Geometric Design & Vertical Alignment

Highway and Traffic Engineering

Introduction

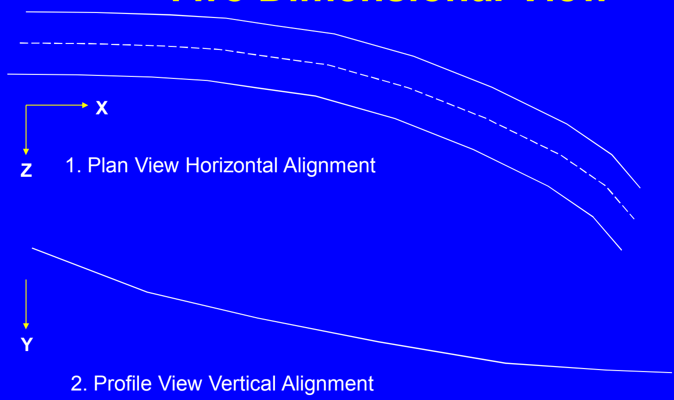
- It includes the specific design elements of a highway, such as
  - the number of lanes,
  - lane width,
  - median type and width,
  - length of freeway acceleration and deceleration lanes,
  - need for truck climbing lanes for highways on steep grades,
  - and radii required for vehicle turning.

- All these elements and the performance characteristics of vehicles play an important role.
- Physical dimensions of vehicles affect a number of design elements such as the
  - radii required for turning
  - Height of highway overpass
  - Lane width

Principles of Highway Alignment

- The alignment of a highway is a three dimensional problem with measurement in x, y and z direction.
- It is a bit complicated, therefore the alignment problem is typically reduced to two dimensional alignment as shown in figure on next slide.

Highway Alignment Two Dimensional View

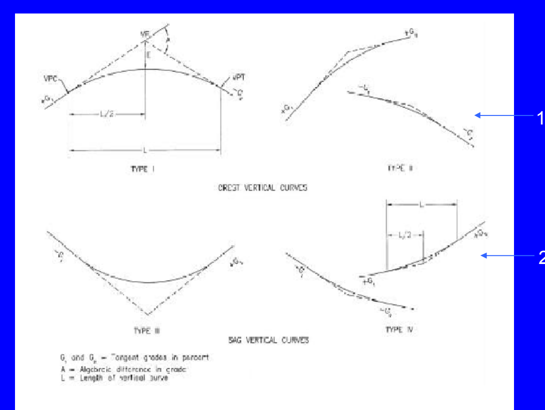


Vertical Alignment

- The objective of vertical alignment is
  - to determine the elevation of highway points to ensure proper roadway drainage and
  - an acceptable level of safety.
- The primary objective of vertical alignment lies in the transition of roadway elevation between two grades.

- This transition is achieved by the means of a vertical curve. These curves can be classified into;
  - Crest Vertical Curves
  - Sag Vertical Curves

Sag and Crest Vertical Curves



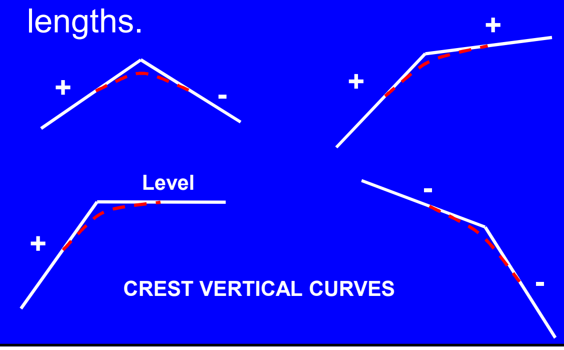
Where

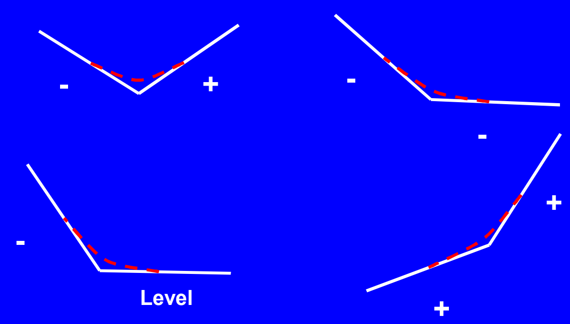
- G1 Initial roadway grade(initial tangent grade)
- G2 Final roadway grade
- A Absolute value of the difference in grades
- L Length of vertical curve measured in a horizontal plane
- PVC Initial point of the vertical curve

- PVI Point of vertical intersection (intersection of initial and final grades)
- PVT Final point of the vertical curve

- Vertical curves are almost arranged such that half of the curve length is positioned before the PVI and half after and are referred as equal tangent vertical curves.
- A circular curve is used to connect the horizontal straight stretches of road, a parabolic curve is usually used to connect gradients in the profile alignment.

- It provides a constant rate of change of slope and implies equal curve lengths.





SAG VERTICAL CURVES

Vertical Curve

- For a vertical curve, the general form of the parabolic equation is;

  $$Y = ax^2 + bx + c \quad \text{—— 1}$$

  where, 'y' is the roadway elevation of the curve at a point 'x' along the curve from the beginning of the vertical curve (PVC).

  'C' is the elevation of the PVC since x=0 corresponds the PVC

Slope of Curve

- To define 'a' and 'b', first derivative of equation 1 gives the slope.

$$\frac{dy}{dx} = 2ax + b \quad \text{—— 2}$$

- At PVC, x=0;

$$\frac{dy}{dx} = b$$

or

$$G = \frac{dy}{dx} \quad \text{—— 3}$$

$$G_1 = b$$

Where G_1 is the initial slope.

- Taking second derivative of equation1, i.e. rate of change of slope;

$$\frac{dy^2}{dx^2} = 2a \qquad \text{—— 4}$$

- The rate of change of slope can also be written as;

$$\frac{dy^2}{dx^2} = \frac{G_2 - G_1}{L} \qquad \text{—— 5}$$

$2a = \frac{G_2 - G_1}{L}$

- Equating equations 4 and 5

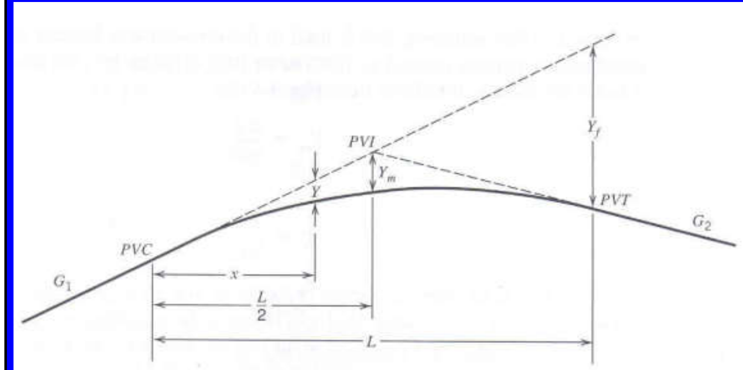
$$2a = \frac{G_2 - G_1}{L} \qquad \text{—— 6}$$

- or

$$a = \frac{G_2 - G_1}{2L} \qquad \text{—— 7}$$

Fundamentals of Vertical Curves

- For vertical curve design and construction, offsets which are vertical distances from initial tangent to the curve are important for vertical curve design.



Offsets for equal-tangent vertical curves.

G_1 = initial roadway grade in percent or ft/ft (m/m) (this grade is also referred to as the initial tangent grade, viewing Fig. from left to right),

G_2 = final roadway grade in percent or ft/ft (m/m),

PVC = point of the vertical curve (the initial point of the curve),

PVI = point of vertical intersection (intersection of initial and final grades),

PVT = point of vertical tangent, which is the final point of the vertical curve (the point where the curve returns to the final grade or, equivalently, the final tangent),

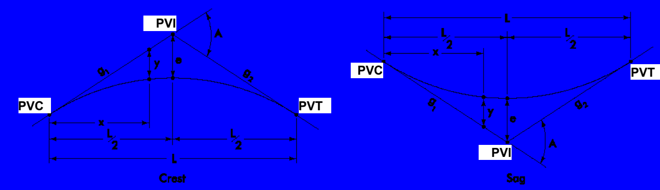
L = length of the curve in stations or ft (m) measured in a constant-elevation horizontal plane,

x = distance from the PVC in ft (m),

Y = offset at any distance x from the PVC in ft (m),

Y_m = midcurve offset in ft (m), and

Y_f = offset at the end of the vertical curve in ft (m).



A = Algebraic difference in gradients, $g_2 - g_1$.

L = Total length of vertical curve.

K = Rate of vertical curvature.

l_1 = Length of curve 1 (unsymmetrical vertical curve only).

l_2 = Length of curve 2 (unsymmetrical vertical curve only).

PVC = The Vertical Point of Curvature.

PVT = The Vertical Point of Tangency.

PVI = The Vertical Point of Intersection.

x = Horizontal distance to any point on the curve from the PVC

x_t = Turning point, which is the minimum or maximum point of the curve.

e = Middle ordinate, which is the vertical distance from the VPI to the arc.

y = Vertical distance at any point on the curve to the tangent grade.

r = Rate of change of grade.

E_{VPC} = Elevation of **PVC**

E_{VPT} = Elevation of **PVT**

E_x = Elevation of a point on the curve at a distance x from the **PVT**

E_t = Elevation of the turning point.

- A vertical curve also simplifies the computation of the high and low points or crest and sag vertical curves respectively, since high or low point does not occur at the curve ends PVC or PVT.
- Let 'Y' is the offset at any distance 'x' from PVC.

- Y_m is the mid curve offset & Yt is the offset at the end of the vertical curve.
- From an equal tangent parabola, it can be written as;

$$y = \frac{A}{200L}x^2 \qquad 8$$

where 'y' is the offset in feet and 'A' is the absolute value of the difference in grades(G2-G1, in %), 'L' is length of vertical curve in feet and 'x' is distance from the PVC in feet.

Putting the value of x=L in eq. 8

$$y_m = \frac{A}{200L}(\frac{L}{2})^2$$

$$y_m = \frac{AL}{800}$$

$$y_f = \frac{A}{200L} * L^2$$

$$y_f = \frac{AL}{200}$$

- First derivative van be used to determine the location of the low point, the alternative to this is to use a k-value which is defined as

$$k = \frac{L}{A}$$

where 'L' is in feet and 'A' is in %.

- This value 'k' can be used directly to compute the high / low points for crest/ sag vertical curves by

$x=kG_1$

where 'x' is the distance from the PVC to the high/ low point. 'k' can also be defined as the horizontal distance in feet required to affect a 1% change in the slope.

Stopping Sight Distance

- It is the sum of;
  - Vehicle stopping distance &
  - Distance traveled during perception / reaction time

Vehicle Stopping Distance

- Vehicle stopping distance is calculated by the following formula

$$d = \frac{v_1^2}{2g(f \pm G)}$$

where		
	V_1	initial speed of vehicle
	f	friction
	G	percent grade divided by 100

Distance Traveled During Perception/ Reaction Time

- It is calculated by the following formula

$$d_r = V_1 * t_r$$

where		
	V_1	Initial Velocity of vehicle
	t_r	time required to perceive and react to the need to stop

- Hence formula for the Stopping sight distance will be;

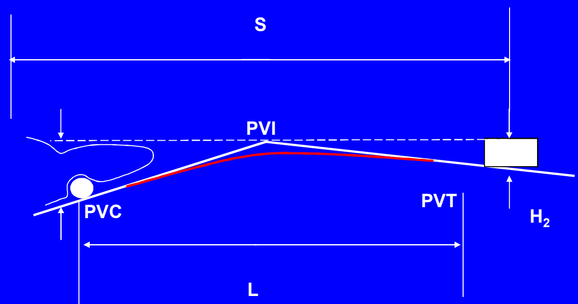
$$SSD = \frac{V_1^2}{2g(f \pm G)} + V_1 t_r$$

SSD and Crest Vertical Curve

- In providing the sufficient SSD on a vertical curve, the length of curve 'L' is the critical concern.
- Longer lengths of curve provide more SSD, all else being equal, but are most costly to construct.
- Shorter curve lengths are relatively inexpensive to construct but may not provide adequate SSD.

- In developing such an expression, crest and sag vertical curves are considered separately.
- For the crest vertical curve case, consider the diagram.

SSD and Crest Vertical Curve



S = Sight distance (ft)

H_1= height of driver's eye above road-way surface (ft)

L = length of the curve (ft)

H_2= height of roadway object (ft)

A = difference in grade

Lm= Minimum length required for sight distance.

Minimum Length of the Curve

- For a required sight distance S is calculated as follows;
- If S>L

$$Lm = 2S - \frac{200(\sqrt{H_1} + \sqrt{H_2})^2}{A}$$

- S<L

$$Lm = \frac{AS^2}{200200(\sqrt{H_1} + \sqrt{H_2})^2}$$

- For the sight distance required to provide adequate SSD, standard define driver eye height H_1 is 3.5 ft and object height H_2 is 0.5 ft. S is assumed is equal to SSD. We get

SSD > L $$Lm = 2SSD - \frac{1329}{A}$$

SSD < L $$Lm = \frac{ASSD^2}{1329}$$

- Working with the above equations can be cumbersome.
- To simplify matters on crest curves computations, K- values, are used.

L = K*A

where k is the horizontal distance in feet, required to affect 1 percent change in slope.

TABLE 3.2
Design Controls for Crest Vertical Curves Based on Minimum and Desirable Stopping-Sight Distance

Design Speed (mph)	Assumed Speed for Condition (mph)	Coefficient of Friction f	Stopping-Sight Distance, Rounded for Design (ft)	Rate of Vertical Curvature, K[a] [length (ft) per percent of A]	
				Computed[b]	Rounded for Design
20	20–20	0.40	125–125	8.6– 8.6	10– 10
25	24–25	0.38	150–150	14.4– 16.1	20– 20
30	28–30	0.35	200–200	23.7– 28.8	30– 30
35	32–35	0.34	225–250	35.7– 46.4	40– 50
40	36–40	0.32	275–325	53.6– 73.9	60– 80
45	40–45	0.31	325–400	76.4–110.2	80–120
50	44–50	0.30	400–475	106.6–160.0	110–160
55	48–55	0.30	450–550	140.4–217.6	150–220
60	52–60	0.29	525–650	180.2–302.2	190–310
65	55–65	0.29	550–725	227.1–394.3	230–400
70	58–70	0.28	625–850	282.8–530.9	290–540

Source: American Association of State Highway and Transportation Officials, "A Policy on Geometric Design of Highways and Streets," Washington, D.C., 1984.

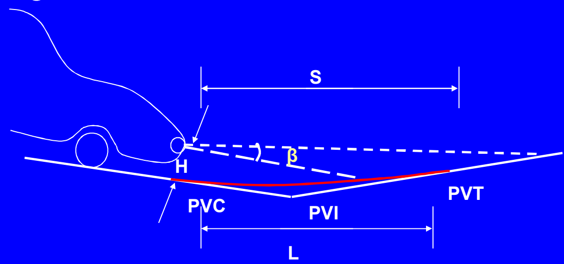
[a] Different K-values for the same speed result from using unequal coefficients of friction.

[b] Using computed values of stopping-sight distance.

SSD and Sag Vertical Curve

- Sag vertical curve design differs from crest vertical curve design in the sense that sight distance is governed by night time conditions, because in daylight, sight distance on a sag vertical curve is unrestricted.
- The critical concern for sag vertical curve is the headlight sight distance which is a function of the height of the head light above the road way, H, and the inclined upward angle of the head light beam, relative to the horizontal plane of the car, β.

- The sag vertical curve sight distance problem is illustrated in the following figure.



- By using the properties of parabola for an equal tangent curve, it can be shown that minimum length of the curve, Lm for a required sight distance is ;
- For S>L $$Lm = 2S - \frac{200(H + S\tan\beta)}{A}$$
- For S<L $$Lm = \frac{AS^2}{200(H + S\tan\beta)}$$

- For the sight distance required to provide adequate SSD, use a head light height of 2.0 ft and an upward angle of 1 degree.
- Substituting these design standards and S = SSD in the above equations;
- For SSD>L $$Lm = 2SSD - \frac{400 + 3.5SSD}{A}$$
- For SSD<L $$Lm = \frac{ASSD^2}{400 + 3.5SSD}$$

- As was the case for crest vertical curves, K-values can also be computed for sag vertical curves.
- Caution should be exercised in using the k-values in this table since the assumption of G=0 percent is used for SSD computations.

TABLE 3.3
Design Controls for Sag Vertical Curves Based on Minimum and Desirable Stopping-Sight Distance

Design Speed (mph)	Assumed Speed for Condition (mph)	Coefficient of Friction f	Stopping-Sight Distance, Rounded for Design (ft)	Rate of Vertical Curvature, K[a] [length (ft) per percent of A]	
				Computed[b]	Rounded for Design
20	20–20	0.40	125–125	14.7– 14.7	20– 20
25	24–25	0.38	150–150	21.7– 23.5	30– 30
30	28–30	0.35	200–200	30.8– 35.3	40– 40
35	32–35	0.34	225–250	40.8– 48.6	50– 50
40	36–40	0.32	275–325	53.4– 65.6	60– 70
45	40–45	0.31	325–400	67.0– 84.2	70– 90
50	44–50	0.30	400–475	82.5–105.6	90–110
55	48–55	0.30	450–550	97.6–126.7	100–130
60	52–60	0.29	525–650	116.7–153.4	120–160
65	55–65	0.29	550–725	129.9–178.6	130–180
70	58–70	0.28	625–850	147.7–211.3	150–220

Source: American Association of State Highway and Transportation Officals, "A Policy on Geometric Design of Highways and Streets," Washington, D.C., 1984.

[a] Different K-values for the same speed result from using unequal coefficients of friction.

[b] Using computed values of stopping-sight distance.