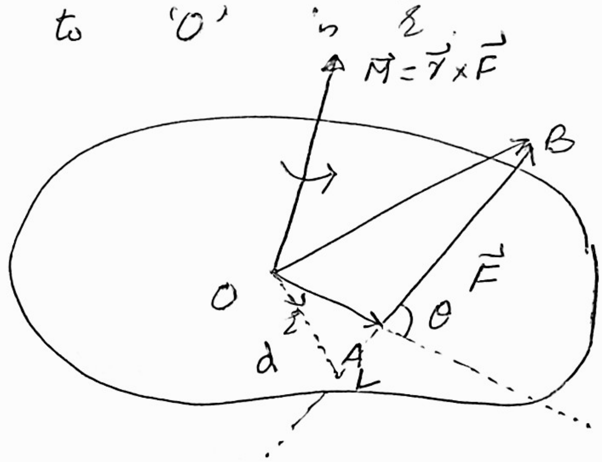
MOMENT OF A FORCE ABOUT A POINT

The tendency of a force to turn a body about a point is called the moment of that force.

EXPRESSION FOR THE MOMENT OF A FORCE

Consider a rigid body (R.B.) which can rotate about a point O. Let a force $\vec{F}$ is applied on the R.B at point A whose position vector with respect to 'O' is $\vec{r}$.



Let the body rotates about the point 'O' under the action of this force. Let $\vec{M}$ be the moment of this force, then

$$\vec{M} = \vec{r} \times \vec{F}$$

$$= |\vec{r}|\,|\vec{F}| \sin\theta \,\hat{n}$$

$$= rF \sin\theta \,\hat{n}$$

where $rF\sin\theta$ represents the magnitude of the moment vector $\vec{M}$ and $\hat{n}$ represents the direction of moment vector.

Again

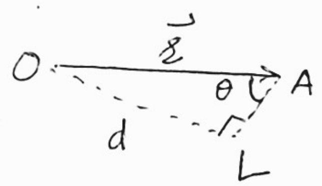
$$\vec{M} = rF\sin\theta\,\hat{n}$$

$$|\vec{M}| = (rF\sin\theta)|\hat{n}|$$

$$M = rF\sin\theta$$

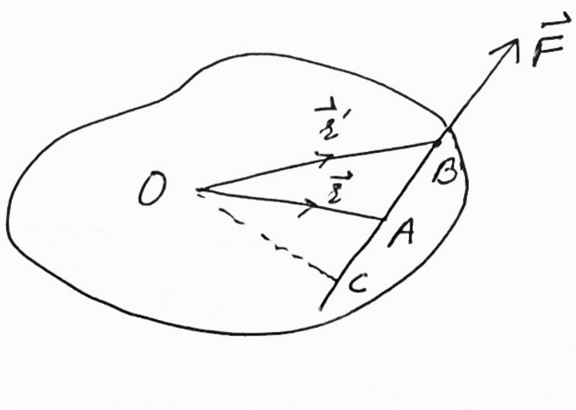
Now draw a perpendicular from point 'O' on the line of action of the force $\vec{F}$ which meets it at L

$$M = rF\sin\theta$$
$$= F(r\sin\theta)$$
$$= Fd$$



MOMENT OF FORCE IS INDEPENDENT OF CHOICE OF POINT OF APPLICATION OF THE FORCE ON LINE OF ACTION OF THE FORCE

Consider a R.B which rotates about the point 'O'



Let a force $\vec{F}$ is applied on the R.B at point A whose position vector with respect to A is $\vec{r}$

Let $\vec{M}$ be the moment of this force

Then

$$\vec{M} = \vec{r} \times \vec{F}$$

Let the same force $\vec{F}$ is applied on the R.B at point B, where p.v. w.r.t 'O' is $\vec{r}'$. Let $\vec{M}'$ be the moment of this force, then

$$\vec{M}' = \vec{r}' \times \vec{F}$$

$$\vec{M}' = (\vec{r} + \vec{AB}) \times \vec{F}$$

$\vec{OB} = \vec{OA} + \vec{AB}$

$\vec{r}' = \vec{r} + \vec{AB}$

$$= \vec{r} \times \vec{F} + \vec{AB} \times \vec{F}$$

$$= \vec{M} + 0$$

$\because \vec{AB}$ and $\vec{F}$ are collinear

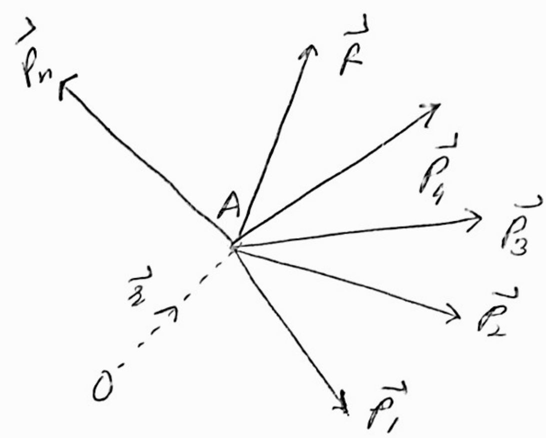
$$\vec{M}' = \vec{M}$$

Moment of force acting at point B = Moment of force acting at point A

Clearly the moment of the force about some fixed point is independent of choice of the point on the line of action of the force.

VARIGNON'S THEOREM

<u>Statement</u> The moment about a point 'O' of the resultant of a system of concurrent forces is equal to the sum of the moments of various forces about the same point 'O'



<u>Proof</u> Consider a system of concurrent forces $\vec{P_1}, \vec{P_2}, \vec{P_3}, \ldots \vec{P_n}$ acting at point A, whose position vector w.r.t 'O' is $\vec{r}$.

Let $\vec{R}$ be the resultant of forces

then $\vec{R} = \vec{P_1} + \vec{P_2} + \vec{P_3} + \ldots + \vec{P_n}$ $\longrightarrow (1)$

Now moment of $\vec{P_1}$ about 'O' $= \vec{G_1} = \vec{r} \times \vec{P_1}$

" " $\vec{P_2}$ " " $= \vec{G_2} = \vec{r} \times \vec{P_2}$

" " $\vec{P_3}$ " " $= \vec{G_3} = \vec{r} \times \vec{P_3}$

. - - - - - - - - - - - - - - - -

. . - - - - - - - - - - - - - - .

- - - - - - - - - - - - - - - -

Moment of $\vec{P_n}$ about 'O' $= \vec{G_n} = \vec{r} \times \vec{P_n}$

Let $\vec{G}$ be the moment of the resultant force $\vec{R}$ about 'O', then

$$\vec{G} = \vec{r} \times \vec{R}$$

$$= \vec{r} \times (\vec{P_1} + \vec{P_2} + \vec{P_3} + \cdots + \vec{P_n})$$

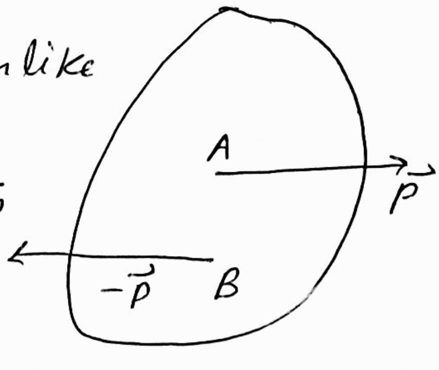
$$= \vec{r} \times \vec{P_1} + \vec{r} \times \vec{P_2} + \vec{r} \times \vec{P_3} + \cdots + \vec{r} \times \vec{P_n}$$

$$= \vec{G_1} + \vec{G_2} + \vec{G_3} + \cdots + \vec{G_n} \quad \text{by Eq.(2)}$$

COUPLE

Def. Two equal unlike, parallel forces whose line of action is not same form a couple.

Consider the two equal, unlike parallel forces $\vec{P}$ and $-\vec{P}$ acting through the points A and B, whose line of action is not same, so they form a couple $(\vec{P}, -\vec{P})$

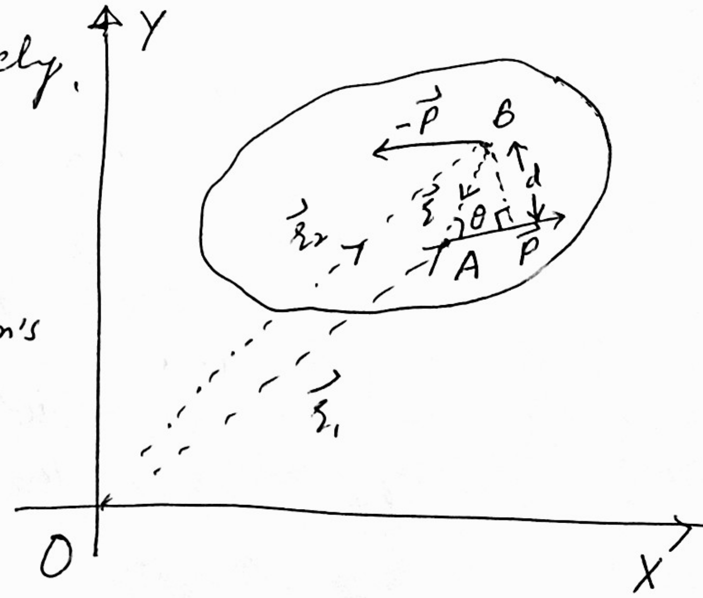


* By the equilibrium law, this force system is not in equilibrium because the forces do not do not act along the same line.

Therefore, a system of two equal and opposite parallel forces can not be replaced or balanced by a single force. Such a system is called a couple.

EXPRESSION FOR MAGNITUDE OF THE MOMENT OF A COUPLE

Consider a couple $(\vec{P}, -\vec{P})$ acting on a R.B. The force $\vec{P}$ acts at a point A and force $-\vec{P}$ acts at point B, whose p.v. w.r.t 'O' are $\vec{r}_1$ and $\vec{r}_2$ respectively.



Let G be the moment of this couple, then according to Varignon's theorem

$$\vec{G} = \vec{r}_1 \times \vec{P} + \vec{r}_2 \times (-\vec{P})$$

$$= \vec{r}_1 \times \vec{P} - \vec{r}_2 \times \vec{P}$$

$$= (\vec{r}_1 - \vec{r}_2) \times \vec{P}$$

$$\vec{G} = \vec{r} \times \vec{P}$$

$$= |\vec{r}|\,|\vec{P}| \sin\theta\, \hat{n}$$

$$|\vec{G}| = rP\sin\theta|\hat{n}|$$

$$G = rP\sin\theta$$

$$\vec{OA} = \vec{OB} + \vec{BA}$$

$$\vec{r}_1 = \vec{r}_2 + \vec{r}$$

$$\vec{r}_1 - \vec{r}_2 = \vec{r}$$

Now draw a perpendicular from point B on the line of action of force $\vec{P}$ which meets it at point L.

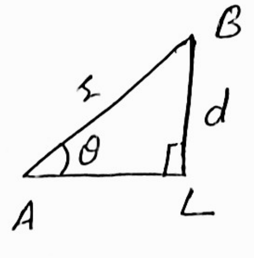
(BL = d is called arm of the couple)

In this way we get a right angled triangle ALB

$$G = rP\sin\theta$$

$$= P\,(r\sin\theta)$$

$$G = Pd$$



$$\frac{d}{r} = \sin\theta$$

$$d = r\sin\theta$$

is the expression for the magnitude of the moment of the couple $(\vec{P}, -\vec{P})$.