ft that currently separate the two level sections. It is determined that the crest and sag curves should be connected (the *PVT* of the crest and *PVC* of the sag) with a constant-grade section that has the lowest grade possible. Again using a 60-mi/h design speed, determine, for the crest and sag vertical curves, the stationing and elevation of the *PVC*s and *PVT*s given that the westernmost level section ends at station 0 + 00 and elevation 100 ft. (Note that A must now be determined and will not be equal to 2.)

3.28 Due to crashes at a railroad crossing, an overpass (with a roadway surface 26 ft above the existing road) is to be constructed on an existing level highway. The existing highway has a design speed of 50 mi/h. The overpass structure is to be level, centered above the railroad, and 180 ft long. What length of the existing level highway must be reconstructed to provide an appropriate vertical alignment?

3.29 A section of a freeway ramp has a +4.0% grade and ends at station 127 + 00 and elevation 138 ft. It must be connected to another section of the ramp (which has a 0.0% grade) that is at station 162 + 00 and elevation 97 ft. It is determined that the crest and sag curves required to connect the ramp should be connected (the *PVT* of the crest and *PVC* of the sag) with a constant-grade section that has the lowest grade possible. Design a vertical alignment to connect between these two stations using a 50-mi/h design speed. Provide the lengths of the curves and constant-grade section.

3.30 A tangent section of highway has a −1.0% grade and ends at station 4 + 75 and elevation 82 ft. It must be connected to another section of highway that has a −1.0% grade and that begins at station 44 + 12 and elevation 131.2 ft. The connecting alignment should consist of a sag curve, constant-grade section, and crest curve, and be designed for a speed of 50 mi/h. What is the lowest grade possible for the constant-grade section that will complete this alignment?

3.31 A roadway has a design speed of 50 mi/h, and at station 105 + 00 a +3.0% grade roadway section ends and at station 125 + 00 a +2.0% grade roadway section begins. The +3.0% grade section of highway (at station 105 + 00) is at a higher elevation than the +2.0% grade section of highway (at station 125 + 00). If a −4% constant-grade section is used to connect the crest and sag vertical curves that are needed to link the +3.0 and +2.0% grade sections, what is the elevation difference between stations 105 + 00 and 125 + 00? (The entire alignment, crest and sag curves, and constant-grade section must fit between stations 105 + 00 and 125 + 00.)

3.32 A sag curve and crest curve connect a −3.5% tangent section of highway (to the west) with a +2.5% tangent section of highway (to the east). The +2.5% tangent section is at a higher elevation than the −3.5% tangent section. The two tangent sections are separated by 1150 ft of horizontal distance. If the design speed of the curves is 50 mi/h, what is the common grade between the sag and crest curves (G_2 of sag and G_1 of crest, from west to east), and what is the elevation difference between the PVC_s and PVT_c?

3.33 A level section of highway is to be connected to a section of highway with a −5% grade. The level highway section ends at station 108 + 40 (elevation 865 ft) and is to connect with the −5% section of highway at station 139 + 20 (elevation 758 ft). Using a design speed of 50 mi/h, determine the stations and elevations of the *PVC*s, *PVI*s, and *PVT*s of the two vertical curves required to connect the highway segments, as well as the length of the constant grade section (connecting grade is to be as small as possible).

Horizontal Curves (Section 3.4)

3.34 You are asked to design a horizontal curve for a two-lane road. The road has 12-ft lanes. Due to expensive excavation, it is determined that a maximum of 34 ft can be cleared from the road's centerline toward the inside lane to provide for stopping sight distance. Also, local guidelines dictate a maximum superelevation of 0.08 ft/ft. What is the highest possible design speed for this curve?

3.35 A horizontal curve on a two-lane highway (10-ft lanes) is designed for 50 mi/h with a 6% superelevation. The central angle of the curve is 35 degrees and the *PI* is at station 482 + 72. What is the station of the *PT* and how many feet have to be cleared from the lane's shoulder edge to provide adequate stopping sight distance?

3.36 A horizontal curve on a single-lane highway has its *PC* at station 123 + 70 and its *PI* at station 130 + 90. The curve has a superelevation of 0.06 ft/ft and is designed for 70 mi/h. What is the station of the *PT*?

3.37 A horizontal curve is being designed through mountainous terrain for a four-lane road with lanes that are 10 ft wide. The central angle (Δ) is known to be 40 degrees, the tangent distance is 520 ft, and the stationing of the tangent intersection (*PI*) is 2600 + 00. Under specified conditions and vehicle speed, the roadway surface is determined to have a coefficient of side friction of 0.08, and the curve's superelevation is 0.09 ft/ft. What is the stationing of the *PC* and *PT* and what is the safe vehicle speed?

3.38 A new interstate highway is being built with a design speed of 70 mi/h. For one of the horizontal curves, the radius (measured to the innermost vehicle path) is tentatively planned as 2500 ft. What rate of superelevation is required for this curve?

3.39 On a roadway with two 12-ft lanes, a horizontal curve is designed for 35 mi/h with a 4% superelevation. It is known that $\Delta = 2\Delta_s$. The *PI* of the curve is at station 30 + 00. What is the station of the *PT* of the curve?

3.40 A developer is having a single-lane raceway constructed with a 200-mi/h design speed. A curve on the raceway has a radius of 4500 ft, a central angle of 30 degrees, and *PI* stationing at 1125 + 10. If the design coefficient of side friction is 0.20, determine the superelevation required at the design speed (do not ignore the normal component of the centripetal force). Also, compute the degree of curve, length of curve, and stationing of the *PC* and *PT*.

3.41 A horizontal curve is being designed for a new two-lane highway (12-ft lanes). The *PI* is at station 250 + 50, the design speed is 65 mi/h, and a maximum superelevation of 0.07 ft/ft is to be used. If the central angle of the curve is 38 degrees, design a curve for the highway by computing the radius and stationing of the *PC* and *PT*.

3.42 You are asked to design a horizontal curve with a 40-degree central angle (Δ = 40) for a two-lane road with 11-ft lanes. The design speed is 70 mi/h and superelevation is limited to 0.06 ft/ft. Give the radius, degree of curvature, and length of curve that you would recommend.

3.43 For the horizontal curve in Problem 3.42, what distance must be cleared from the inside edge of the inside lane to provide adequate stopping sight distance?

3.44 A horizontal curve on a single-lane freeway ramp is 400 ft long, and the design speed of the ramp is 45 mi/h. If the superelevation is 10% and the station of the *PC* is 18 + 25, what is the station of the *PI* and how much distance must be cleared from the center of the lane to provide adequate stopping sight distance?

3.45 A freeway exit ramp has a single lane and consists entirely of a horizontal curve with a central angle of 90 degrees and a length of 628 ft. If the distance cleared from the centerline for sight distance is 19.4 ft, what design speed was used?

3.46 A horizontal curve on a two-lane highway (12-ft lanes) has *PC* at station 123 + 80 and *PT* at station 129 + 60. The central angle is 35 degrees, the superelevation is 0.08, and 20.6 ft is cleared (for sight distance) from the inside edge of the innermost lane. Determine a maximum safe speed (assuming current design standards) to the nearest 5 mi/h.

3.47 A horizontal curve was designed for a four-lane highway for adequate SSD. Lane widths are 12 ft, and the superelevation is 0.06 and was set assuming maximum f_s. If the necessary sight distance required 52 ft of lateral clearance from the roadway centerline, what design speed was used for the curve?

Combined Vertical and Horizontal Curves (Section 3.5)

3.48 A section of highway has vertical and horizontal curves with the same design speed. A vertical curve on this highway connects a +1% and a +3% grade and is 420 ft long. If a horizontal curve on this highway is on a two-lane section with 12-ft lanes and has a central angle of 37 degrees and a superelevation of 6%, what is the length of the horizontal curve?

3.49 A section of a two-lane highway (12-ft lanes) is designed for 75 mi/h. At one point a vertical curve connects a −2.5% and +1.5% grade. The *PVT* of this curve is at station 36 + 50. It is known that a horizontal curve starts (has *PC*) 294 ft before the vertical curve's *PVC*. If the superelevation of the horizontal curve is 0.08 and the central angle is 38 degrees, what is the station of the *PT*?

3.50 Two straight sections of freeway cross at a right angle. At the point of crossing, the east-west highway is at elevation 150 ft and has a constant +5.0% grade (upgrade in the east direction), and the north-south highway is at elevation 125 ft and has a constant −3.0% grade (downgrade in the north direction). Design a 90-degree ramp that connects the northbound direction of travel to the eastbound direction of travel. Design the ramp for the highest design speed (to nearest 5 mi/h) with the constraint that the minimum allowable value of *D* is 8.0. (Assume that the *PC* of the horizontal curve is at station 15 + 00, and the vertical curve *PVI*s are at the *PC* and *PT*.) Give the stationing and elevations of the *PC*, *PT*, *PVC*s, and *PVT*s.

3.51 A crest vertical curve and a horizontal curve on the same highway have the same design speed. The equal-tangent vertical curve connects a +3% initial grade with a +1% final grade and has a *PVC* at 101 + 78 and a *PVT* at 106 + 72, The horizontal curve has a *PI* at 150 + 10 and a central angle of 75 degrees. If the superelevation of the horizontal curve is 8% and the road has two 12-ft lanes, what is the stationing of the *PT*?

Multiple Choice Problems (Multiple Sections)

3.52 A 400-ft equal-tangent sag vertical curve has its *PVC* at station 100 + 00 and elevation 500 ft. The initial grade is –4.0% and the final grade is +2.5%. Determine the elevation of the lowest point of the curve.

a) 495.077 ft
b) 495.250 ft
c) 485.231 ft
d) 492.043 ft

3.53 A horizontal curve is being designed around a pond with a tangent length of 1200 ft and central angle of 0.5211 radians. If the *PI* is at station 145 + 00, determine the station of *PT*.

a) 168 + 45.43
b) 156 + 45.43
c) 173 + 94.00
d) 156 + 72.72

3.54 A car is traveling over a 1400-ft vertical curve. One of the passengers decides to calculate the current offset from the PVC. By looking at the onboard navigation device, the passenger knows that the car is 750 feet from the PVC. The initial grade is +5.5% while the final roadway grade is +3.0%. What is the current offset?

a) 4.38 ft
b) 17.50 ft
c) 17.08 ft
d) 5.02 ft

3.55 You are designing a highway to AASHTO guidelines on rolling terrain where the design speed will be 65 mi/h. At one section, a +1.25% grade and a –2.25% grade must be connected with an equal-tangent vertical curve. Determine the minimum length of curve that can be designed while meeting SSD requirements.

a) 864.30 ft
b) 645.00 ft
c) 674.74 ft
d) 673.43 ft

3.56 A car is traveling downhill on a suburban road with a grade of 4% at a speed of 35 mi/h. Determine the required stopping sight distance.

a) 149.29 ft
b) 245.97 ft
c) 233.84 ft
d) 261.26 ft

3.57 A tow truck is searching a city street at 40 mi/h for illegally parked vehicles. It travels over an equal tangent vertical curve with an initial grade of +4.0% and final grade of –2.0%. If the height of the driver's eye is 6.0 ft and the driver spots a car 450 ft away with a height of 4.0 ft, what is the minimum length of the vertical curve for this situation?

a) 562.94 ft
b) 1304.15 ft
c) 240.07 ft
d) 306.85 ft