$$\therefore \quad \oint_C (\vec{r}\cdot\vec{r}\,')\, d\vec{l}\,' = -\int_S \vec{r}\times d\vec{a}\,' = -\vec{r}\times\int_S d\vec{a}\,'$$

$$= \left[\frac{1}{2}\oint_C \vec{r}\,'\times d\vec{l}\,'\right]\times\vec{r}$$

$$\therefore \quad \vec{A}_{dip}(\vec{r}) = \frac{\mu_0 I}{4\pi}\,\frac{1}{r^3}\left(\frac{1}{2}\oint_C \vec{r}\,'\times d\vec{l}\,'\right)\times\vec{r}$$

$$= \frac{\mu_0}{4\pi}\,\frac{1}{r^3}\underbrace{\left(\frac{I}{2}\oint_C \vec{r}\,'\times d\vec{l}\,'\right)}_{\equiv\, I\vec{a}\,'}\times\vec{r}$$

$$\vec{m} = I\vec{a}\,' = \frac{I}{2}\oint_C \vec{r}\,'\times d\vec{l}\,'$$ ≡ Magnetic dipole momen
magnetic moment of the
circuit.

In general, the magnetic dipole moments associated with line, surface, and volume currents:

$$\vec{m} = \frac{1}{2}\int_C \vec{r}\,' \times \vec{I}(\vec{r}\,')\, d\vec{l}\,' \qquad \text{If start with: } \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_C \frac{\vec{I}(\vec{r}\,')\, dl'}{|\vec{r}-\vec{r}\,'|}$$

For line currents

$$\vec{m} = \frac{1}{2}\int_S \vec{r}\,' \times \vec{K}(\vec{r}\,')\, da' \qquad \text{If start with: } \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_S \frac{\vec{K}(\vec{r}\,')\, da'}{|\vec{r}-\vec{r}\,'|}$$

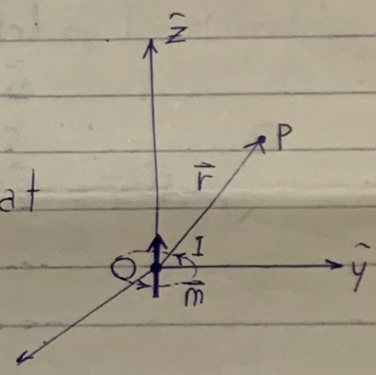
For surface currents (of course, $da' = da'_{\perp}$)

$$\vec{m} = \frac{1}{2}\int_V \vec{r}\,' \times \vec{J}(\vec{r}\,')\, d\tau' \qquad \text{If start with: } \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{r}\,')\, d\tau'}{|\vec{r}-\vec{r}\,'|}$$

For volume currents

$$\therefore \quad \vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m} \times \vec{r}}{r^3}$$

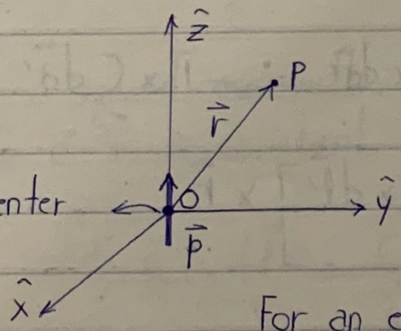
For a magnetic dipole located at the origin.



In general, $\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m} \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$; $\vec{m} \neq \vec{m}(\vec{r}\,')$

i.e., * $\vec{m}$ is constant vector.

Recall;



For an electric dipole located / placed at the origin,

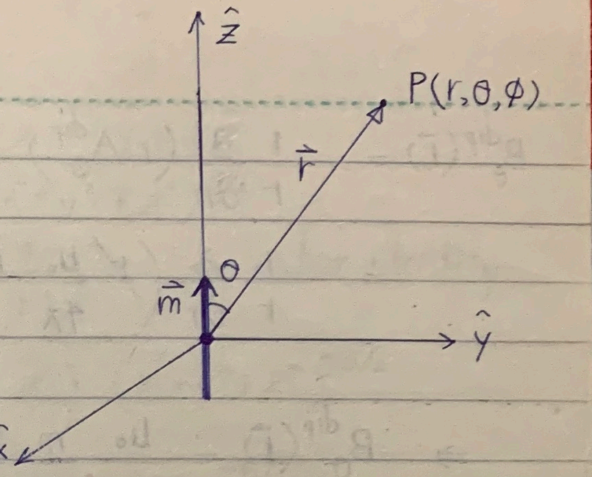
$$V_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3} \quad ; \quad \vec{p} \equiv \text{electric dipole moment}$$

In general, $V_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$; $\vec{p} \neq \vec{p}(\vec{r}\,')$

i.e, $\vec{p}$ is constant ve

* $\vec{m} = \frac{I}{2}\oint_C \vec{r}\,' \times d\vec{l}\,' \Rightarrow \vec{r}\,'$ dependance will be integrated out ; I is steady current

i.e., $I \neq I(\vec{r}\,', t) \Rightarrow I$ is constant $\Rightarrow \vec{m}$ is constant vector



$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

$$\vec{m} = m\hat{z} = m(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

$$\vec{r} = r\hat{r}$$

$$\vec{m}\times\vec{r} = (m\cos\theta\,\hat{r} - m\sin\theta\,\hat{\theta})\times r\hat{r}$$

$$= mr\cos\theta\underbrace{(\hat{r}\times\hat{r})}_{=0} - mr\sin\theta\underbrace{(\hat{\theta}\times\hat{r})}_{=-\hat{\phi}}$$

$$= mr\sin\theta\,\hat{\phi}$$

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi} \equiv \underbrace{\vec{A}_{dip}(r,\theta)}_{\text{explicit dependance on } r \,\&\, \theta.}$$

$$A_{dip}(\vec{r}) \sim \frac{1}{r^2}$$

Compare with: $V_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$; $V_{dip}(\vec{r}) \sim \frac{1}{r^2}$

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi} = A_\phi^{dip}(r,\theta)\hat{\phi} \text{ or } A_\phi^{dip}(\vec{r})\hat{\phi}$$

$$A_r^{dip}(\vec{r}) = 0 = A_\theta^{dip}(\vec{r})$$

$$\vec{B}_{dip}(\vec{r}) = \vec{\nabla}\times\vec{A}_{dip}(\vec{r}) = B_r^{dip}(\vec{r})\hat{r} + B_\theta^{dip}(\vec{r})\hat{\theta} + B_\phi^{dip}(\vec{r})\hat{\phi}$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi^{dip}) - \cancel{\frac{\partial A_\theta^{dip}}{\partial\phi}}^{=0}\right]\hat{r}$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\cancel{\frac{\partial A_r^{dip}}{\partial\phi}}^{=0} - \frac{\partial}{\partial r}(rA_\phi^{dip})\right]\hat{\theta} + \frac{1}{r}\left[\cancel{\frac{\partial}{\partial r}(rA_\theta^{dip})} - \cancel{\frac{\partial A_r^{dip}}{\partial\theta}}^{=0}\right]\hat{\phi}$$

$$\therefore\ B_r^{dip}(\vec{r}) = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\phi^{dip})$$

$$= \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\right) = \frac{\mu_0}{4\pi}\frac{m}{r^3\cancel{\sin\theta}}\underbrace{\frac{\partial}{\partial\theta}(\sin^2\theta}_{=2\cancel{\sin\theta}\cos\theta}$$

$$B_r^{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{m}{r^3}(2\cos\theta)$$

$$B_\theta^{dip}(\vec{r}) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r A_\phi^{dip}\right)$$

$$= -\frac{1}{r}\frac{\partial}{\partial r}\left(\cancel{r}\,\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^{\cancel{2}}}\right) = -\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r}\underbrace{\frac{\partial}{\partial r}\left(\frac{1}{r}\right)}_{\equiv -1/r^2}$$

$$\Rightarrow \quad B_\theta^{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{m}{r^3}(\sin\theta)$$

and $\quad B_\phi^{dip}(\vec{r}) = 0$

$$\therefore \quad \vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{m}{r^3}2\cos\theta\,\hat{r} + \frac{\mu_0}{4\pi}\frac{m}{r^3}\sin\theta\,\hat{\theta}$$

$$\Rightarrow \quad \boxed{\vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{m}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right) = \vec{B}_{dip}(r,\theta)}$$

in spherical coordinate system.

explicit dependance on r & θ.

Note: $\quad B_{dip}(\vec{r}) \sim \frac{1}{r^3}$

Compare with: $\quad \vec{E}_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right) \equiv \vec{E}_{dip}(r,\theta)$

$$E_{dip}(\vec{r}) \sim \frac{1}{r^3}$$