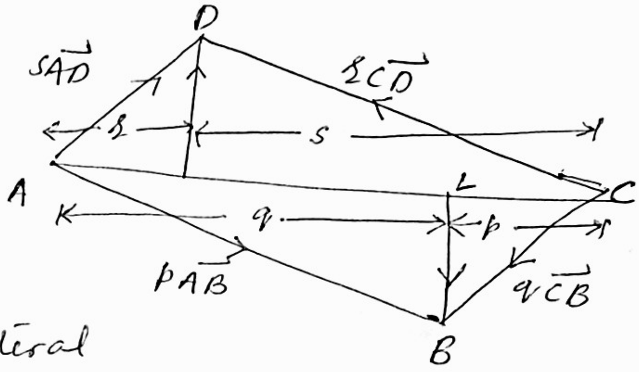
Exercise Set 2

Q1 If the forces $p\vec{AB}$, $q\vec{CB}$, $r\vec{CD}$, $s\vec{AD}$ act along the sides of a plane quadrilateral are in equilibrium show that $pr = qs$.

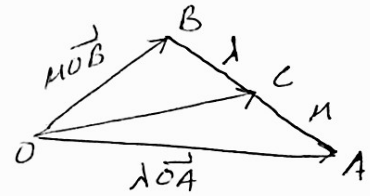
SOLUTION



Consider the forces $p\vec{AB}$, $q\vec{CB}$, $r\vec{CD}$, $s\vec{AD}$ acting along the sides of a plane quadrilateral ABCD. The system is in equilibrium under the action of these forces.

To prove $pr = qs$

Applying (λ, μ) theorem to the concurrent forces $p\vec{AB}$ and $q\vec{CB}$.



$\lambda\vec{OA} + \mu\vec{OB} = (\lambda+\mu)\vec{OC}$

or $\lambda\vec{AO} + \mu\vec{BO} = (\lambda+\mu)\vec{CO}$

$$p\vec{AB} + q\vec{CB} = (p+q)\vec{LB}$$

s.t $AL : LC = q : p$

i.e $\dfrac{AL}{LC} = \dfrac{q}{p} \longrightarrow (1)$

Also applying (λ, μ) theorem to the concurrent forces $s\vec{AD}$ and $r\vec{CD}$

$$s\vec{AD} + r\vec{CD} = (s+r)\vec{MD}$$

s.t $AM : MC = r : s$

i.e $\dfrac{AM}{MC} = \dfrac{r}{s} \longrightarrow (2)$

Now we are left with the two forces $(p+q)\vec{LB}$ and $(r+s)\vec{MD}$. Since the system is in equilibrium under the action of these two forces, this is possible only when these forces are equal in magnitude opposite in direction and their line of action must be same. This is possible only when

$$L = M$$

Using in (1)

$$\frac{AM}{MC} = \frac{q}{p} \longrightarrow (3)$$

From (2) & (3),

$$\frac{r}{s} = \frac{q}{p}$$

$$pr = qs \quad \text{as required.}$$

Q 10

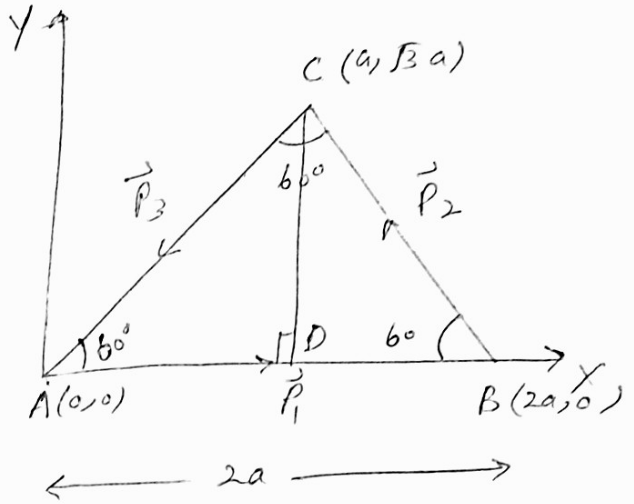
A system of forces acts on a plate in the form of an equilateral triangle of side $2a$. The moments of the forces about the three vertices are G_1, G_2, G_3 respectively. Find the magnitude of the resultant.

Soln.

Let the system of forces be equivalent to three forces P_1, P_2, P_3 acting along the sides of an equilateral triangle ABC of side $2a$.

Let D be the mid point of the side AB.

Let G_1, G_2, G_3 be the moments of forces about A, B, C respectively.



We know that

$$G' = G - xY + yX$$

At A(0,0)

$$G_1' = G$$

Also $G' = G - xY + yX$

At B(2a, 0)

$$G_2 = G_1 - (2a)Y + (0)X$$

or $2aY = G_1 - G_2$

$$Y = \frac{1}{2a}(G_1 - G_2)$$

and $G' = G - xY + yX$

At $C(a; \sqrt{3}a)$

$$G_3 = G_1 - a\frac{1}{2a}(G_1 - G_2) + (\sqrt{3}a)X$$

or $2G_3 = 2G_1 - G_1 + G_2 + 2\sqrt{3}aX$

$$2G_3 = G_1 + G_2 + 2\sqrt{3}a X$$

or $2\sqrt{3}aX = 2G_3 - G_1 - G_2$

or $X = \frac{1}{2\sqrt{3}a}(2G_3 - G_1 - G_2)$

Let R be the magnitude of resultant force, Then

$$R = \sqrt{X^2 + Y^2}$$

$$= \sqrt{\left[\frac{1}{2\sqrt{3}a}(2G_3 - G_1 - G_2)\right]^2 + \left[\frac{1}{2a}(G_1 - G_2)\right]^2}$$

$$= \sqrt{\frac{1}{12a^2}(4G_3^2 + G_1^2 + G_2^2 - 4G_1G_3 + 2G_1G_2 - 4G_2G_3) + \frac{1}{4a^2}(G_1^2 + G_2^2 - 2G_1G_2)}$$

$$R = \sqrt{\frac{1}{12a^2}(4G_3^2 + G_1^2 + G_2^2 - 4G_1G_3 + 2G_1G_2 - 4G_2G_3 + 3G_1^2 + 3G_2^2 - 6G_1G_2}$$

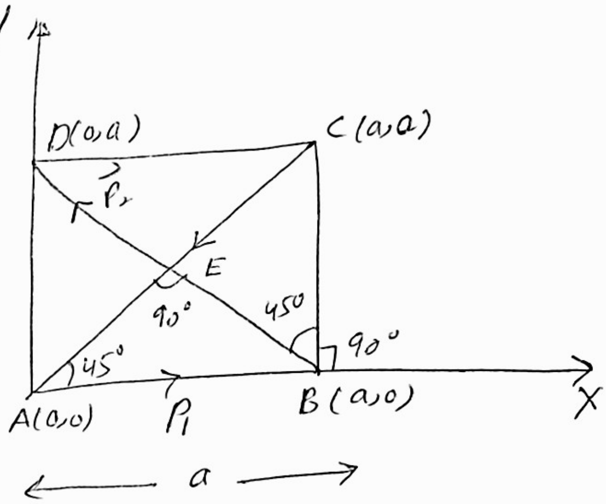
$$= \sqrt{\frac{1}{12a^2}(4G_1^2 + 4G_2^2 + 4G_3^2 - 4G_1G_2 - 4G_2G_3 - 4G_1G_3)}$$

$$= \sqrt{\frac{G_1^2 + G_2^2 + G_3^2 - G_1G_2 - G_2G_3 - G_1G_3}{3a^2}}$$

Q11 A couple of moment G acts on a square board ABCD of side a. Replace the couple by the forces acting along AB, BD, CA.

Solution

Consider a couple of moment G is acting on a square board ABCD whose each side is a.



Let the couple of moment G is replaced by the forces $\vec{P_1}, \vec{P_2}, \vec{P_3}$ along the sides AB, BD, CA respectively.

Resolving the forces along the coordinate axes.

$$X = P_1 \cos 0° + P_2 \cos 135° - P_3 \cos 45°$$

$$= P_1(1) + P_2\left(-\frac{1}{\sqrt{2}}\right) - P_3\left(\frac{1}{\sqrt{2}}\right)$$

$$= P_1 - \frac{1}{\sqrt{2}}P_2 - \frac{1}{\sqrt{2}}P_3$$

$$Y = P_1 \sin 0° + P_2 \sin 135° - P_3 \sin 45°$$

$$= P_1(0) + P_2\left(\frac{1}{\sqrt{2}}\right) - P_3\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}(P_2 - P_3)$$

Since the system of forces is equal to a single couple of moment G,

So $G \neq 0$, $R = 0$

$$R = 0 \Rightarrow X = 0, Y = 0$$

$$X = 0 \Rightarrow P_1 - \frac{1}{\sqrt{2}} P_2 - \frac{1}{\sqrt{2}} P_3 = 0$$

$$P_1 - \frac{1}{\sqrt{2}} (P_2 - P_3) = 0$$

$$P_1 = \frac{1}{\sqrt{2}} (P_2 + P_3) \longrightarrow (1)$$

and $Y = 0 \Rightarrow \frac{1}{\sqrt{2}} (P_2 - P_3) = 0$

or $P_2 - P_3 = 0$

$$P_2 = P_3 \longrightarrow (2)$$

Taking the moment of forces about 'A'.

Let 'G' be the moment of the resultant force, then according to varignon's theorem

$$G = P_1 (0) + P_2 |\overline{AE}| + P_3 (0)$$

$$G = P_2 \frac{a}{\sqrt{2}}$$

$$P_2 = \frac{\sqrt{2} G}{a}$$

So $P_3 = \frac{\sqrt{2} G}{a}$, $\because P_2 = P_3$ by (2)

Using values in (1)

$$P_1 = \frac{1}{\sqrt{2}} \left(\frac{\sqrt{2} G}{a} + \frac{\sqrt{2} G}{a} \right) = \frac{1}{\sqrt{2}} \frac{2\sqrt{2} G}{a} = \frac{2G}{a}$$

So $P_1 = \frac{2G}{a}$, $P_2 = P_3 = \frac{\sqrt{2} G}{a}$