Example 1

If Three forces are represented in magnitude, direction and position by the sides of a triangle taken as order, they are equivalent to a couple.

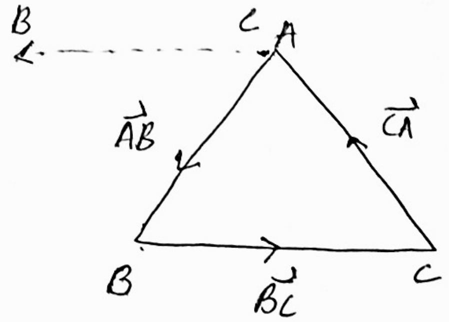
SOLUTION

Consider the forces $\vec{AB}$, $\vec{BC}$ and $\vec{CA}$ are represented by magnitude direction and position by the sides of a triangle ABC taken in order.

To prove (i) Three forces are equal to a couple

(ii) The moment of this couple is equal to twice the area of triangle ABC

from triangular law,



$\vec{CA} + \vec{AB} = \vec{CB}$

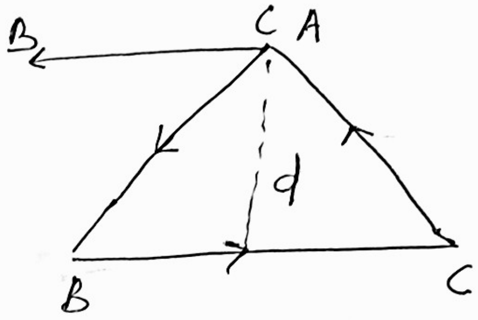
So $\vec{CB}$ is the resultant of forces $\vec{CA}$ and $\vec{AB}$ which must pass through the point A.

Now we are left with the two forces

(i) $\vec{BC}$ (through B and C)

(ii) $\vec{CB}$ (Through point A)

These two forces are equal in magnitude, opposite in direction and their line of action is not same. So They form a couple $(\vec{BC}, -\vec{BC})$. Let d be the arm of this couple and G be the magnitude of the moment of this couple, then



$$G = |\vec{BC}| d$$

Now area of triangle

$$= \frac{1}{2} |\vec{BC}| d$$

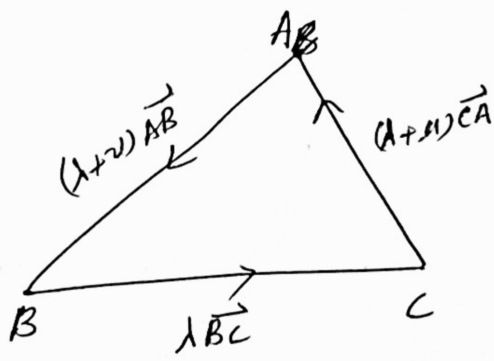
$$= \frac{1}{2} G$$

$$\Rightarrow G = 2 \text{ (Area of triangle } ABC)$$

<u>EXAMPLE 2</u> Forces act along the sides BC, CA, AB of a triangle. Show that they are equivalent to a couple only if the forces are propotional to the sides.

<u>SOLUTION</u>

Let the forces $\lambda\vec{BC}$, $(\lambda+\mu)\vec{CA}$ and $(\lambda+\nu)\vec{AB}$ act along the sides of the triangle ABC, then



$$\lambda\vec{BC} + (\lambda+\mu)\vec{CA} + (\lambda+\nu)\vec{AB}$$

$$= \lambda(\vec{BC} + \vec{CA} + \vec{AB}) + \mu\vec{CA} + \nu\vec{AB}$$

or $\lambda\vec{BC} + (\lambda+\mu)\vec{CA} + (\lambda+\nu)\vec{AB}$

$$= \text{a couple} + \mu\vec{CA} + \nu\vec{AB}$$

The system is equivalent to a couple only if

$$\mu \vec{CA} + \nu \vec{AB} = 0$$

which holds only if $\mu = \nu = 0$

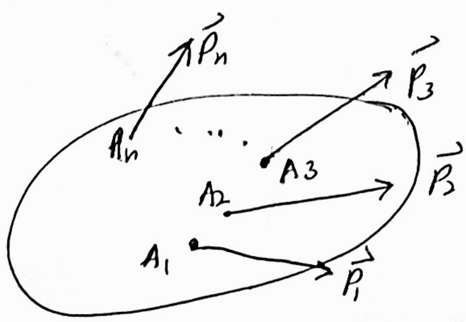
Hence forces along sides of triangle are $\lambda \vec{BC}$, $\lambda \vec{CA}$ and $\lambda \vec{AB}$.

Example 3 (Do it yourself)

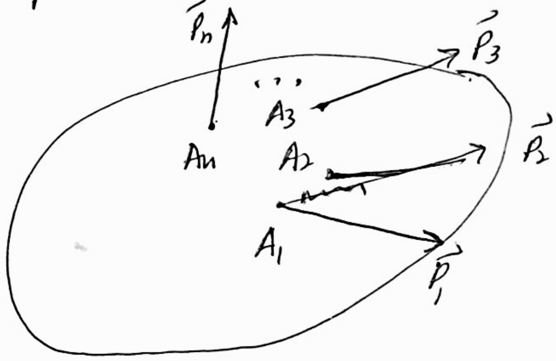
Reduction of a system of coplanar forces to one force and one couple.

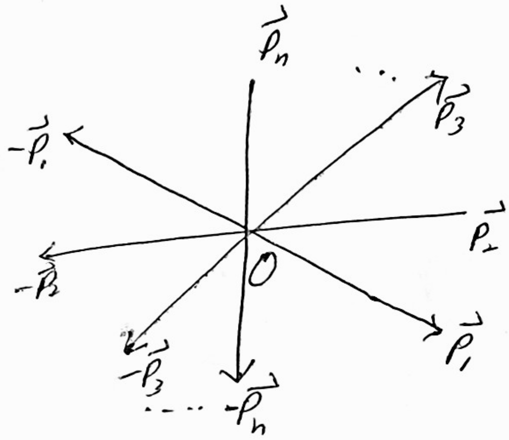
Consider a system of coplanar forces $P_1, P_2, \ldots P_n$ acting through the points $A_1, A_2, \ldots A_n$ of the R.B respectively.

To prove This system of coplanar forces can be reduced into a single force $\vec{R}$ and single couple of moment G.



Introducing the forces $\vec{P_1}, \vec{P_2}, \dots \vec{P_n}$ and the forces $-\vec{P_1}, -\vec{P_2}, \dots -\vec{P_n}$ through the point 'O'.





Now the forces through the points $A_1, A_2, \dots A_n$ and the forces $-\vec{P_1}, -\vec{P_2}, \dots -\vec{P_n}$ through points O form the couples $(\vec{P_1}, -\vec{P_1}), (\vec{P_2}, -\vec{P_2}) \dots (\vec{P_n}, -\vec{P_n})$ say of moments $G_1, G_2, \dots G_n$

These coplanar couples can be composed into a single couple of moment G

Then

$$G = G_1 + G_2 + G_3 + \dots + G_n$$

The concurrent forces can be composed into a single resultant force $\vec{R}$.

$$\vec{R} = \vec{P_1} + \vec{P_2} + \vec{P_3} + \dots + \vec{P_n}$$

Therefore a system of coplanar forces can be reduced into a single force $\vec{R}$ and a single couple of moment G.

SPECIAL CASES

(i) If $\vec{R}=0$, $\vec{G}=0$ The system of coplanar forces is said to be in complete equilibrium.

(ii) If $\vec{R}=0$, $\vec{G}\neq 0$
The system of coplanar forces is equal to a single couple of moment G.

(iii) $\vec{R}\neq 0$, $G=0$ The system of coplanar forces is equal to a single force $\vec{R}$.

(iv) $\vec{R}\neq 0$, $G\neq 0$ The system of coplanar forces is reduced into a single force $\vec{R}$ and a single couple of moment G.

These cases show that if a coplanar system of forces is not in equilibrium, it can be reduced to either a single force (when $\vec{R}\neq 0$) or a single couple (when $\vec{R}=0$)