Chapter 14

Boundary Layer Flows

Lec10#

14.1 Introduction

All fluids have viscosity, that stickiness we are familiar with when we touch honey or molasses. Just as we pick up a delicious film when we dip into honey, so an airplane moving through air is sheathed with a film of air that is forever dragged along on its flight. A submarine is cloaked with a similar mantle, only thicker because of the submarine's slower speed in a fluid more viscous than air. There is nothing at all mysterious about this mantle: we define it as the region where shear stresses exist. Outside this layer, therefore, the shear stresses will be zero.

Take some oil and pour it on a flat horizontal board. Then lift the board slowly to a near vertical position and study the resultant flow of oil. You will see the frictional stresses that act whenever a viscous fluid moves over a surface. The stresses work their way from the boundary into the flow to retard it. Thus we would expect a loss of energy from the flow. To maintain a given flow, we must resupply energy, most often from some upstream source like a pump.

Prandtl showed that the differences in velocity of the fluid particles between the object and a moving fluid were confined to a region he called *Grenzschicht*, or *boundary layer*. Outside of this layer, the fluid flows as if there were no frictional resistance. Within the layer the distribution of velocity is similar to that shown in Fig. 14.1. Figure 14.1a is a typical laminar flow velocity profile, Fig. 14.1b is a typical turbulent flow velocity profile, Fig. 14.1c depicts conditions for separation, and Fig. 14.1d shows a profile of the separated region and the wake. The region between the dash-line and the wing surface represents the boundary layer.

The boundary layer is defined as the region where the shear stresses are finite. It is the shear stress on the surface that produces the friction drag force* on the body. The drag force due to friction is quite simple to calculate if the flow is laminar. It is not so simple, however, to calculate the drag force for turbulent flow.

We know that the fluid flow of a continuous medium is fully described by the Navier-Stokes equation. Only a few explicit solutions of these equations are known. In many cases it is extremely difficult even to determine approximations. We want to consider the following very simple problem: can we solve the Navier-Stokes equation for the two-dimensional steady flow of an incompressible fluid around a simply con-

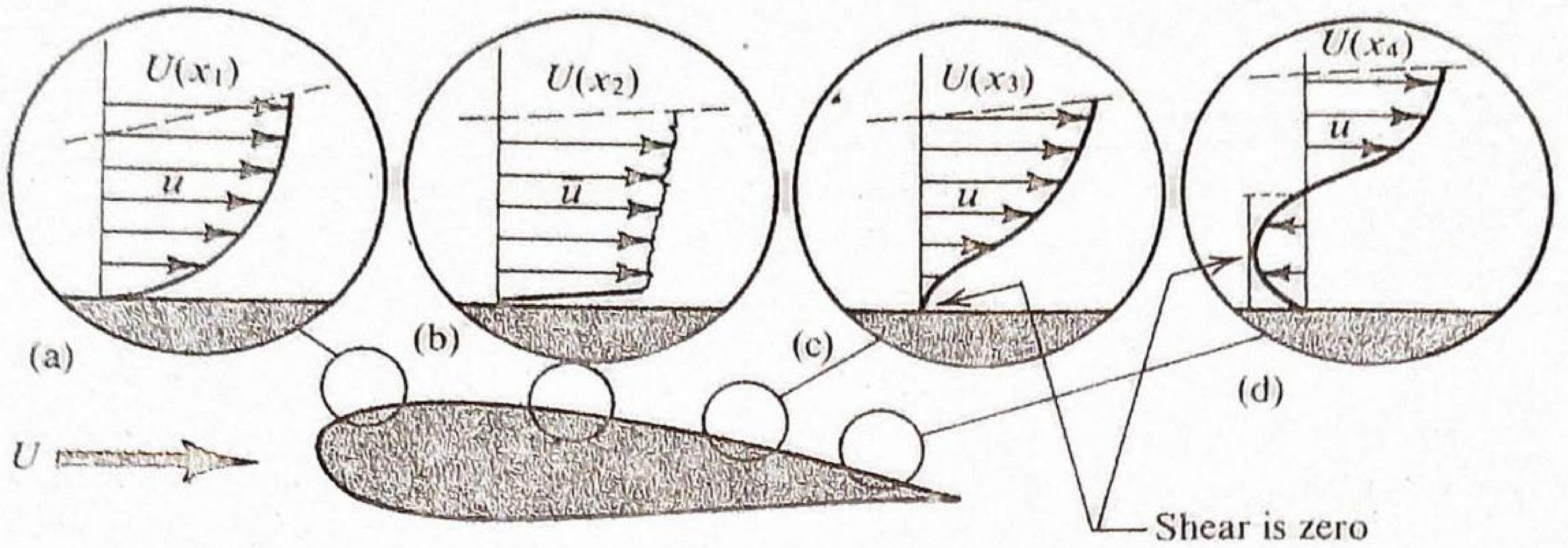

Figure 14.1 Velocity distributions in the boundary layer over an airfoil section. (Source: Lectures on viscous flows by Professor T. Sarpkaya. Reproduced here by written permission of Professor T. Sarpkaya.)

nected finite body which has a smooth continuous boundary? To solve this problem Prandtl simplified the Navier-Stokes equation by omitting certain terms that were small when compared to other terms. The result of his analysis, the *boundary layer* equations, are simpler than the Navier-Stokes equation, and need to be discussed in some detail.

14.1.1 Reynolds' Experiment

In order to gain some comprehension of what transpires in the transition from laminar to turbulent flow, we construct the classical experiment performed by Osborne Reynolds. Reynolds was the first to distinguish laminar from turbulent flow, showing that the fluid often passes from one into the other, just as a river moves from quiet pools to white rapids back to languid flows. Reynolds conducted his experiments on the flow of water through a glass tube in a manner shown in Fig. 14.2.

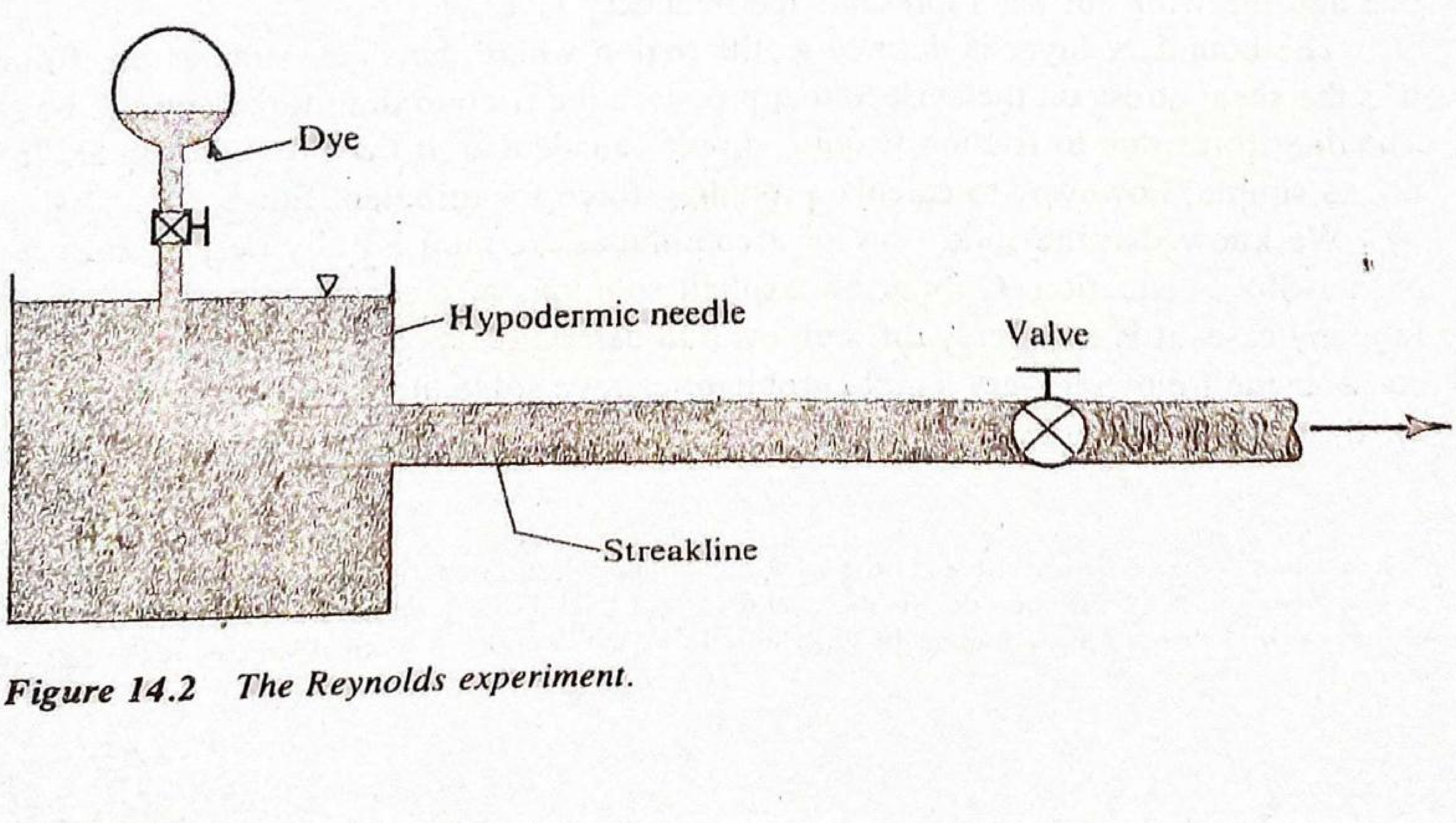

Figure 14.2 The Reynolds experiment.

A glass tube is mounted horizontally with one end placed in a large reservoir and the other end open to the atmosphere. The flow rate through the tube is controlled by a valve. A smooth bell mouth entrance is provided at the entrance to the tube along with a dye jet arrangement to allow a fine stream of dye to be injected into the stream and enter the tube with minimal disturbances. For low velocity flows the dye streakline moves, generating a straight line through the tube in a steady stable manner. Such a behavior is characteristic of a *laminar flow* through a pipe. Now, if the flow rate is gradually increased, the dye streakline will begin to waver until suddenly it bursts. The streakline is diffused downstream of the burst, indicating that the fluid and the dye are mixing. When this condition is reached, the flow has changed to *turbulent flow*, the chief characteristic of which is a violent interchange of macroscopic momentum that completely disrupts the orderly movement observed when the flow was laminar. (The photographs of Fig. 11.6 show this clearly.) By carefully controlling the experiment, Reynolds was able to obtain a value of Reynolds number $R_D = 12{,}000$ before turbulence was seen to occur. Many experimentalists since Reynolds have reproduced his results. Some have been able to maintain laminar flow up to Reynolds number 50,000. These upper critical Reynolds numbers have no practical significance in engineering design, since in the majority of cases, we cannot isolate the fluid flow from disturbances (such as external vibrations and inherent flow turbulence) that cause the flow to trip from laminar into turbulent flow.

On the other hand, the lower critical Reynolds number is of practical utility in fluid dynamics design. It is the highest value of the Reynolds number that can be tolerated and still allow the external disturbances (which might precipitate turbulence) to be dampened out. Even though transition occurs in the pipe between $R_D = 2300$ and $R_D = 4000$, we will assume in our calculations the lower value of $R_D = 2300$ to be the upper limit of laminar flow in a pipe.

In the laminar flow regime the pressure losses are directly proportional to the average velocity, whereas in turbulent flow the losses are proportional to the velocity to a power varying from 1.7 to 2.0. All this takes place in an internal flow. An analogous phenomenon occurs in boundary layer flows.

14.2 The Boundary Layer Concept

In 1904 Prandtl originated the concept of a boundary layer. He hypothesized that for fluids having relatively small viscosity, the effect of internal friction in the fluid is significant only in a narrow region surrounding solid boundaries or bodies where the fluid adheres to the boundary. Thus, close to a body in a region called the boundary layer is where shear stresses exert an increasingly larger effect on the fluid as we move toward the solid boundary, because of the increased velocity gradient $\partial u/\partial y$ as $y \to 0$. But outside the boundary layer where the effect of the shear stresses on the flow is small compared to values inside the boundary layer (since the velocity gradient $\partial u/\partial y$ is negligible), the fluid particles experience no vorticity, and the flow is similar to potential flow. Hence, the "surface" at the boundary layer is a rather fictitious surface dividing a rotational and irrotational flow. Note that *fluid can pass through this "surface" of boundary layer*.

The classical procedure that illustrates many of the characteristics of the boundary layer is to place a semi-infinite flat plate in a uniform stream of infinite extent. Such a geometry is shown in Fig. 14.3.

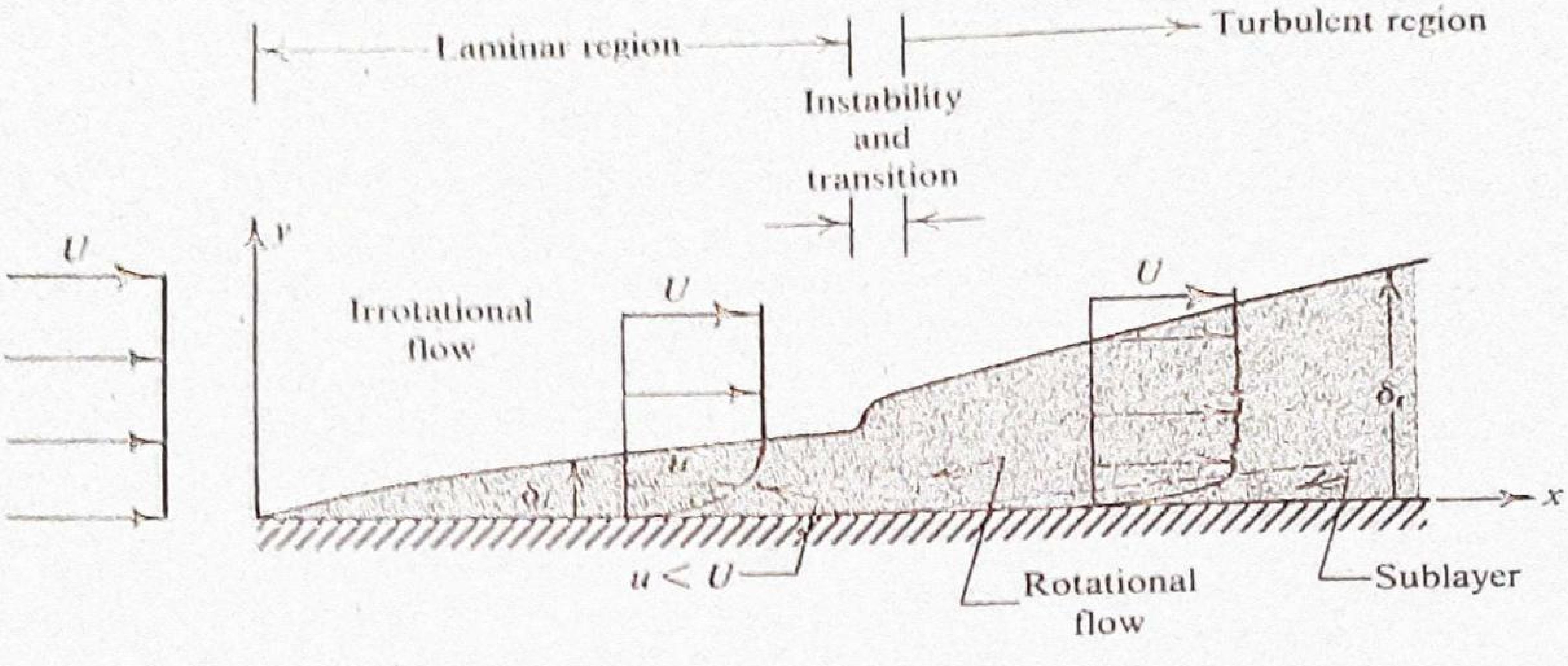

Figure 14.3 Detail of flow over a flat plate.

Experimental observations indicate that the fluid on the plate has zero velocity. As the vertical distance increases, so does the velocity, which attains very nearly the free stream velocity a short distance, δ, away from the plate. As with Reynolds' pipe flow experiment, the boundary layer has a region where the flow can become turbulent, as indicated in Fig. 14.3. (Experiments indicate that the flow in the boundary layer on a flat plate becomes turbulent at a Reynolds number $R_x = Ux/\nu$ between 5×10^5 and 10^7.)

Figure 14.3 shows three distinct regions of flow: laminar, transitional, and turbulent flow. To see how a fluid parcel behaves in these three zones, consider a rectangular fluid parcel following the streamline in Fig. 14.4. Outside the boundary layer, the parcel moves in pure translation since the stresses on the bottom and top faces are identical. In the laminar boundary layer, the fluid parcel follows a rectilinear path, but, since unequal shear stresses cause the upper face to move faster than the lower face the parcel deforms, and we observe both translation and rotation. In the transition region, the path changes from rectilinear to curvilinear. Oscillations build up, and the path becomes unstable. When the parcel reaches the turbulent region, the path is undefined and three-dimensional, and the parcel rotates unpredictably.

The boundary layer thickness of each region is different, and so one of the first tasks before us is to estimate the thickness of the laminar region. This can be accomplished quite simply by considering the boundary layer as that region where the viscous force per unit volume is of the same order of magnitude as the inertial force per unit volume. The thickness of the boundary layer is denoted by δ, and is in the y-direction. Since we are concerned solely with the boundary layer of the flow, the order of

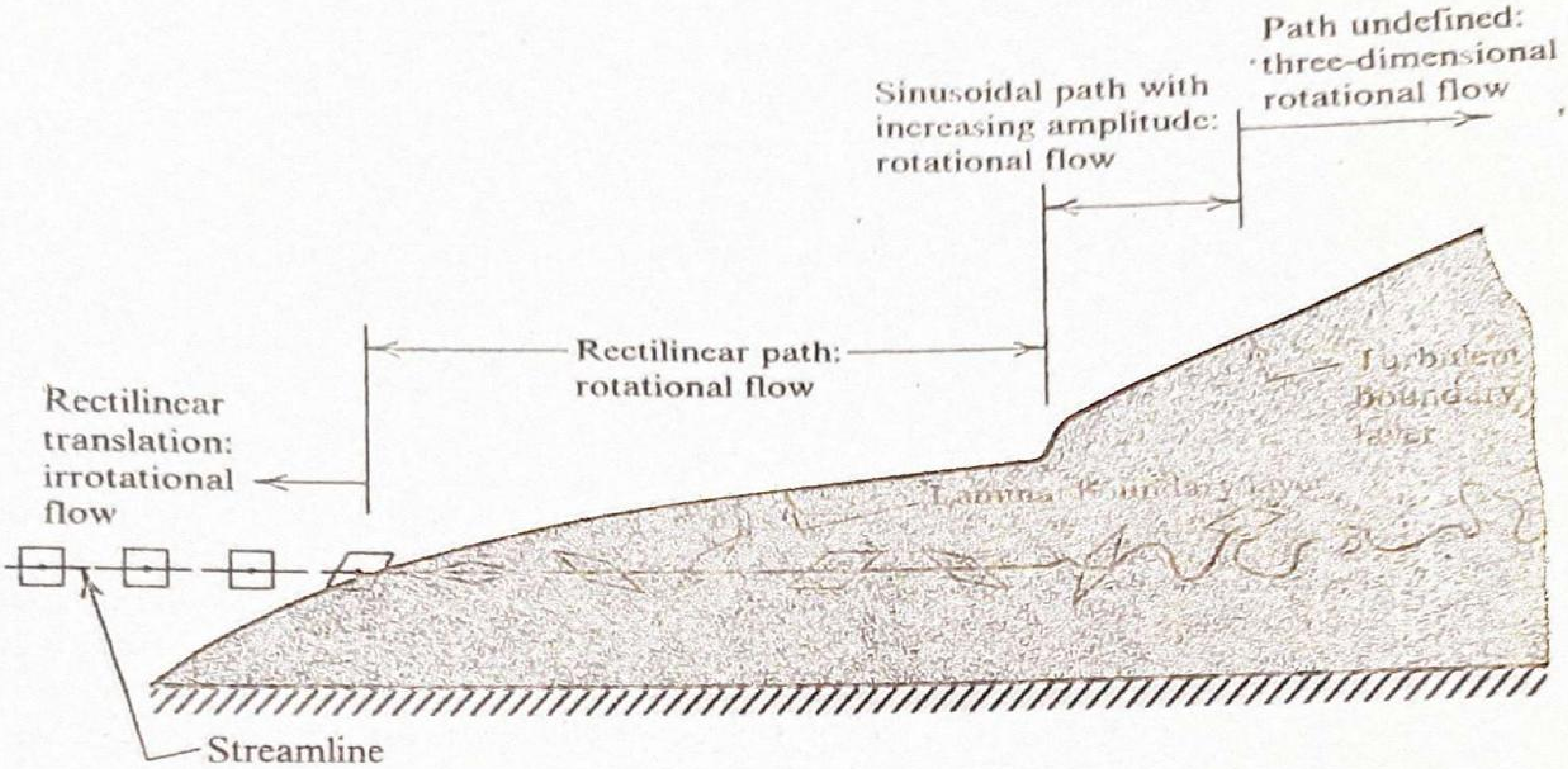

Figure 14.4 Behavior of a fluid parcel traveling along a streamline through a boundary layer along a flat plate.

magnitude of y is δ. The largest value of the velocity component u is the free-stream velocity U. The inertial force per unit volume is $\rho u(\partial u/\partial x)$, and for laminar flow the viscous force per unit volume is

$$\frac{\partial p_{xy}}{\partial y} = \mu\frac{\partial^2 u}{\partial y^2}$$

if the flow is parallel. Making the following estimations on orders of magnitude:

$$u \sim U,\ y \sim \delta \tag{14.1}$$

$$\frac{\partial u}{\partial x} \sim \frac{U}{x}$$

$$\frac{\partial u}{\partial y} \sim \frac{U}{\delta}$$

$$\frac{\partial^2 u}{\partial y^2} \sim \frac{U}{\delta^2} \tag{14.2}$$

and setting

$$\rho u\,\frac{\partial u}{\partial x} \sim \mu\,\frac{\partial^2 u}{\partial y^2}$$

we get

$$\rho \frac{U^2}{x} \sim \mu \frac{U}{\delta^2} \tag{14.3}$$

or solving for the dimensionless boundary layer thickness, we get

$$\frac{\delta}{x} \sim \sqrt{\frac{\mu}{\rho U x}} \sim \frac{1}{\sqrt{R_x}} = \frac{k}{\sqrt{R_x}} \tag{14.4}$$

The constant of proportionality k of Eq. (14.4) turns out to be 5.0 and will be found by solving the laminar boundary layer equation to be presented in the next section. Examination of Eq. (14.4) shows that the boundary layer thickness for laminar flow over a flat plate is inversely proportional to the square root of the density and free-stream velocity, and proportional to the square root of the dynamic viscosity and the distance from the leading edge of the flat plate. (This should be compared with the result obtained for the suddenly accelerated flat plate, Eq. (9.44).) Thus, at the leading edge of the flat plate, we expect and obtain zero thickness. Also, the more viscous a fluid is, the thicker the boundary layer. Thus at supersonic and hypersonic speeds, imagine the tremendous *magnitude* of the shear stresses that move the fluid from zero velocity to the velocity of the body. And these stresses must act in an infinitesimally thin region no thicker than a hair filament.