Chapter 19

Two-Port Networks.

Introduction:-

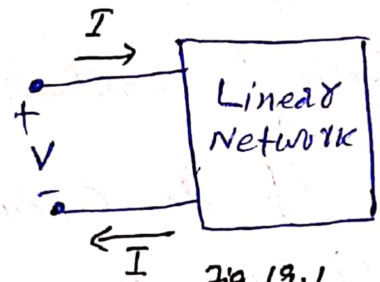


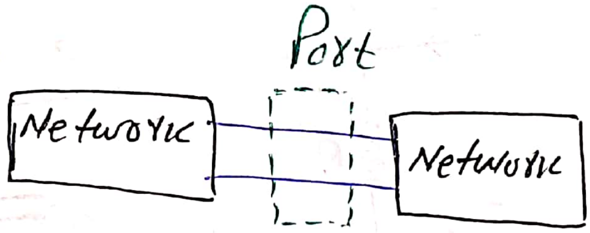
Fig 19.1
(one-port N/w)

What is Port ?

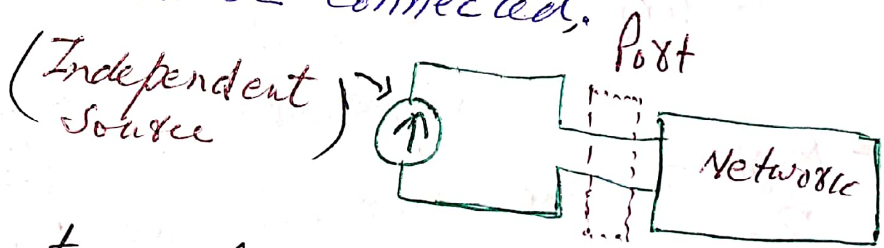
"A Pair of terminals through a current may enter or leave a network is known as a Port".

"OR"

Port:- "It is a Pair of terminals which connects the electrical circuits or networks to the external circuits".



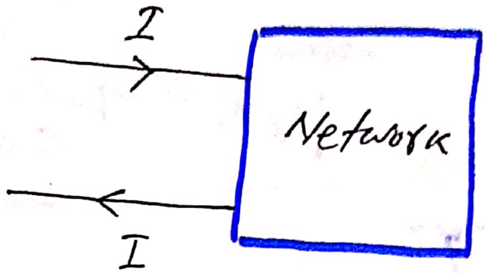
So, through this Port, any external network or any independent source can be connected, to the existing Network



But Any two terminals can not be called as a Port, and for that, they need to satisfy the Port Condition.

Port Condition :- "If the current (I) enter through the one terminal of the Port, then the same current (I)

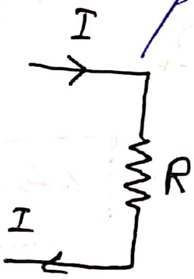
Should leave through the Second terminal of the Port.



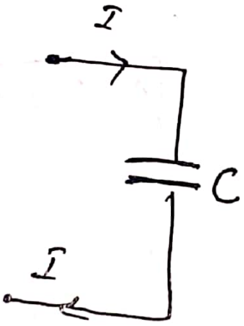
So, If the pair of terminal Satisfy this Condition then they Can be Called as a **Port.**

So, if the circuit or the electrical network Contains one Such Port then it Called the **one-Port network**

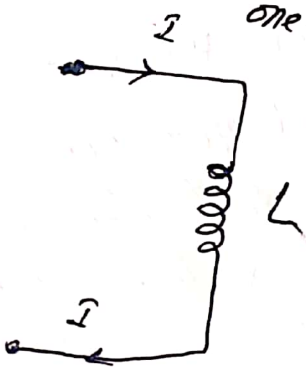
Note: All the basic elements like the Resistor, the Capacitor, and the Inductor are the example of one-Port N/W.



(Resistor)



(Capacitor)



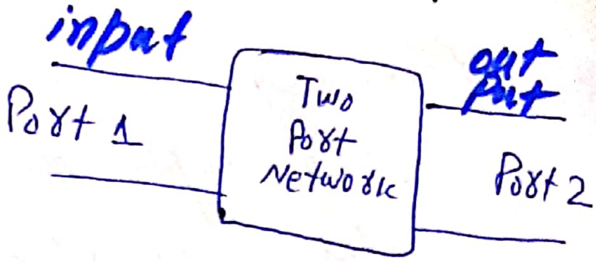
(Inductor)

⇒ Most of the circuits we have dealt with So far are two-terminal or one-Port circuits.

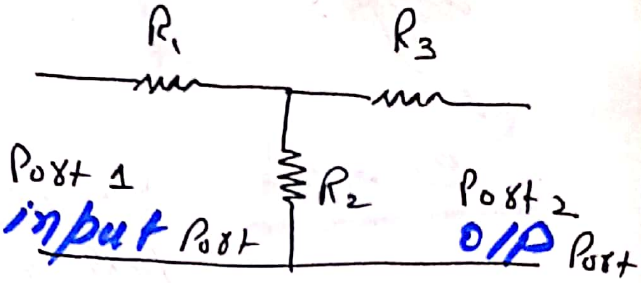
⇒ We have also Studied four-terminal or two-port circuits involving, Transformers, op-Amp & transistors & (Fillets),

Definition of Two-Port Network

" A two-port network is an electrical network with two separate ports for input and output.
if the electrical circuit OR the network, which contains two ports, then it is called the two-Port network."



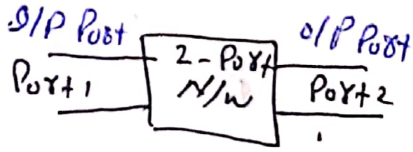
Example: This simple T-network is an example of two-Port network.



T-Network

⇒ In the two-Port network, Port 1 is often considered as the **input** Port, while the second Port (Port 2) is considered as the **output** port.

By using the two-port network model, we can isolate the portion of larger circuits, So, here basically this two port network is regarded as the black box, and the property of this network is specified by some Parameters.



"Black Box" A bunch of components, we ~~k~~ don't know what is inside

Note: Our study of two-port network is for at least two reasons.

First Such networks are useful in communication (Signal & system), Control system (Linear Control System), Power system, and Electronics (EDC / ECD).
For example they are used in electronics to model transistor and to facilitate cascaded design.

Second, Knowing the parameters of a two-port network enables us to treat as a "black box", when embedded with in a larger network.
"This allows the response of the network to the signal applied to the ports to be calculated easily, without solving for all the internal voltage & currents in the Network"

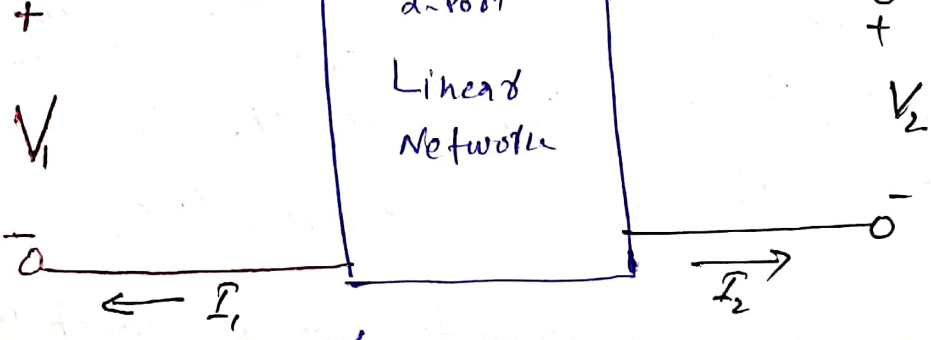


Fig. 19.1 (two-Port network)
Fig. 19.1 (b)

LTI system (Linear Time Invarient)

To, characterize a two-port network requires that we relate the terminals quantities V_1, V_2, I_1 & I_2 in fig 19.1(b), out of which two are independent.* The Various terms that relates these voltages & currents are called Parameters.

We will assume that the two-port circuit contains no independent source, although they can contains dependent sources.

The Analysis method, we will discuss require the following condition be met

1. Linearity

2. No independent source inside the network

3. No energy stored inside the network (Zero initial condition)

4. I_1 entering $= I_1$ leaving & I_2 entering $= I_2$ leaving (Satisfies the port condition).

⇒ Generally, the n/w is analyzed in s-Domain

$$\overbrace{\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad}^{\text{Independent Variables}}$$

** ⇒ $V_1 = f(I_1, I_2)$ ⇒ V_1 & V_2 can be expressed in term of independent variable I_1 & I_2

$V_2 = f(I_1, I_2)$

Dependent — Independent

⇒ ECD ⇒ $V_1 = f(I_1, V_2)$, $I_2 = f(I_1, V_2)$ — V_2 & I_1 indep, V_1 & I_2 depen and based on which variables are independent variables, the six different models or the six different parameters can be defined for this two-port N/w.

1. Z-Parameters or Impedance Parameters
2. Y-Parameters or Admittance Parameters
3. h-Parameters or Hybrid Parameters
4. ABCD Parameters or Transmission Parameters
5. Inverse Hybrid Parameters
6. Inverse Transmission Parameters.

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19.2.1 Impedance Parameters or Z-Parameters

Impedance & admittance parameters are commonly used in the synthesis (Combination of Components) of filter.

The are also useful in the design & analysis of impedance-matching networks & power distribution networks.

Z- Parameters

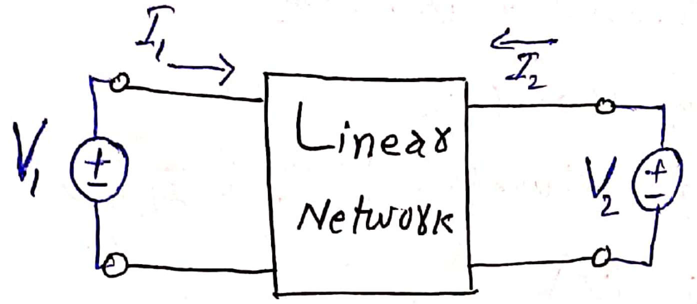


Fig (19.2) (a)
The Linear two-port Network
(a) driven by voltage Source

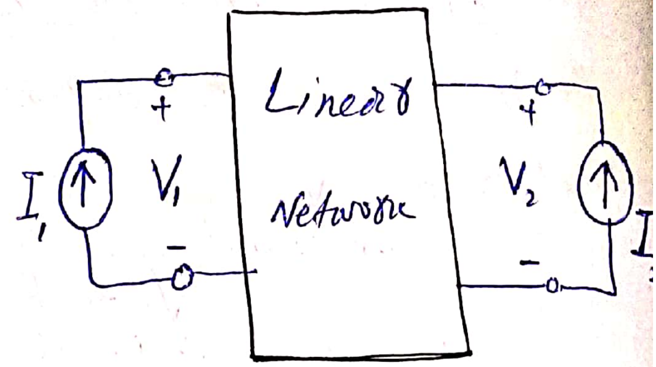


fig 19.2 (b)
(b) Driven by current Source

From either Fig. 19.2 (a) or (b), the terminal voltage can be related to the terminal current as

$V = ZI$
$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$$\left.\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}\right\} \longrightarrow 19.1$$

or in Matrix form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow V = ZI$$

$$[V] = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \longrightarrow 19.2$$

$$[V] = [Z][I]$$

Where the Z- term are called the impedance Parameters or Simply Z- Parameters.

⇒ We have four variables V_1, V_2, I_1 & I_2,
⇒ V_1 & I_1 for the Port 1 while
⇒ V_2 & I_2 for the Port 2
⇒ Out of the four variables, the two variables are independent variables, while the remaining two are the

dependent variables, So, for the Z-Parameter the I_1 & I_2 are the independent variables, while the V_1 & V_2 are the dependent variables,

That means V_1 & V_2 are the function of I_1 & I_2 &

$$V_1 = f(I_1, I_2)$$

$$V_2 = f(I_1, I_2)$$

they can be expressed in term of the I_1 & I_2

Be Remember, two-port Network contains only linear elements (i.e) that means the relation of this V_1 & V_2 with I_1 & I_2 is also linear.

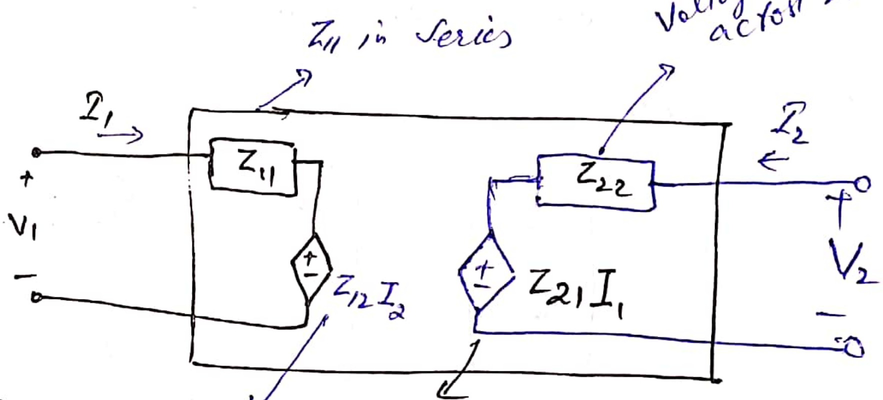
So for Z-Parameter

by KVL

$V_1 = Z_{11} I_1 + Z_{12} I_2$

This current controlled voltage source is depends on current (I_2).

I_2. It is current control voltage source whos voltage depends on I_2



Depends on another source (This is dependent voltage source),

(Equivalent ckt)

Similarly

$V_2 = Z_{21} I_1 + Z_{22} I_2$

→ This voltage source is depend on I_1

where Z_{11}, Z_{12}, Z_{22} & Z_{21} are Proportionality constants & known as the Z-Parameters.

$Z_{21} \times I_1$ is the current-controlled voltage source, So this is the Equivalent circuit of the 2-Port network in term of Z-Parameters.

$$[V] = [Z][I]$$

where V is the matrix of the dependent variables.

I is the matrix of the independent variables

Z is the matrix of this Z-Parameter

So Now, we let's see, how we can find the individual Z-Parameters

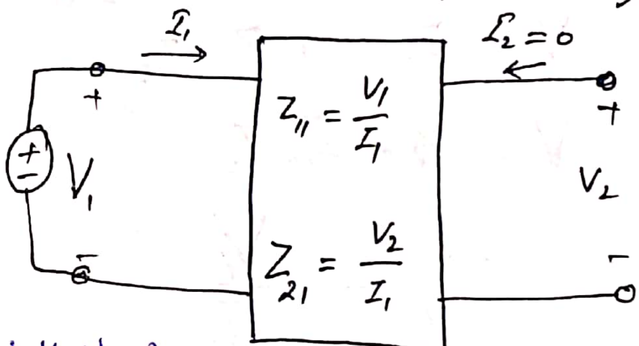
$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow 19.1 \text{ (a)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow 19.1 \text{ (b)}$$

So, the individual Z Parameters can be found by setting one of the Port Currents to Zero.

(1) When $\boxed{I_2 = 0}$ that mean the Port 2 is o.c (open circuit)

So in this Condition the Eq 19.1(a) becomes



$$V_1 = Z_{11} I_1 + 0$$

$$Z_{11} = \frac{V_1}{I_1}\Bigg|_{I_2 = 0}$$

→ It has unit of Impedance & infact that is, why it is Represented by the symbol Z, here ~~it's~~ in the numerator, the Voltage is on the Port-1, while in the denominator the current on the Port-1 Side. & infact thats why the Subscript is Z_{11}

for finding Z_{11}, the Port is open circuit, so, this Parameter is known as the open circuit driving Point impedance / I/P Impedance.

Similarly when $I_2 = 0$, the Eq 19.1 (b) ⇒ 2nd expression

$$V_2 = Z_{21} I_1 + 0$$

$$Z_{21} = \frac{V_2}{I_1}\Bigg|_{I_2 = 0}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$\therefore$ Z_{21} is open circuit transfer impedance from Port 2 to Port 1.

So, here in Z_{21}, the voltage is the Numerator is on the output side or from the Port 2, while the current is in the denominator is on the Port 1 side. So, this Parameter is known as the open circuit forward transfer impedance, because it relates the voltage on the output side with the current on the input side.

Similarly $\boxed{I_1 = 0}$ $\Rightarrow$ when the Port 1 is open circuit (o.c) or $\boxed{I_1=0}$

Eq 19.1 (a) becomes

$$V_1 = Z_{11} \underset{I_1=0}{I_1} + Z_{12} I_2$$

$$V_1 = 0 + Z_{12} I_2$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

(o.c Port 1)

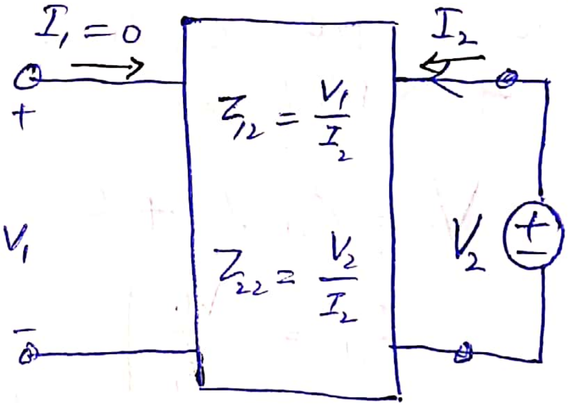


Fig 19.3 (b)

Z_{12} is open circuit transfer impedance from Port 1 to Port 2.

So this (Z_{12}) Parameter relates the input voltage with the output current, when port 1 is open-circuited, so this Parameter is known as the open circuit Reverse transfer impedance.

Now Consider Eq 19.1(b), $I_1 = 0$,

19.1 (b) $\Rightarrow$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_2 = 0 + Z_{22} I_2 \qquad \boxed{I_1 = 0}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$ When $I_1 = 0$

So (Z_{22}) Parameter relates the output voltage with the output current, when Port 1 is open-circuited, so, it is known as the open circuit driving Point impedance or simply it is known as the output impedance of the two-port network when the Port-1 is open-circuited.

All Four Parameters

$$\boxed{\begin{array}{ll} Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2 = 0} & Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2 = 0} \\ \\ Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_2 = 0} & Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1 = 0} \end{array}} \longrightarrow 19.3$$

Since the Z-Parameter are obtained by open-circuiting the in the input or output port, they are also called the open-circuit impedance Parameters. Specifically

Z_{11} = open-circuit input Impedance

Z_{12} = open-circuit transfer impedance from port 1 to Port 2

Z_{21} = open-circuit transfer impedance from Port 2 to Port 1

Z_{22} = open-circuit output Impedance.

Properties of 2-Port N/W.

(i) $Z_{11} = Z_{22}$ Symmetrical

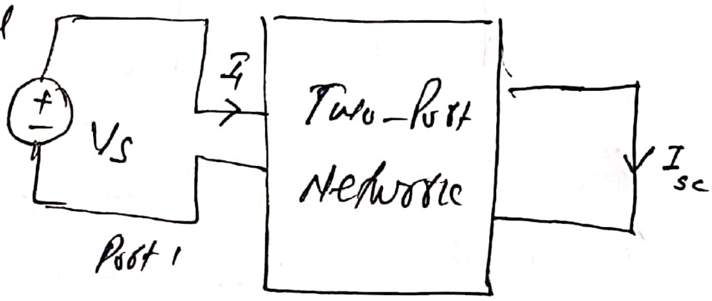
(ii) $Z_{12} = Z_{21}$ Reciprocal.

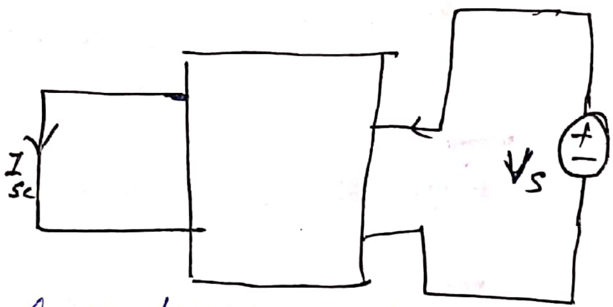
Now, there are certain Properties of the two-port network, which frequently also occur in the Practical networks, and using these Properties the network can be analyzed quite easily. These Properties are **Reciprocity** and **Symmetry**

Reciprocal

" what is Reciprocal Network "

Let we have applied the supplied voltage V_s at Port 1 & we are measuring the short circuit current at the Port 2. Now if the N/w is reciprocal then we should get the same short circuit current at Port 1, when the supply voltage V_s at the Port 2.





or in other words

A Two-Port network is said to be a Reciprocal network, if the ratio of excitation at one Port to the response at other port is same, if the excitation and the response are interchanged.

Condition for Reciprocity

if $\boxed{Z_{12} = Z_{21}}$

So, The two-Port network is Said to be reciprocal if Z_{12} is equal to Z_{21}, so if we know the Z-Parameters of any network & if this Condition gets satisfied then we can say that the given two-Port network is the reciprocal Network.

So Now, Lets Prove this Condition (for understanding).

Let say we have applied the supply the Voltage V_s at Port 1 we are measuring the short Circuit Current at the Port 2

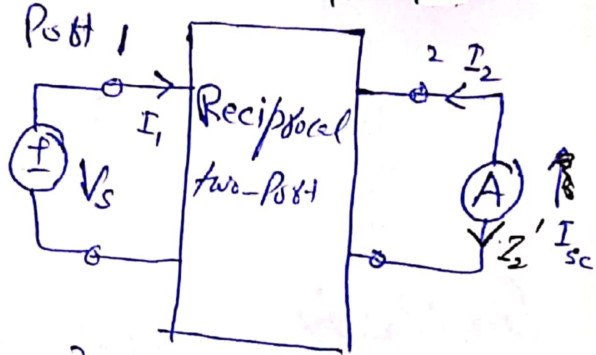


fig 19.4 (a)

Here

$V_1 = V_s$

$I_2 = -I_2'$, $V_2 = 0$

So Port is short-circuited, So the Voltage $\boxed{V_2 = 0}$

Eq 19.1 ⇒

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_s = Z_{11} I_1 - Z_{12} I_2'$$

Similarly

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow 1st$$

$V_2 = 0$ ⇒

$$0 = Z_{21} I_1 - Z_{22} I_2' \longrightarrow 2nd$$

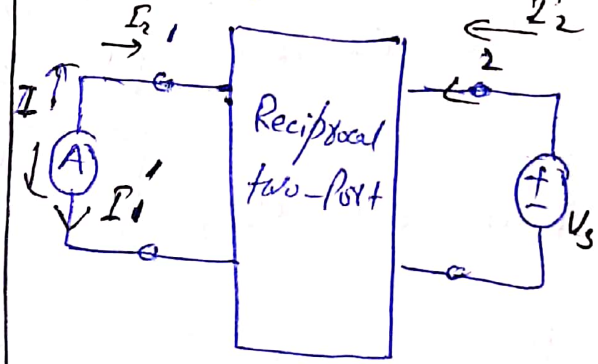


fig 19.4 (Interchanging a Voltage Source at one Port with an ideal Ammeter at the other Port Produces the Same reading in a Reciprocal 2-Port

we are intrested to find the ratio of V_s & I_2'. That means the ratio of excitation (V_s) to the response, so,

Let find the value of I_1 in term of I_2', Consider the 2nd Expression

$$Z_{21} I_1 - Z_{22} I_2' = 0$$

$$Z_{21} I_1 = Z_{22} I_2'$$

$$I_1 = \left(\frac{Z_{22}}{Z_{21}}\right) \cdot I_2'$$

Now we will Put the value of I_1 in I^{st} Expression

$$V_s = Z_{11} I_1 - Z_{12} I_2'$$

$$V_s = Z_{11}\left(\frac{Z_{22}}{Z_{21}}\right) \cdot I_2' - Z_{12} I_2'$$

LCM

$$V_s = \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}\right) \times I_2'$$

$$\frac{V_s}{I_2'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} \longrightarrow (1)$$

Similarly Consider the fig 19.4 (b) ⇒ Port 1 / I_1 is S.C & Excitation applied to Port 2

In this Condition $\boxed{V_1 = 0}$ due to I_1' short circuit

$\boxed{V_2 = V_s}$ & $\boxed{I_1 = -I_1'}$

Putting all these values in Eq 19.1 ⇒

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \Rightarrow 0 = -Z_{11} I_1' + Z_{12} I_2 \longrightarrow (2)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \Rightarrow V_s = -Z_{21} I_1' + Z_{22} I_2 \longrightarrow (3)$$

Now once again here we are intrested to finding the ratio of Voltage (V_s) & I_1'. i.e the ratio of Excitation to the Response

First we will find the value of I_2 from Eq (3)

Eq (3) $\Rightarrow$ $$Z_{11} I_1' = Z_{12} I_2$$

$$I_2 = \left(\frac{Z_{11}}{Z_{12}}\right) I_1' \longrightarrow (4)$$

Putting the value of I_2 in Eq (2) to get $\left(\frac{V}{I}\right)$

Eq (2) $\Rightarrow$ $$V_s = -Z_{21} I_1' + Z_{22} I_2$$

$$V_s = -Z_{21} I_1' + Z_{22} \times \left(\frac{Z_{11}}{Z_{12}}\right) I_1'$$

$$V_s = \left(\frac{-Z_{12} Z_{21} + Z_{11} Z_{22}}{Z_{12}}\right) I_1'$$

$$V_s = \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{12}}\right) I_1'$$

The ratio

$$\frac{V_s}{I_1'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{12}} \longrightarrow (5)$$

So if the two-Port Network is reciprocal then these two ratios of the Eq No. (1) & Eq (5) should be equal. $\boxed{Z_{21} = Z_{12}}$ if this condition is satisfied, we can say N/w is Recip.

$$\Rightarrow \frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

Putting Values

$$\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{12}} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$

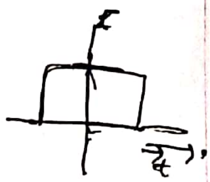
if $\boxed{Z_{12} = Z_{21}}$

$$\boxed{Z_{12} = Z_{12}}$$

that means if this condition gets satisfied then we can say the Network is Reciprocal

2) Symmetrical Network

When $Z_{11} = Z_{22}$, the two Port network is said to be symmetrical. This Implies that the network has mirror like Symmetry about Some center line; that is a line can be found that divides the network into two similar halves.



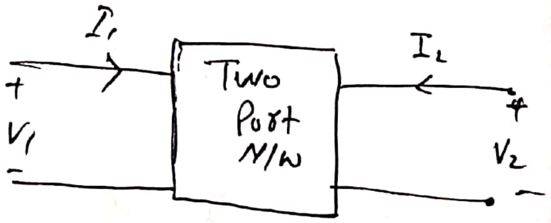
"OR"

Detail Symmetrical Network

"A two-Port network is said to be Symmetrical network, if the ratio of voltage to the current at one Port is same as the ratio of the voltage to the current at the other Port, with one of the Port open circuited.

i.e

$$\frac{V_1}{I_1}\Big|_{I_2=0} = \frac{V_2}{I_2}\Big|_{I_1=0}$$



Input Impedance = O/P Impedance

$$Z_{11} = Z_{22}$$

Prove

Let Assume $I_2 = 0$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \Rightarrow \frac{V_1}{I_1} = Z_{11}\Big|_{I_2=0}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \Rightarrow I_1 = 0 \quad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{V_2}{I_2}\Big|_{I_1=0} = Z_{22}$$

both Ratio Should be equal

$$\boxed{Z_{11} = Z_{22}}$$

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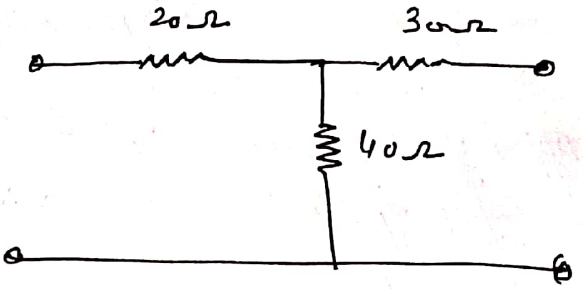
Example 19.1

Determine the Z Parameter for the circuit in Fig 19.7



To determine Z_{11} & Z_{21}, we apply a Voltage Source V_1 to the input port and leave the output port open as fig 19.8(a)

then

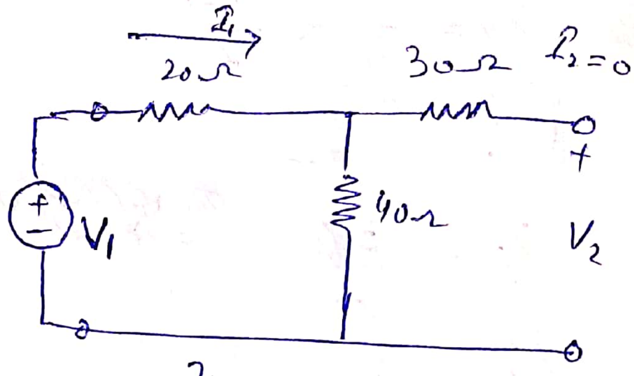


Fig 19.8(a)

$$Z_{11} = \frac{V_1}{I_1} =$$

$$Z_{11} = \frac{(20+40)\cancel{I_1}}{\cancel{I_1}}$$

$$\boxed{Z_{11} = 60\ \Omega}$$

i.e Z_{11} is the input impedance at port 1.

$$Z_{21} = \frac{V_2}{I_1} = \frac{40\cancel{I_1}}{\cancel{I_1}}$$ → Voltage across 40 Ω resistor

$$\boxed{Z_{21} = 40\ \Omega}$$

check

To find Z_{12} & Z_{22}, we apply a Voltage V_2 to the output port and leave the input port as open. Fig 19.8(b)

$$Z_{12} = \frac{V_1}{I_2} = \frac{40\cancel{I_2}}{\cancel{I_2}} = 40\ \Omega$$

$$\boxed{Z_{12} = 40\ \Omega}$$

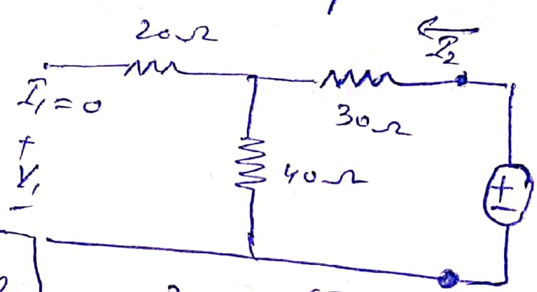


Fig 19.8(b)

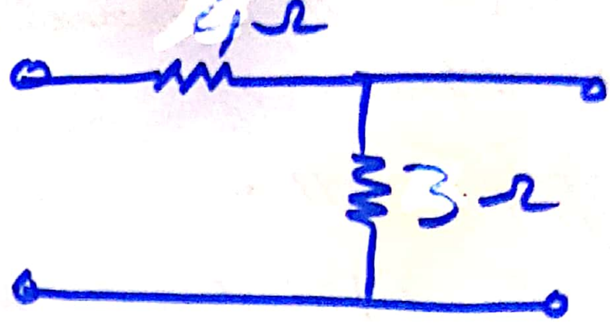
$$Z_{22} = \frac{V_2}{I_2} = \frac{(30+40)\cancel{I_2}}{\cancel{I_2}}$$

$$\boxed{Z_{21} = Z_{12}}$$ → Reciprocal

$$Z_{22} = 70\ \Omega$$

$$\Rightarrow [Z] = \begin{bmatrix} 60\ \Omega & 40\ \Omega \\ 40\ \Omega & 70\ \Omega \end{bmatrix}$$

P.P 19.1 Find the Z-Parameters of the two-port Network in Fig. 19.9 (18)



Similar fashion as Example 19.1

$I_2 = 0$

Apply Voltage Source V_1 to the input port

$V_1 = (4+3)I_1$

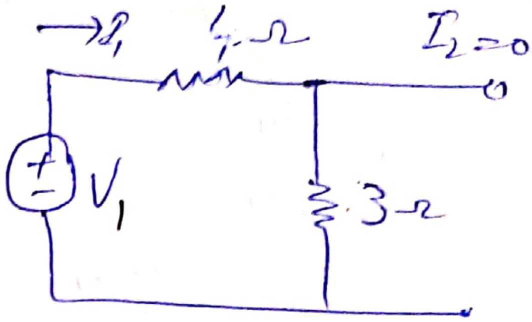
$Z_{11} = \frac{V_1}{I_1} = \frac{7\cancel{I_1}}{\cancel{I_1}}$

$\boxed{Z_{11} = 7\,\Omega}$

The Voltage V_2

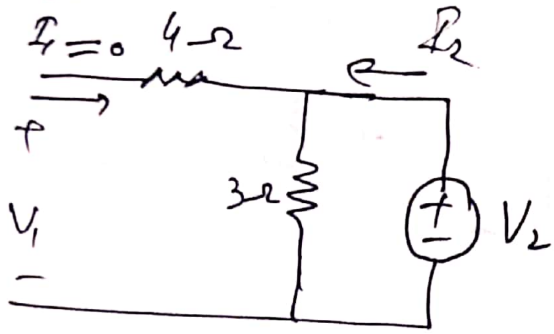
$Z_{21} = \frac{3\cancel{I_1}}{\cancel{I_1}}$

$\boxed{Z_{21} = 3\,\Omega}$



To determine Z_{12} & Z_{22}, apply V_2 to the O/P & Leave the input port as open Then

Apply KVL



$V_2 = 3I_2$

$Z_{22} = \frac{V_2}{I_2} = \frac{3\cancel{I_2}}{\cancel{I_2}}$

$\boxed{Z_{22} = 3\,\Omega}$

Ex. 19.1 SAME

The voltage V_1 is

$V_1 = 3I_2$

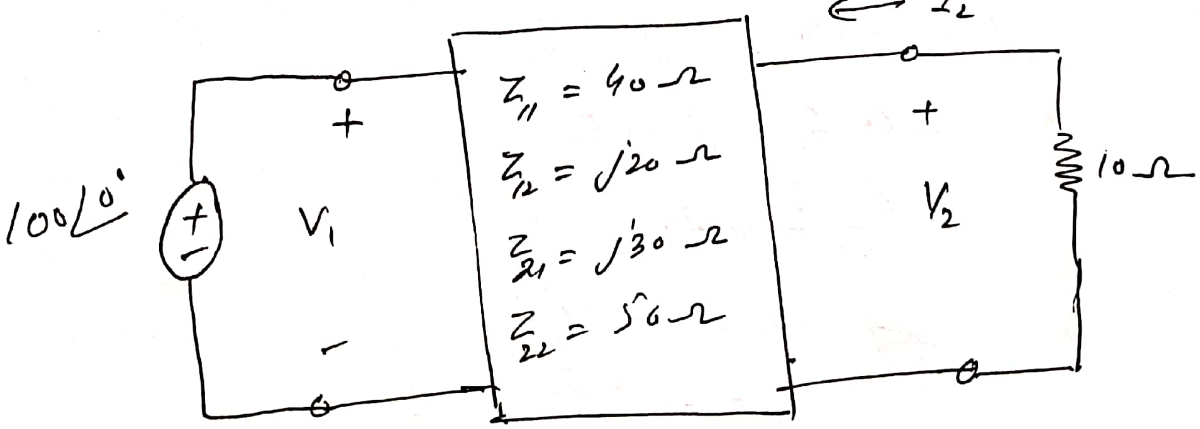
$Z_{12} = \frac{V_1}{I_2} = \frac{3\cancel{I_2}}{\cancel{I_2}}$

$\boxed{Z_{12} = 3\,\Omega}$

⇒ The Z-Parameter for the two-Port N/W is

$$Z = \begin{bmatrix} 7\,\Omega & 3\,\Omega \\ 3\,\Omega & 3\,\Omega \end{bmatrix}$$

Example 19.2 Find I_1 & I_2 in the circuit in Fig 19.10

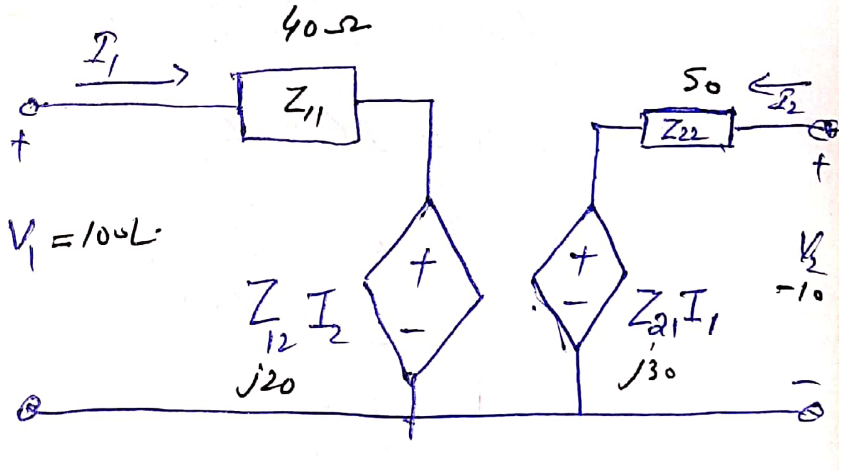


Solution

This not a reciprocal network because $Z_{12} \neq Z_{21}$. we may use the Equivalent circuit in Fig 19.5(b)

Eq 19.1 ⟹ $V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$



Substituting the given Z- Parameter in Eq No. (1) & (2) respectively

$$\Rightarrow \quad V_1 = 40 I_1 + j20 I_2 \longrightarrow (3)$$

$$V_2 = j30 I_1 + 50 I_2 \longrightarrow (4)$$

Since we are looking for I_1 & I_2, we Substitute

$$V_1 = 100\angle 0° \quad \& \quad V_2 = -10 I_2$$

Substitute the value of V_1 & V_2 in Eq (3) & 4.

→ Devices that delivers Power have Negative Power Values

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Eq (3) $\Rightarrow$

$$100 = 40 I_1 + j20 I_2 \longrightarrow (5)$$

Eq (4) $\Rightarrow$ $$-10 I_2 = j30 I_1 + 50 I_2$$

$$-10 I_2 - 50 I_2 = j30 I_1$$

$$-60 I_2 = j30 I_1$$

$$-\frac{60}{30} I_2 = j I_1$$

$$j I_1 = -2 I_2$$

$$I_1 = -\frac{2 I_2}{j} \times \frac{j}{j}$$

$$\boxed{I_1 = j2 I_2} \longrightarrow (6)$$

Putting the value of I_1 in Eq (5)

Eq 5 $\Rightarrow$ $$100 = 40(j2 I_2) + j20 I_2$$

$$100 = j80 I_2 + j20 I_2$$

$$100 = j100 I_2$$

$$I_2 = \frac{100}{j100} = -j$$

$$\boxed{I_2 = -j}$$

From Eq (6) $\Rightarrow$ $I_1 = j2(I_2)$

Putting the value of I_2

$$I_1 = j2(-j)$$

$$\boxed{I_2 = 1\angle -90^\circ} \text{ A}$$

$$I_1 = -j^2 2$$

$$\boxed{I_1 = 2} \Rightarrow \boxed{I_1 = 2\angle 0^\circ}$$

Practice Problem 19.2

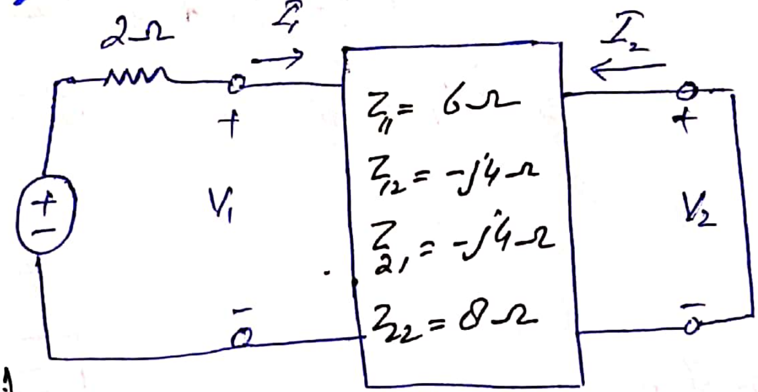
(Example 19.2 Easy & Same as PP19.2 Do your self)

21

Calculate I_1 & I_2 in the two-Port of given circuit

Solution

The given Network is a Reciprocal Network thus, T-equivalent ckt of



$$V_1 = Z_{11} I_1 + Z_{12} I_2 + 2I_1 \quad \& \quad V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$2\angle 30 = 6I_1 - j4 I_2 \rightarrow (1) \qquad 0 = -j4I_1 + 8I_2 \rightarrow (2)$$

Eq (2) becomes

$$8I_2 = j4 I_1$$

$$I_2 = \frac{j4}{8} I_1 \Rightarrow \frac{j}{2} I_1$$

$$\boxed{I_2 = \frac{j}{2} I_1}$$

Putting the value of I_2 in Eq (1)

$$2\angle 30 = 6I_1 - j4\left(\frac{j}{2} I_1\right) + 2I_1$$

$$2\angle 30 = 6I_1 - j^2 \frac{4}{2} I_1 + 2I_1$$

$$2\angle 30 = 6I_1 + 2I_1 + 2I_1$$

$$2\angle 30^\circ = 8I_1 + 2I_1$$

$$I_1 = \frac{2\angle 30}{10}$$

$$\boxed{I_1 = 0.2\angle 30^\circ} \Rightarrow \boxed{I_1 = 200\angle 30}$$

Similarly Short V_1 & use V_2 as Source.

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$\boxed{V_1 = 0}$ & $\boxed{V_2 = V_s}$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + 2I_1$$

$$0 = 6I_1 + (-j4) I_2 + 2I_1$$

$$0 = 8I_1 - j4 I_2 \qquad 8I_1 = j4 I_2$$

$$I_1 = \frac{j4}{8} I_2$$

$$I_1 = j\frac{1}{2} I_2$$

$$\boxed{I_1 = \frac{j}{2} I_2}$$

IV.

2nd Eq

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_2 = -j4 I_1 + 8 I_2$$

Putting $I_1 = \frac{j}{2} I_2$

$$V_2 = -j4\left(\frac{j}{2} I_2\right) + 8I_2$$

$$V_2 = -j^2 2 I_2 + 8I_2$$

$$V_2 = 2I_2 + 8I_2$$

Suppose $\boxed{V_2 = 1V}$

$1\angle 120^\circ$ v.

$$1\angle 120^\circ = 10 I_2 =$$

$$I_2 = \frac{1}{10} \angle 120^\circ$$

$$I_2 = 0.1 \angle 120^\circ$$

$\Longrightarrow$ $\boxed{I_2 = 100\, mA}$

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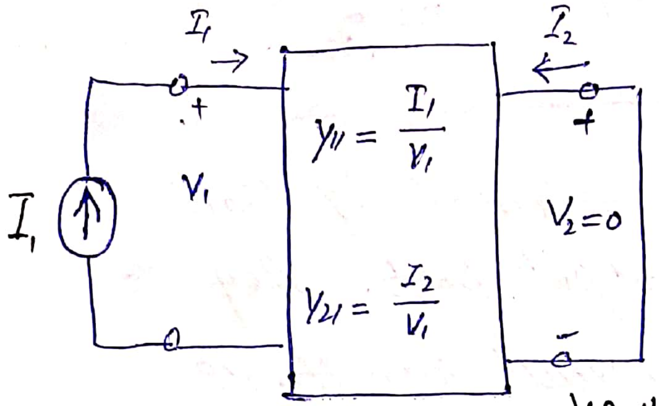
19.3 Admittance Parameters

In either 19.12 (a) / b, the terminal currents can be expressed in term of the terminal voltage as



port 2 is short circuit

Fig 19.12 (a)
Finding y_{11} & y_{21}

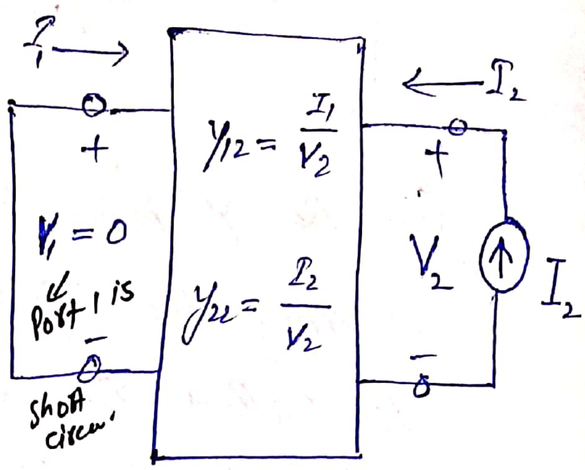


Fig 19.12 (b) Finding y_{12} & y_{22}.

$$\left.\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}\right\} \rightarrow (19.8)$$

or In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][I] \longrightarrow 19.9$$

The y term are known as the admittance Parameter or simply Y-Parameters & have unit of Siemens.

The values of the Parameter can be determine by setting $\boxed{V_1 = 0}$ input port short-circuited.

or $V_2 = 0$ (output port short-circuited). Eqn (24)

Then

$$\boxed{\begin{aligned} y_{11} &= \left.\frac{I_1}{V_1}\right|_{V_2=0} & y_{12} &= \left.\frac{I_1}{V_2}\right|_{V_1=0} \\ y_{21} &= \left.\frac{I_2}{V_1}\right|_{V_2=0} & y_{22} &= \left.\frac{I_2}{V_2}\right|_{V_1=0} \end{aligned}} \rightarrow \text{Eq. 18.10}$$

Since the Y-Parameter are obtained by short-circuiting the input or output port, they are also called the "short-circuit admittance parameters", specifically (Short-circuit Driving Point admittance, OR

y_{11} = Short-circuit Input Admittance

y_{12} = Short-circuit transfer admittance from Port 2 to Port 1

y_{21} = Short-circuit transfer admittance from Port 1 to Port 2

y_{22} = Short-circuit output admittance.

Equivalent circuit

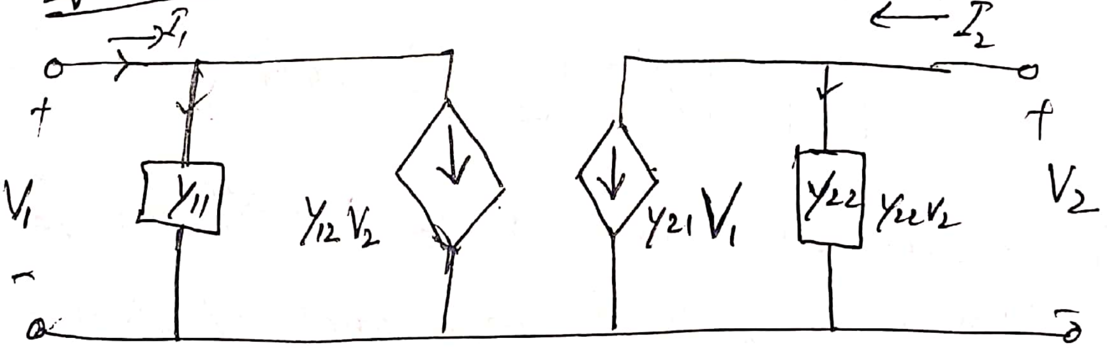


Fig. 18.13 (General Equivalent Circuit)

Extra **Detail of Y- Parameter** (25)

In two Port N/w there are four variables

$V_1 \; I_1$ $\quad$ $V_2 \; I_2$ $\quad$ 2- are Independent & 2- are dependent

Port 1 $\leftarrow \underbrace{V_1 \; I_1}$ $\quad$ $\underbrace{V_2 \; I_2} \rightarrow$ Port 2

For Y- Parameter the voltage V_1 & V_2 are the Independent variable, while I_1 & I_2 are the dependent variable

$$I_1 = f(V_1, V_2)$$

$$I_2 = f(V_1, V_2)$$

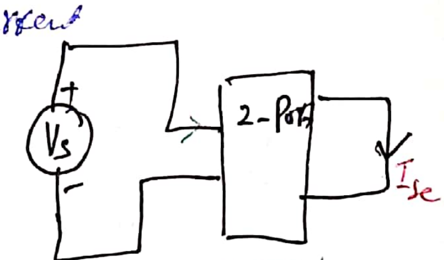
that mean I_1 & I_2 are the function of V_1 & V_2.

incoming current $\leftarrow$ $I_1 = Y_{11} V_1 + Y_{12} V_2$ & $I_2 = Y_{21} V_1 + Y_{22} V_2$

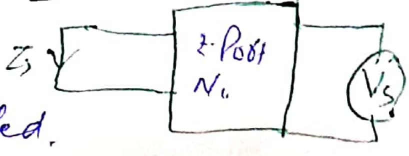
It is voltage controlled current source $\quad$ (VCCS)

So, here $Y_{12} \cdot V_2$, It is dependent source & to be precise it is Voltage-Controlled current Source (VCCS) in this source, the value of the current depends on the voltage at Port 2.

Reciprocal Network $\quad$ $\boxed{Y_{12} = Y_{21}}$ Same current

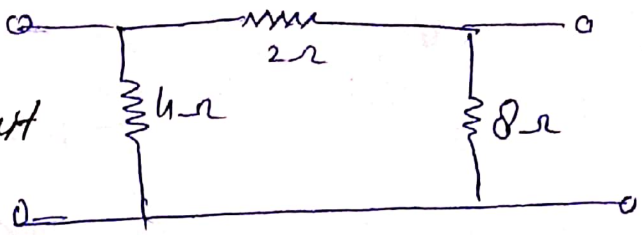


"A Network is said to be a Reciprocal network, if the ratio of excitation at one port to the response at another port is same, if the excitation and the response are interchanged.



Example 19.3

obtain the Y-Parameter for the π network shown in Fig 19.14



Solution

To find Y_{11} & Y_{21}, short-circuit the output port and connect a current source I_1 to the input port as Fig 19.15(a)

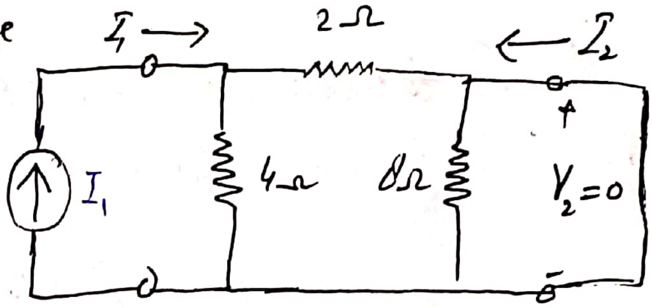


Fig 19.15 (Finding Y_{11} & Y_{21})

Since 8Ω resistor is short-circuited, the 2Ω resistor is in parallel with 4Ω resistor.

$$2 \parallel 4 = \frac{2\times4}{6} = \frac{8}{6} = \frac{4}{3}$$

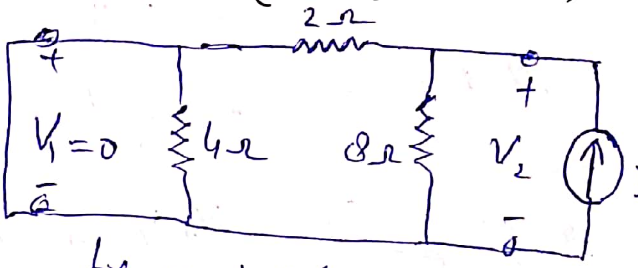


Fig 19.5 b (finding Y_{12} & Y_{22})

$$V_1 = I_1(4 \parallel 2)$$

$$V_1 = \frac{4}{3} I_1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{\cancel{I_1}}{\frac{4}{3}\cancel{I_1}} = \frac{3}{4}\, S \rightarrow \text{Siemen (unit)}$$

$$\boxed{Y_{11} = \frac{3}{4}\, S} \Rightarrow \boxed{Y_{11} = 0.75\, S}$$

By C.D.R

$$-I_2 = \frac{4}{4+2} \times I_1$$

$$\boxed{-I_2 = \frac{2}{3} I_1} \Rightarrow \boxed{I_2 = -\frac{2}{3} I_1}$$

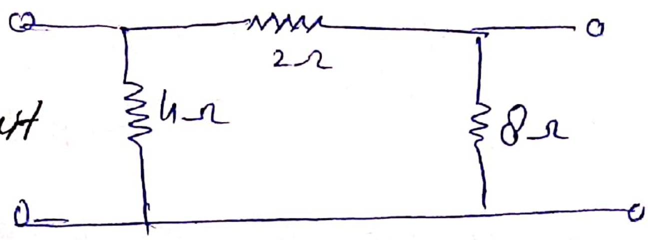
$$Y_{21} = \frac{I_2}{V_1} = \frac{-\frac{2}{3}\cancel{I_1}}{\frac{4}{3}\cancel{I_1}} = -\frac{2}{4} = -\frac{1}{2} = -0.5\, S$$

$$\boxed{Y_{21} = -0.5\, S}$$

Example 19.3

obtain the y-Parameter for the π network Shown in Fig 19.14

Solution



To find Y_{11} & Y_{21}, Short-circuit the output port and Connect a Current Source I_1 to the input port as Fig 19.15(a)

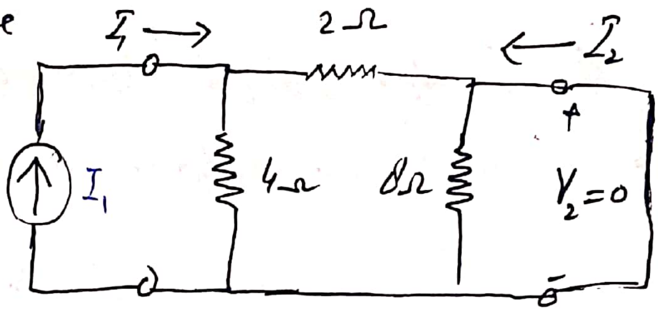


Fig 19.15 (finding Y_{11} & Y_{21})

Since 8Ω resistor is Short-circuited, the 2Ω resistor is in Parall with 4Ω resistor.

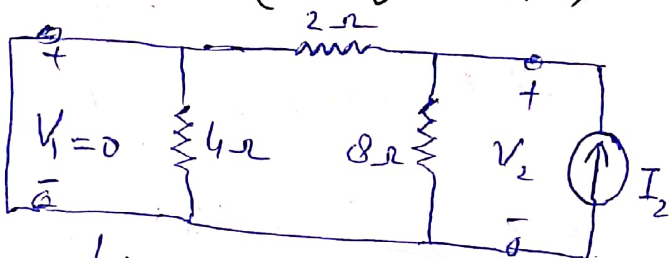


Fig 19.5 b (finding Y_{12} & Y_{22})

$$2 \| 4 = \frac{2\times4}{6} = \frac{8}{6} = \frac{4}{3}$$

$$V_1 = I_1 (4\|2)$$

$$V_1 = \frac{4}{3} I_1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{\cancel{I_1}}{\frac{4}{3}\cancel{I_1}} = \frac{3}{4}\ S \rightarrow \text{Siemen (unit)}$$

$$\boxed{Y_{11} = \frac{3}{4}\ S} \Rightarrow \boxed{Y_{11} = 0.75\ S}$$

By C.D.R

$$-I_2 = \frac{4}{4+2} \times I_1$$

$$\boxed{-I_2 = \frac{2}{3} I_1} \Rightarrow \boxed{I_2 = -\frac{2}{3} I_1}$$

$$Y_{21} = \frac{I_2}{V_1} = \frac{-2/3\ \cancel{I_1}}{\frac{4}{3}\cancel{I_1}} = -\frac{2}{4} = -\frac{1}{2} = -0.5\ S$$

$$\boxed{Y_{21} = -0.5\ S}$$

To get Y_{12} & Y_{22}, Short circuit the input port and connect a current source I_2 to the output port as Fig 19.15(b).

The 4-Ω resistor is short-circuited so that the 2Ω & 8Ω are in parallel.

$$8 \| 2 = \frac{8 \times 2}{8+2} = \frac{16}{10} = \frac{8}{5}$$

$$V_2 = I_2 (8 \| 2)$$

$$V_2 = \frac{8}{5} I_2 \quad \Rightarrow Y_{22} = \frac{I_2}{V_2} = \frac{\cancel{I_2}}{\frac{8}{5}\cancel{I_2}} = 5/8$$

$$\boxed{Y_{22} = 0.625\ S}$$

By C.D.R

$$-I_1 = \frac{8}{8+2} \times I_2$$

$$-I_1 = \frac{8}{10} I_2$$

$$\boxed{I_1 = -\frac{4}{5} I_2}$$

$$\Rightarrow \quad Y_{12} = \frac{I_1}{V_2} = \frac{-\frac{4}{5}\cancel{I_2}}{\frac{8}{5}\cancel{I_2}} = -0.5\ S$$

$$\boxed{Y_{12} = -0.5\ S}$$

Y:

⇒ Analysis

$Y_{21} = Y_{12}$ The ckt are Reciprocal.

$-0.5\ S = -0.5\ S$

[P.P. 19.3 Try yourself]

Example 19.4

Determine the y-Parameter for the two-port network Shown in figure 19.17

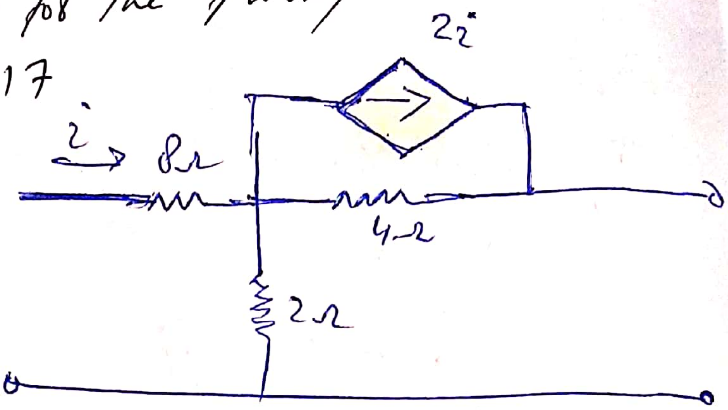
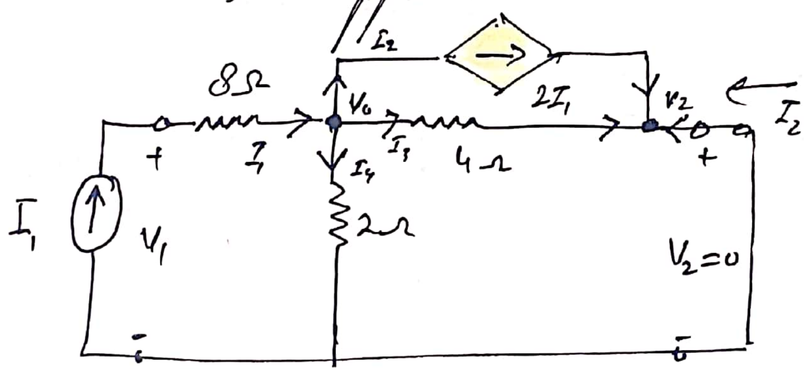


fig 19.17

Same Procedure as we did in Previous examples.

To get y_{11} & y_{21}, we use circuit in fig 19.18(a) in which Port 2 is short circuited & Current Source is applied to Port 1.

At Node 1



$I_1 = I_2 + I_3 + I_4$

$$\frac{V_1 - V_0}{8} = 2I_1 + \frac{V_0}{2} + \frac{V_0 - 0}{4}$$

$V_2 = 0$

But $I_1 = \dfrac{V_1 - V_0}{8}$

LCM

$$\boxed{\frac{2V_0 + V_0}{4} = \frac{3V_0}{4}}$$

$$I_1 = 2I_1 + \frac{V_0}{2} + \frac{V_0}{4}$$

$$0 = 2I_1 - I_1 + \frac{3V_0}{4}$$

$$0 = (I_1) + \frac{3V_0}{4}$$

$$0 = \left(\frac{V_1 - V_0}{8}\right) + \frac{3V_0}{4}$$

taking LCM

$$0 = \frac{V_1 - V_0 + 6V_0}{8}$$

$$V_1 - V_0 + 6V_0 = 0$$

$$V_1 + 5V_0 = 0$$

$$\boxed{V_1 = -5V_0} \longrightarrow (1)$$

As we know that $I_1 = \dfrac{V_1 - V_0}{8}$

$$\boxed{V_1 = -5V_0}$$

$$I_1 = \frac{-5V_0 - V_0}{8} = -\frac{6V_0}{8}$$

$$\boxed{I_1 = -0.75V_0}$$

&

$$y_{11} = \frac{I_1}{V_1} = \frac{\cancel{-}0.75\cancel{V_0}}{\cancel{-}5\cancel{V_0}}$$

$$\Longrightarrow \boxed{y_{11} = 0.15\ S}$$

At Node #2

Fig 19.18(a)

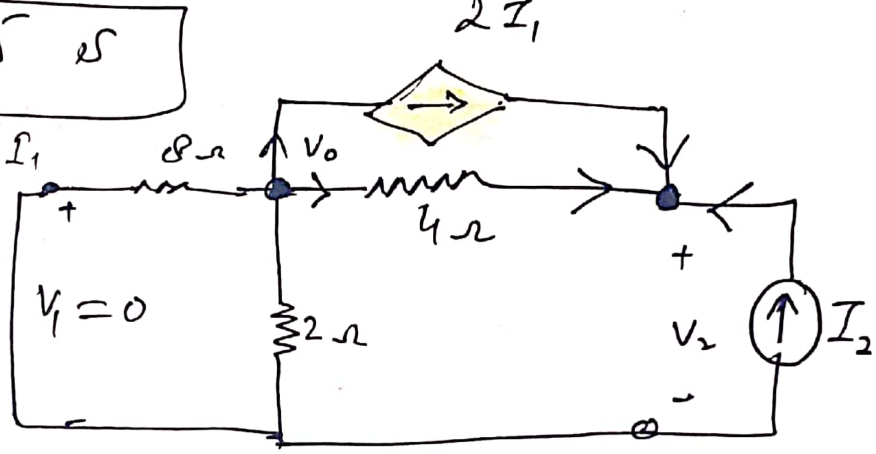


Fig 19.18 (finding y_{12} & y_{22})

$$\frac{V_0 - 0}{4} + 2I_1 + I_2 = 0$$

$$-I_2 = 2(-0.75V_0) - \left(\frac{+1V_0}{4}\right)$$

$$-I_2 = -1.5V_0 - 0.25V_0$$

$$\cancel{-}I_2 = \cancel{-}1.25V_0$$

$$\boxed{I_2 = 1.25V_0}$$

$$y_{21} = \frac{I_2}{V_1} = \frac{1.25\cancel{V_0}}{-5\cancel{V_0}} = -0.25\ S$$

$$\boxed{y_{21} = -2.5S}$$

Similarly, we get y_{12} & y_{22} using Fig 19.18 (b) 30

at Node 1

$V_1 = 0$

$$\frac{0 - V_0}{8} = 2I_1 + \frac{V_0}{2} + \frac{V_0 - V_2}{4}$$

But = $I_1 = \frac{0 - V_0}{8}$

$$\Rightarrow I_1 = \frac{-V_0}{8}$$

$$0 = -\frac{V_0}{8} + \frac{V_0}{2} + \frac{V_0 - V_2}{4}$$

$$0 = \frac{-V_0 + 4V_0 + 2V_0 - 2V_2}{8} \quad \text{Lcm}$$

$$+5V_0 - 2V_2 = 0$$

$$+2V_2 = +5V_0$$

$$V_2 = \frac{5}{2}$$

$$\boxed{V_2 = 2.5V_0}$$

Here

$$y_{12} = \frac{I_1}{V_2} = \frac{-V_0/8}{2.5V_0} = -0.05\,S$$

$$\boxed{y_{12} = -0.05\,S}$$

at Node 2

$$\frac{V_0 - V_2}{4} + 2I_1 + I_2 = 0$$

$$-I_2 = 0.25V_0 - \frac{1}{4}(2.5V_0) - 2\left(\frac{V_0}{8}\right)$$

$$I_2 = -0.625V_0$$

$$y_{22} = \frac{I_2}{V_2} = \frac{0.625\cancel{V_0}}{2.5\cancel{V_0}} = \underline{0.25\,S}$$

$$\boxed{y_{12} \neq y_{21}}$$

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19.4 Hybrid Parameter or h-Parameters

⇒ The Z & Y Parameters of two-Port network do not always exist. So there is need for developing another Set of Parameters.

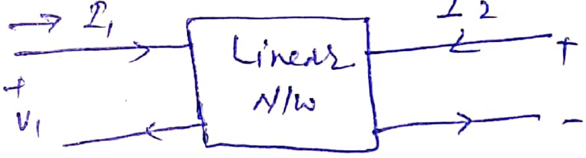
Thus the third Set of Parameter is based on making V_1 & I_2 the dependent variable & I_1 & V_2 are independent variable

H-Parameter have mixed dimension, So they are called hybrid Parameter

$$V_1 = f(V_2, I_1)$$
$$I_2 = f(V_2, I_1)$$

⇒ Input Impedance
out put Impedance
voltage Gain
current Gain

The hybrid Parameter are generally used to determine amplifier characteristic Parmeters Such as voltage gain, input resistance / input impedance, output resistance / out put impedance etc. Thus



$V_1 = h_{11} I_1 + h_{12} V_2$ ⟶ (VCVC)

$I_2 = h_{21} I_1 + h_{22} V_2$ ⟶ (CCCS)

The h-Parameter equivalent network of a Two Port N/w

$h_{11} I_1$ is voltage drop across h_{11} &

$h_{12} V_2$ is the VCVS (voltage Controlled Voltage Source)

The Value of this voltage Source depend on voltage at Port 2 (V_2).

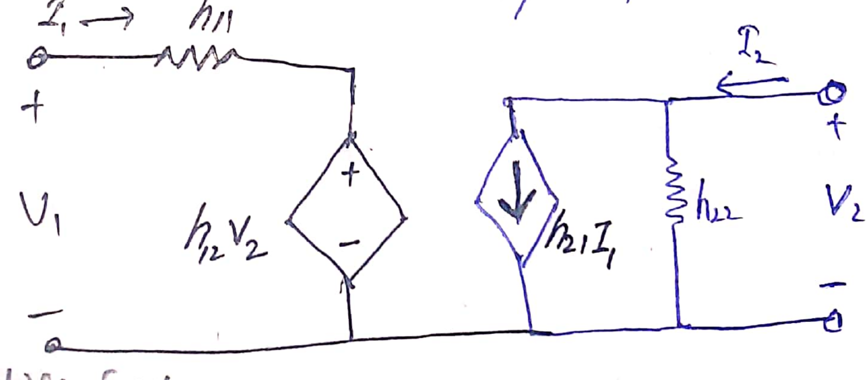


Fig. 19.20

Determination of h-Parameters

(1) ⟹ The Parameter h_{11} & h_{21} may be determine by Short circuiting the output terminals of given circuit.

While on the other hand, h_{12} & h_{22} may be determine by open circuiting the input terminal.

→ **Determination of h_{11} & h_{21}**

① $\boxed{V_2 = 0}$

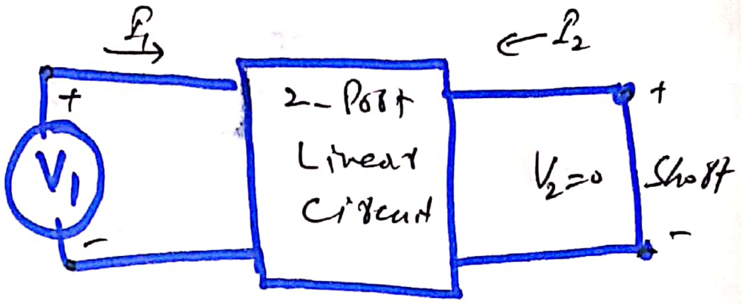


Fig (Output Short circuited)

Considering the Equation

$$V_1 = h_{11} \cdot I_1 + \cancel{h_{12} V_2} \xrightarrow{} 0 \quad (V_2 = 0)$$

$$V_1 = h_{11} I_1 + 0$$

$$\boxed{h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}}$$

∴ h_{11} is ratio of voltage to current (I/P side) & it has unit of Ω (Ohm)

h_{11} is called input resistance or h_{11} is short-circuit input impedance

Similarly

$$I_2 = h_{21} I_1 + \cancel{h_{22} V_2} \rightarrow 0$$

$$I_2 = h_{21} I_1 \Rightarrow \boxed{h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}}$$

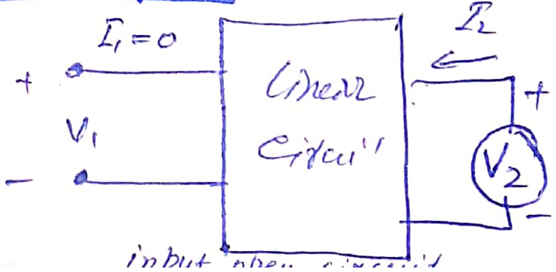
(unit less)

h_{21} is the ratio of o/p current to input current. It has no unit, The h_{21} is called the forward current gain with output short circuited

∴ h_{21} is known as short circuit forward current gain

→ **Determination of h_{12} & h_{22}**

⟹ These are determined by open circuiting the input terminal $\boxed{I_1 = 0}$



input open circuit

$$V_1 = 0 + h_{12} V_2$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}$$

O/P Port is O.C, h_{12} is ratio of I/P voltage to O/P voltage, It is called Reverse voltage gain (It is unitless parameter)

&

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0 \ (O.C.P.1)}$$

⇒ It has unit of Admittance (open circuit O/P admittance).

All 4-h Parameters

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} \qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$

Summary

h_{11} = Short-circuit input Impedance

h_{12} = open-circuit Reverse voltage gain

h_{21} = Short-circuit forward Current gain

h_{22} = open-circuit output Admittance.

In BJT,

$$\boxed{\begin{aligned} h_i &= h_{11} \\ h_r &= h_{12} \\ h_f &= h_{21} \\ h_o &= h_{22} \end{aligned}}$$

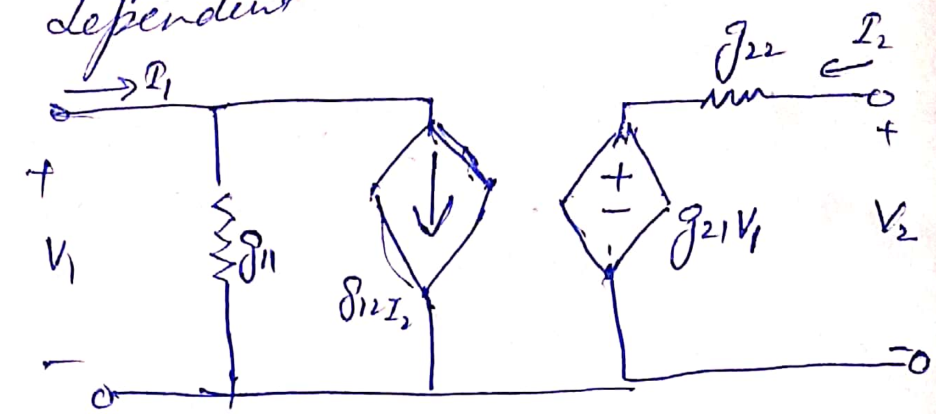
Condition for Reciprocity $\boxed{h_{12} = -h_{21}}$

&

Condition for Symmetry $\boxed{h_{11}h_{22} - h_{12}h_{21} = 1}$

g- Parameter / Inverse h- Parameter

V_1 & I_2 are independent variable
I_1 & V_2 " dependent "



A set of parameters closely related to the h-parameter are the g parameter, or Inverse-hybrid parameters. These are used to describe the terminal voltage & current -

The g-parameter are frequently used to model the field-effect transistor (FET).

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [g] \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

The value of g- Parameters $\boxed{I_2 = 0}$ O.C $\boxed{V_1 = 0}$ S.C

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0}$$

g_{11} = open circuit I/P admittance

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0}$$

g_{12} = Short-circuit reverse gain

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0}$$

g_{21} = open circuit forward gain

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0}$$

g_{22} = Short circuit O/P impedance

Example 19.5 Find the hybrid parameter for the two-port network of Fig 19.22.

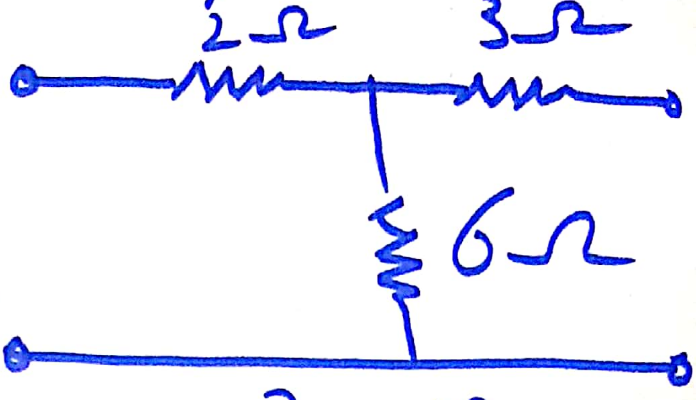


Fig. 19.22

Solution To find h_{11} & h_{21} we short-circuit the output port and connect a current source I_1 to the input port as shown in Fig. 19.23 (a)

$R_2 \| R_3$

$3 \| 6$

$$\Rightarrow = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\,\Omega$$

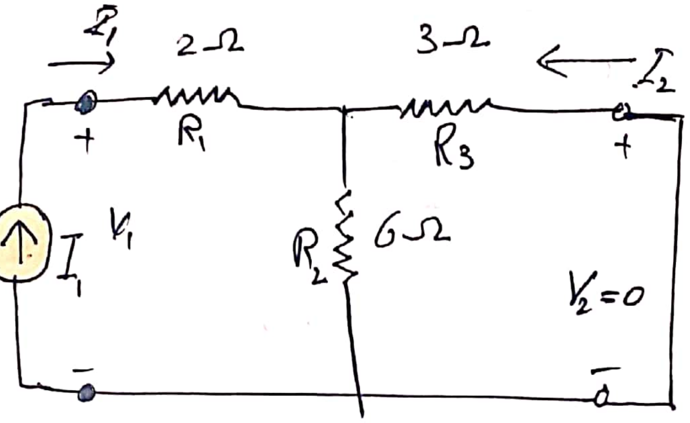
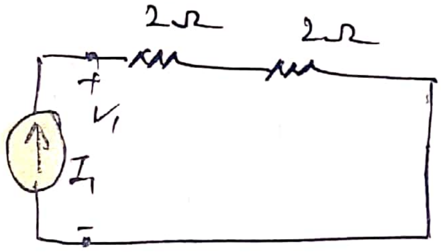


Fig 19.23 (a) Computing h_{11} & h_{21}.

Voltage Across 4 Ω Resistor.

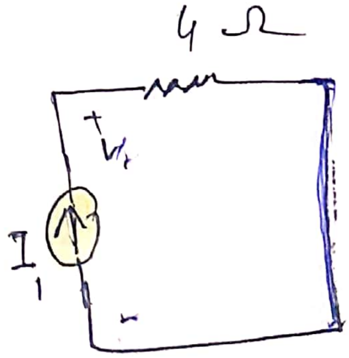
$$V_1 = 4I_1$$



Hence

$$h_{11} = \frac{V_1}{I_1} = \frac{4\cancel{I_1}}{\cancel{I_1}}$$

$$\boxed{h_{11} = 4\,\Omega}$$



Also, from Fig 19.23 (a), we obtain by CDR (Current Division Rule)

$$\boxed{h_{21} = \frac{I_2}{I_1}}$$

$$-I_2 = \frac{6}{6+3} \times I_1$$

$$-I_2 = \frac{6}{9} I_1$$

$$+I_2 = -\frac{2}{3} I_1$$

Here $h_{21} = \dfrac{I_2}{I_1} = \dfrac{-\frac{2}{3}\cancel{I_1}}{\cancel{I_1}}$

$$\boxed{h_{21} = -\frac{2}{3}}$$

To obtain h_{12} & h_{22}, we h_{22}, we open-circuit the input port and connect a voltage source V_2 to the output as in Fig 18.23(b), By VDR (Voltage Division Rule)

$h_{12} = \dfrac{V_1}{V_2}$

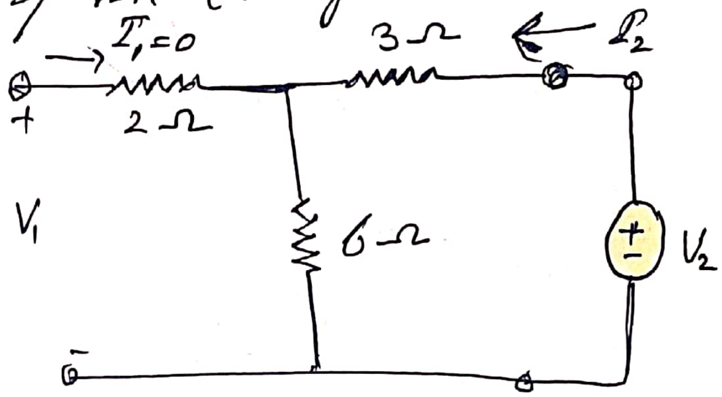


Fig. 18.23(b)
Computing h_{12} & h_{22}

V_1 is the voltage across 6 Ω resistor

$$V_1 = \frac{6}{6+3} \times V_2$$

$$V_1 = \frac{6}{9} V_2 \Rightarrow V_1 = \frac{2}{3} V_2$$

$$h_{12} = \frac{\frac{2}{3}\cancel{V_2}}{\cancel{V_2}} \Rightarrow \boxed{h_{12} = \frac{2}{3}}$$

& $h_{22} = \dfrac{I_2}{V_2}$ for V_2 Apply KVL.

$$V_2 = (3+6)I_2$$

$$V_2 = 9I_2$$

$$h_{22} = \frac{I_2}{V_2} = \frac{\cancel{I_2}}{9\cancel{I_2}} = \frac{1}{9}\,S \Rightarrow \boxed{h_{22} = \frac{1}{9}}\,S$$

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 4 & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{9} \end{bmatrix}$$

Practice Problem 19.6

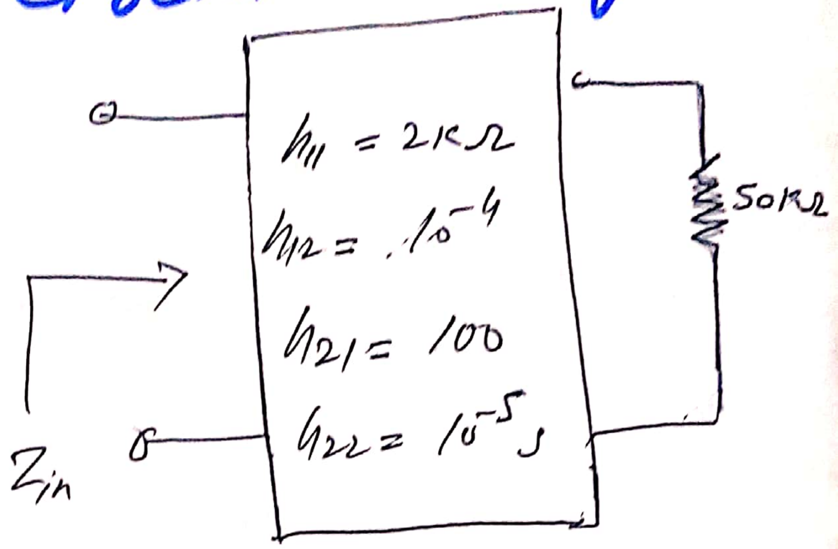
Find the impedance at the input port of the circuit in Fig 18.27

$h_{11} = 2\,k\Omega = 2\times10^{3}\,\Omega$

$h_{12} = 10^{-4}$

$h_{21} = 100$

$h_{22} = 10^{-5}$



Solution

The hybrid parameter based on V_1 & I_2 the dependent variables, Thus

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$$

Substituting the hybrid parameter, we obtain,

Eq (1) $\Rightarrow$ $$V_1 = 2000\, I_1 + 10^{-4} V_2 \longrightarrow (3)$$

$$I_2 = 100\, I_1 + 10^{-5} V_2 \longrightarrow (4)$$

Since from the circuit, the output voltage at output port

$$V_2 = -50{,}000\, I_2$$

Putting the value of V_2 in Eq (4)

By (4) $\Rightarrow$ $I_2 = 100 I_1 + 10^{-5}(-50{,}000) I_2$

$$I_2(1+0.5) = 100 I_1$$

$$I_2 = \frac{100}{1.5} I_1$$

Substituting the values of V_2 & I_2 in Eq (1) we get

$$V_1 = 2000 I_1 + 10^{-4}(-50{,}000) I_2$$

$$V_1 = 2000 I_1 + (-5)\left(\frac{1000}{\cancel{15}_3}\right) I_1$$

$$V_1 = I_1 \left[2000 - \frac{1000}{3}\right]$$

$$\frac{V_1}{I_1} = \frac{6000 - 1000}{3}$$

$$\frac{V_1}{I_1} = \frac{5000}{3} = \frac{5}{3}(1000)$$

Thus the ~~I/P~~ Impedance at the input Port of the Circuit $\boxed{Z_{in} = 1.667\,k\Omega}$

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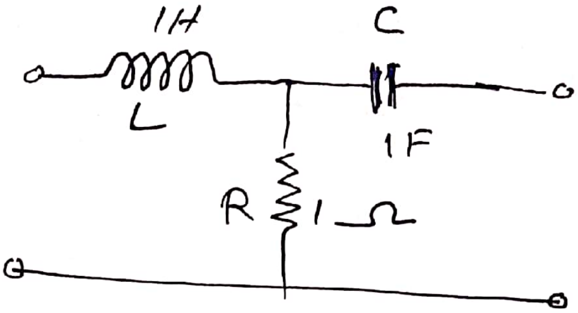
Example 19.7: Find the g-Parameters as function of s for the circuit in Fig 19.28



$$g = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$, $g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$ (o.c)

$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$, $g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}$ (S.C)

⇓ Transfor into S-Domain

Solution In the S-Dom

1H Inductor ⇒ $sL = S \times 1 = S$

1F Capacitor ⇒ $\frac{1}{sC} = \frac{1}{S.1} = 1/S$

1 Ω Resistor ⇒ 1 Ω Remains Same

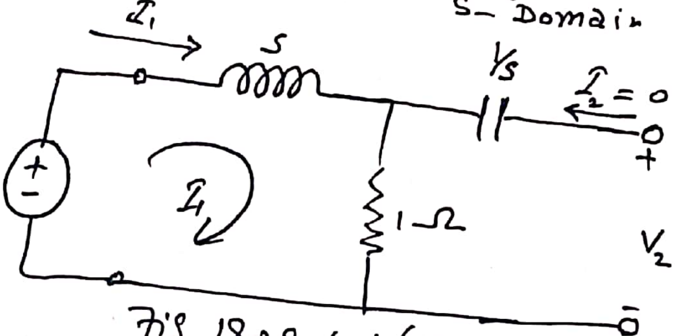


Fig 19.29 (a) ($I_2 = 0$) o.s (o/p)

To get g_{11} & g_{21}, we open-circuit the output port and Connect a Voltage Source V_1 to the input port as in Fig 19.29 (a), from the Fig.

$$g_{11} = \frac{I_1}{V_1} \qquad \text{for } I_1 = \frac{V_1}{S+1} \quad \text{(by KVL)}$$

$$\therefore \quad g_{11} = \frac{\cancel{V_1}/S+1}{\cancel{V_1}} \Rightarrow \boxed{g_{11} = \frac{1}{S+1}}$$

for $g_{21} = \frac{V_2}{V_1}$ for V_2 we Apply V.D.R (Voltage Divider)

We will Find voltage Across 1 Ω resistor

$$\therefore \quad V_2 = \frac{1}{S+1} \times V_1$$

$$V_2 = \frac{V_1}{S+1}$$

$$g_{21} = \frac{\cancel{V_1}/S+1}{\cancel{V_1}}$$

$$\Rightarrow \boxed{g_{21} = \frac{1}{S+1}}$$

To obtain g_{12} & g_{22}, we short-circuit the input port and connect a current source I_2 to the output port as Fig 19.29 (b).

By CDR (current Dividing Rule)

$$-I_1 = \frac{1}{s+1} \times I_2$$

$$I_1 = -\frac{1}{s+1} \times I_2$$

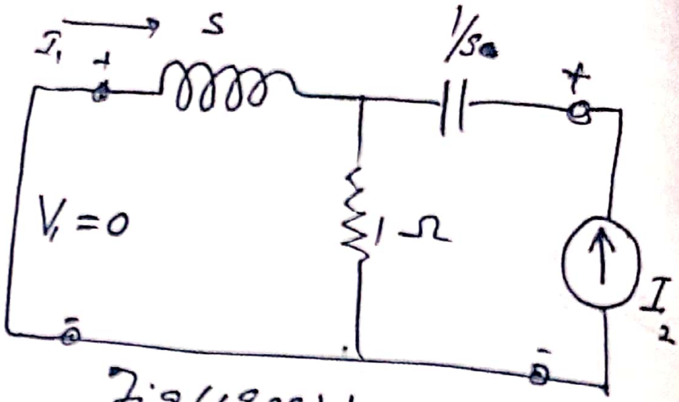


Fig (19.29) b

$$\therefore g_{12} = \frac{I_1}{I_2} = \frac{-\frac{1}{s+1} \times \cancel{I_2}}{\cancel{I_2}}$$

$$\boxed{g_{12} = -\frac{1}{s+1}}$$

& $g_{22} = \frac{V_2}{I_2}$ $\therefore$ for $V_2 = I_2\left(\frac{1}{s} + s \parallel 1\right)$

$$V_2 = I_2\left(\frac{1}{s} + \frac{s}{s+1}\right) \qquad \frac{s \times 1}{s+1} = \frac{s}{s+1}$$

$$V_2 = I_2\left(\frac{s+1+s^2}{s(s+1)}\right)$$

$$V_2 = I_2\left(\frac{s^2+s+1}{s(s+1)}\right)$$

$$g_{22} = \frac{\cancel{I_2}\left(\frac{s^2+s+1}{s(s+1)}\right)}{\cancel{I_2}}$$

$$g_{22} = \frac{s^2+s+1}{s(s+1)}$$

$$[g] = \begin{bmatrix} \frac{1}{s+1} & -\frac{1}{s+1} \\ \frac{1}{s+1} & \frac{s^2+s+1}{s(s+1)} \end{bmatrix}$$

19.5 Transmission Parameters

OR ABCD Parameters or T-Parameter or Cascade Parameter or Chain Parameters

$V_1 = V_S$ = Sending voltage

$V_2 = V_R$ = Receiving voltage

$I_1 = I_S$

$-I_2 = I_R$ (opposite sign) Leaving

V_R & I_R ⇒ Independent Variable

V_S & I_S ⇒ dependent variable.

Fig 19.31.

$-I_2$ is used rather than I_2, because the current is considered to be leaving the network.

Reason (why I_2 -ve)

i) When Cascade two-Port (output to Input)

ii) They are very useful in analysis of Transmission Line

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Practical use of T-Parameter

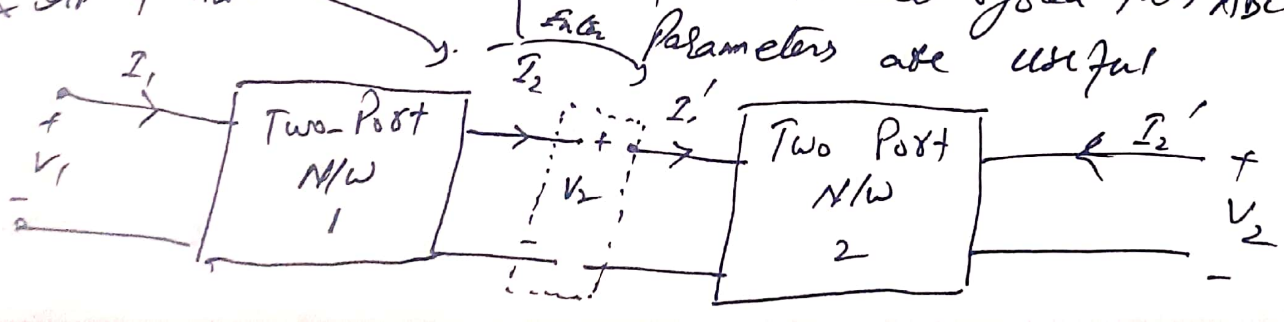
⇒ Used in Telephone System

⇒ Used in Microwave N/WORK

⇒ Used in Radar System

T-Parameter is also known as Cascade or chain Parameters because one two or more System are Cascaded then for analysis of Cascaded System these ABCD Parameters are useful

The I_2 current is o/p of N/w 1 & o/p of N/w 2



To find A & C, let us assume that $\boxed{I_2 = 0}$

$\therefore$ $V_1 = AV_2 - \cancel{BI_2}^{\,0}$

$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}}$$

Similarly $I_1 = CV_2 - 0 \Rightarrow \boxed{C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}}$

To find B & D, let $\boxed{V_2 = 0}$ short circuit

$V_1 = \cancel{AV_2}^{\,0} - BI_2$

$V_1 = -BI_2 \Rightarrow \boxed{B = -\left.\frac{V_1}{I_2}\right|_{V_2 = 0}}$

Similarly

$I_1 = \cancel{CV_2}^{\,0} - DI_2$

$I_1 = -DI_2 \Rightarrow \boxed{D = -\left.\frac{I_1}{I_2}\right|_{V_2 = 0}}$

A = open circuit voltage Ratio

B = Negative short circuit transfer Impedance

C = open circuit ~~trouffer~~ transfer Admittance

D = Negative short-circuit current ratio

where A & D are Dimensionless (Unit less)

B is in Ω (Ohm) where

C is in Siemens (S)

Example 19.8 :-

(Find the Transmission Parameters for the two-port network in Fig 19.32

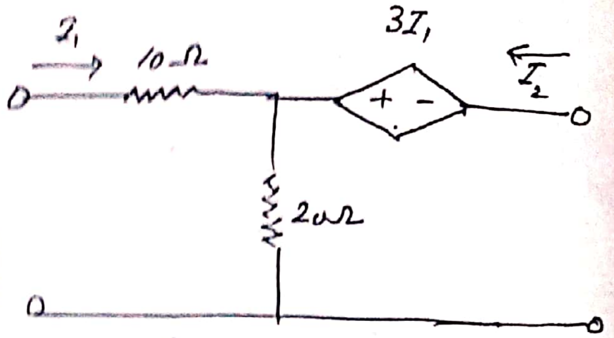


Fig 19.32

Solution

To determine A & C, we leave the output port open circuit as in fig. 33(a), so that $\boxed{I_2 = 0}$ and place a voltage source V_1 at the input port we have

By KVL

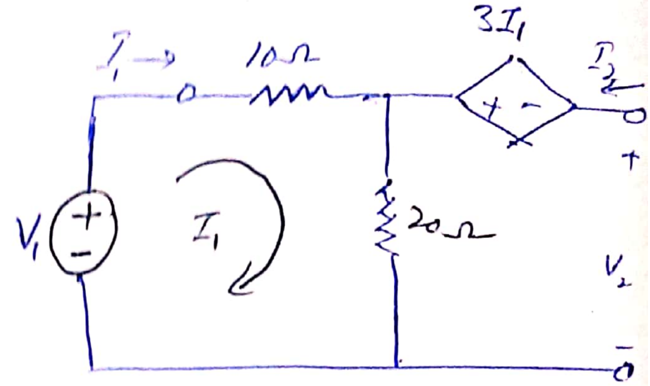


Fig 19.33(a) finding A & C
$I_2 = 0$ (O.C)

$$V_1 = (10+20)I_1$$

$$\boxed{V_1 = 30I_1}$$

Dependent Source

$$\& \quad V_2 = 20I_1 - 3I_1$$

$$\boxed{V_2 = 17I_1}$$

$$\therefore \quad A = \frac{V_1}{V_2}\bigg/_{I_2=0} = \frac{30\cancel{I_1}}{17\cancel{I_1}} = \frac{30}{17}$$

$$\boxed{A = 1.765}$$

&

$$C = \frac{I_1}{V_2} = \frac{\cancel{I_1}}{17\cancel{I_1}} = \frac{1}{17} \Rightarrow \boxed{C = 0.0588\ S}$$

To obtain B & D, we short-circuit the output port, so that $\boxed{V_2 = 0}$ as shown in fig 19.33(b) on next page & place a voltage source V_1 at the input port.

(44)

Apply KCL at Node 1

$$I_1 - I_3 + I_2 = 0 \rightarrow (1)$$

$$\frac{V_1 - V_a}{10} - \frac{V_a}{20} + I_2 = 0 \rightarrow (2)$$

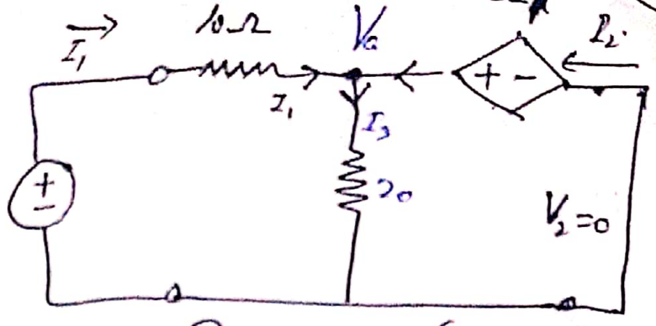


Fig 18.33 (b)
Finding B & D.

But $V_a = 3I_1$

& $I_1 = \left(\frac{V_1 - V_a}{10}\right)$, $I_3 = \frac{V_a}{20}$

In (1) $\Rightarrow$ $I_1 - \frac{3I_1}{20} + I_2 = 0$

$$I_1 - \frac{3I_1}{20} = -I_2$$

LCM $\frac{20I_1 - 3I_1}{20} = -I_2$

$$\frac{17}{20} I_1 = -I_2 \Rightarrow$$

Therefor $D = -\frac{I_1}{I_2} = \cancel{-}\frac{\cancel{I_1}}{\cancel{-}\left(\frac{17}{20}\right)\cancel{I_1}}$

$$\boxed{D = \frac{20}{17}} \Rightarrow \boxed{D = 1.176}$$

For V_1 $\Rightarrow$ $V_a = 3I_1$ & $I_1 = \frac{V_1 - V_a}{10}$

$$I_1 = \frac{V_1 - 3I_1}{10} \Rightarrow 10I_1 = V_1 - 3I_1$$

$\Rightarrow B = -\frac{V_1}{I_2}$

$$\Rightarrow V_1 = 10I_1 + 3I_1$$

$$\boxed{V_1 = 13I_1}$$

$$B = \cancel{-}\frac{13\cancel{I_1}}{\cancel{-}\left(\frac{17}{20}\right)\cancel{I_1}}$$

$$B = \frac{20 \times 13}{17} = \frac{260}{17} \Rightarrow \boxed{B = 15.29\ \Omega}$$