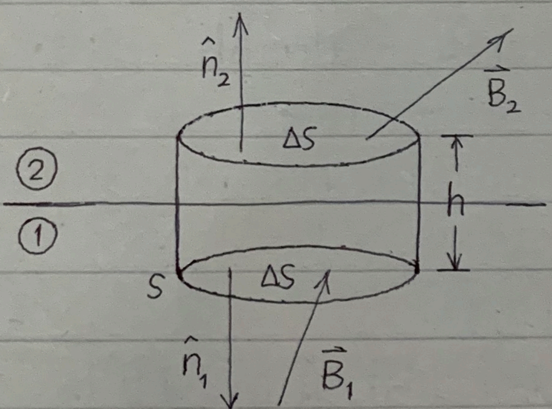
Boundary conditions on the Field Vectors:

Boundary condition on $\vec{B}$:



$\Delta S = \pi r^2$

$r \equiv$ radius of the cylindrical pillbox.

Consider two different media, 1 and 2, in contact, as shown in the Fig., above. Let us construct a small pillbox-shaped surface 'S', which intersects the interface. The height of the pillbox being negligibly small in comparison with the diameter of the bases.

The magnetic flux through any closed surface is zero, i.e., while crossing a boundary.

$$\oint_S \vec{B} \cdot d\vec{a} = 0$$

or $\oint_S \vec{B} \cdot \hat{n}\, da = 0$; $\hat{n} \equiv$ unit vector normal to the area element, 'da'.

where 's' is the entire surface of the pillbox.

$$\underbrace{\int \vec{B}_2 \cdot \hat{n}_2\, da}_{\text{top surface}} + \underbrace{\int \vec{B}_1 \cdot \hat{n}_1\, da}_{\text{bottom surface}} + \underbrace{\int \vec{B} \cdot \hat{n}\, da}_{\text{curved surface}} = 0$$

$$\underbrace{\text{curved surface}}_{0}$$

Since, Area of the curved surface $= 2\pi r h \simeq 0$ as 'h' is negligibly small.

$$\vec{B}_2 \cdot \hat{n}_2 \Delta S + \vec{B}_1 \cdot \hat{n}_1 \Delta S = 0$$

$\vec{B}_1$ = Magnetic field in medium ①.

$\vec{B}_2$ = Magnetic field in medium ②.

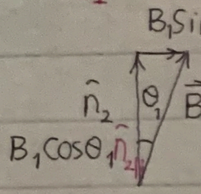
$$\vec{B}_2 \cdot \hat{n}_2 + \vec{B}_1 \cdot \hat{n}_1 = 0$$

From the Fig., it is clear that

$$\hat{n}_1 = -\hat{n}_2$$

$$\therefore \quad \vec{B}_2 \cdot \hat{n}_2 - \vec{B}_1 \cdot \hat{n}_2 = 0$$

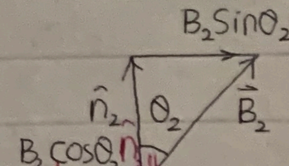
$$\vec{B}_1 = B_1 \sin\theta_1 \hat{n}_{2\perp} + B_1 \cos\theta_1 \hat{n}_{2\parallel}$$



$$(\vec{B}_2 - \vec{B}_1) \cdot \hat{n}_2 = 0 \qquad B_2 \cos\theta_2 - B_1 \cos\theta_1 = 0$$

$$\left[(B_2 \sin\theta_2 - B_1 \sin\theta_1)\hat{n}_{2\perp} + (B_2\cos\theta_2 - B_1\cos\theta_1)\hat{n}_{2\parallel}\right] \cdot \hat{n}_2 = 0$$

$$B_2^{\perp} - B_1^{\perp} = 0$$



$$(\quad)\underbrace{\hat{n}_{2\perp} \cdot \hat{n}_2}_{=0} + (\quad)\underbrace{\hat{n}_{2\parallel} \cdot \hat{n}_2}_{=1} = 0$$

$$\Rightarrow \boxed{B_2^{\perp} = B_1^{\perp}}$$

As, $\vec{\nabla} \cdot \vec{B}(\vec{r}) = 0$ always!

$$\Rightarrow B_2 \cos\theta_2 - B_1 \cos\theta_1 = 0$$

normal to what?

normal to the interface

$$\text{or} \quad B^{\perp}_{above} = B^{\perp}_{below}$$

i.e., the normal component of $\vec{B}$ is continuous across the interface.

Boundary Condition on $\vec{H}$:

A boundary condition on the $\vec{H}$-field may be obtained by applying Ampere's circuital law, to the rectangular loop ABCD. On this path, the lengths $\overline{AB}$ and $\overline{CD}$ will be taken equal to 'ΔL' and the segments AD and BC will be assumed negligibly small. If there is a surface current on the interface, then the current enclosed by the rectangular path is $|\Delta\vec{L} \times \vec{K}|$. $\vec{K}$ = surface current density

Boundary Conditions on $\vec{B}$:

Recall; $\oint_S \vec{E}(\vec{r}) \cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc.}$

$Q_{enc.} = \oint_S \sigma da$; σ is uniform.

$\Rightarrow E_2^{\perp} - E_1^{\perp} = \sigma/\epsilon_0$

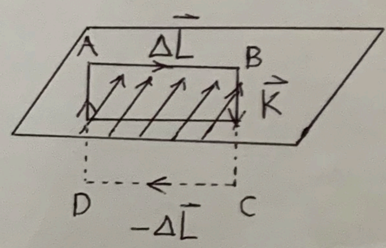
and $\vec{\nabla} \times \vec{E}(\vec{r}) = 0 \Rightarrow \oint_C \vec{E}(\vec{r}) \cdot d\vec{l} = 0 \Rightarrow E_2^{\parallel} - E_1^{\parallel} = 0$.

Now, $\oint_S \vec{B}(\vec{r}) \cdot d\vec{a} = 0 \Rightarrow B_2^{\perp} - B_1^{\perp} = 0$.

$\oint_C \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_0 I_{enc.}$

$I_{enc.} = \int_C K\, dl$; K is uniform.

$= K \Delta L$



$\Rightarrow B_2^{\parallel} - B_1^{\parallel} = \mu_0 K$