Chapter # 13:-

Magnetically Coupled Circuits

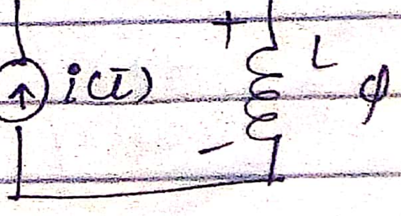
Def:- (Mutual inductance)

"when 2-coils are placed close to each other, the magnetic flux caused by current in one coil links with other coil, so inducing Emf / V in second.

→ for $V = L\,di/dt$.

Faraday's law → $V = N\dfrac{d\phi}{dt}$ —— ①

① × 1:- $V = N\dfrac{d\phi}{dt} \times \dfrac{di}{di}$ $\quad \because 1 = \dfrac{di}{di}$



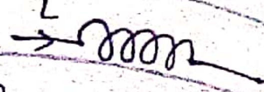
$$\boxed{V = L\dfrac{di}{dt}}$$

$\because L = N\dfrac{d\phi}{di}$

Diff. b/w Self & Mutual inductance:-

→ Self-inductance:-

let we have one coil, and current is flowing through it. Rate of change of current produces emf / voltages within same coil. This phenomenon is self-inductance.



$$V \propto \frac{di}{dt} \Rightarrow V = -L\frac{di}{dt}$$

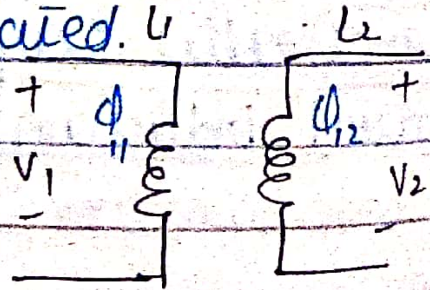
* -ve-sign is due to Lenz's law.

Lenz's law:- According to Lenz's law, generated emf opposes rate of change of current through which it is generated. (cause)

→ Mutual inductance:-

If rate of change of current through one coil produces emf / voltages in another coil placed near it; is called mutual inductance.

→ coils are physically separated.



coil#1 $\boxed{\phi_1 = \phi_{11} + \phi_{12}}$

$$V_1 = N_1\frac{d\phi}{dt}, \quad V_2 = N_2\frac{d\phi}{dt}$$

$$V_1 = N_1\frac{d\phi}{dt} \times \frac{di_1}{di_1}, \quad V_2 = N_2\frac{d\phi}{dt} \times \frac{di_2}{di_2}$$

$$\boxed{V_1 = L_1\frac{di_1}{dt}}, \quad \boxed{V_2 = M_{21}\frac{di_2}{dt}}$$

coil#2 :-

$$\phi_2 = \phi_{21} + \phi_{22}$$

$$V_2 = N_2 \frac{d\phi_2}{dt} \times \frac{di_2}{di_2} \quad ; \quad V_1 = N_1 \frac{d\phi_{21}}{dt} \times \frac{di_2}{di_2}$$

$$V_2 = N_1 \frac{d\phi_2}{di_2} \times \frac{di_2}{dt} \qquad V_1 = N_1 \frac{d\phi_{21}}{di_2} \times \frac{di_2}{dt}$$

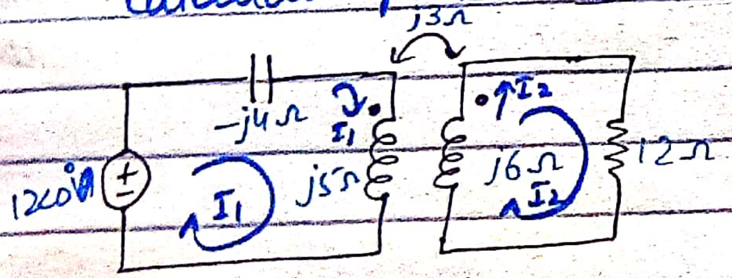
$$\boxed{V_2 = L_2 \frac{di_2}{dt}} \qquad \boxed{V_1 = M_{12} \frac{di_2}{dt}}$$

in coil L_2 in coil L_1

$M_{12} = M_{21} = M \rightarrow$ mutual inductance..

EX# 13.1 :-

calculate phasor currents I_1 & I_2 in ckt:-



KVL at loop# I_1 :-

due to opposite convention (leaving & entering currents)

$$-12 - j4I_1 + j5I_1 - j3I_2 = 0$$

$$\boxed{jI_1 - j3I_2 = 12} \quad \text{— ①}$$

KVL at loop # I_2 :-

$$-j3I_1 + (12+j6)I_2 = 0.$$

$$I_1 = \frac{(12+j6)\,I_2}{j3} = \boxed{(2-j4)\,I_2} \rightarrow \text{put in ①.}$$

$$j(2-j4)I_2 - j3I_2 = 12$$

$$(j2+4-j3)\,I_2 = 12 \Rightarrow (4-j)\,I_2 = 12$$

$$I_2 = \frac{12}{4-j}.$$

$$\boxed{I_2 = 2.91 \angle 14.04^\circ A}$$

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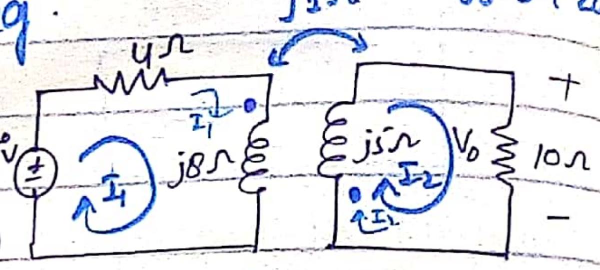
for I_1 :-
$$I_1 = (2-j4)I_2$$
$$I_1 = (2-j4)(2.91\angle 14.04^\circ)$$
$$I_1 = (4.47\angle -63.43)(2.91\angle 14.04)$$
$$\boxed{I_1 = 13.01\angle -49.39^\circ A}$$

Monday 06-04-2021

PP#13.1:- Find V_o in following.



KVL at I_1 :-
$$-120\angle 45^\circ + 4I_1 + j8I_1 + j1I_2 = 0$$
$$(4+j8)I_1 + j1I_2 = 120\angle 45^\circ \quad — (1)$$

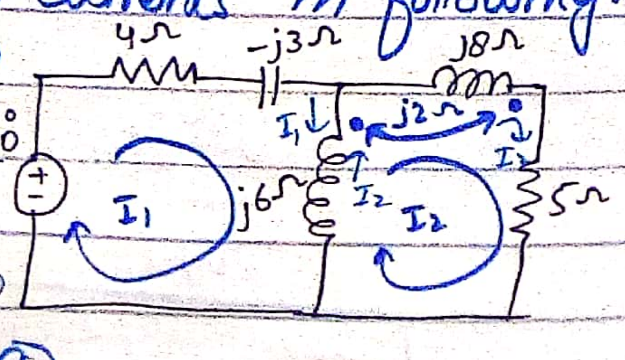
I_1 & I_2 both enters in dotted terminals. (addition of mutual inductive voltages i.e:- j1Ω)

KVL at I_2 :-
$$j5I_2 + 10I_2 + j1I_1 = 0.$$
$$j1I_1 + (10+j5)I_2 = 0$$
or
$$I_1 = -\frac{(10+j5)}{j1} I_2$$
$$\boxed{I_1 = (-5+j10)I_2} \rightarrow \text{put in (1)}$$
$$(1) \Rightarrow (4+j8)[(-5+j10)I_2] + j1I_2 = 120\angle 45^\circ$$
$$-100I_2 + jI_2 = 120\angle 45^\circ$$
$$I_2 = \frac{120\angle 45^\circ}{(-100+j)}$$
$$\boxed{I_2 = 1.199\angle -134.43^\circ A}$$

$\rightarrow$ $V_o = I_2 \times 10$
$$= (1.199\angle -13.43)(10)$$
$$\boxed{V_o = 12\angle -134.4^\circ V}$$

EX#13.2 Calculate Mesh currents in following:

$\because$ I_1 enter in coil # 1 (dotted)
I_2 leave in coil # 2 (dotted end)



KVL at loop # 1 :-
$$-100 + 4I_1 - j3I_1 + j6(I_1 - I_2) - j2I_2 = 0$$
$$\boxed{(4+j3)I_1 - j8I_2 = 100} \quad — (1)$$

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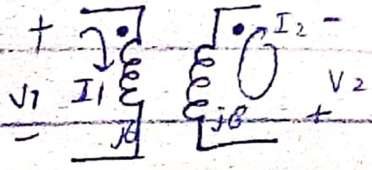
for mesh#2:- We see I_2; both coil in series. I_2 is leaving from dotted terminals in both coils; so.

$L = L_1 + L_2 + 2M$. → uses here.



$V_1 = -j2I_2$

$V_2 = -j2I_1$

$L = L_1 + L_2 + 2M$

$L = L_1 + L_2 - 2M$

KVL at mesh#2:-

$$-j2I_1 - \underbrace{j6I_1 + (L_1 + L_2 + 2M + 5)I_2}_{j6(I_2 - I_1)} = 0.$$

$$-j2I_1 - j6I_1 + (j6 + j8 + 2(j2) + 5)I_2 = 0.$$

$$\boxed{-j8I_1 + (5 + j18)I_2 = 0} \quad -②$$

Cramer's rule:-

$$\Delta = \begin{vmatrix} (4+j3) & -j8 \\ -j8 & (5+j18) \end{vmatrix} = (4+j3)(5+j18) - (-j8)(-j8)$$

$$= \boxed{30 + j87}$$

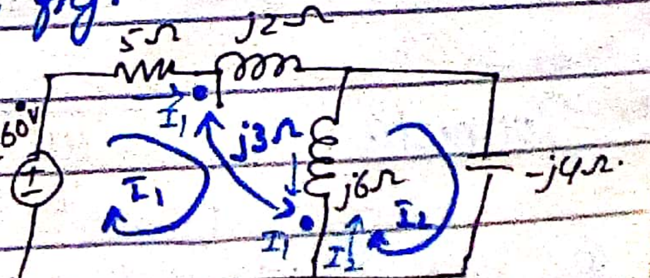
$$\Delta_1 = \begin{vmatrix} 100 & -j8 \\ 0 & 5+j18 \end{vmatrix} = 500 + j1800$$

$$\Delta_2 = \begin{vmatrix} 4+j3 & 100 \\ -j8 & 0 \end{vmatrix} = j800$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{500 + j1800}{30 + j87} = \boxed{20.3 \angle 3.5^\circ A}$$

$$I_2 = \frac{j800}{30 + j87} = \frac{\Delta_2}{\Delta} = \boxed{8.69 \angle 19.025^\circ A}$$

PP#13.2:- Find I_1 & I_2 in fig.



KVL at Mesh #1:-

I_1 & I_2 enters at dotted terminals → +ve mutual inductance.

$$\cancel{-100\angle 60 + 5I_1 + j2I_1 + j6(I_1 - I_2) + j3\Omega = 0}.$$

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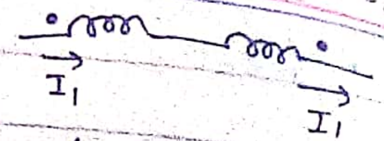
$-100\angle 60^\circ + 5I_1 + (j2 + j6 - 2(j3))\,I_1 -$ $j6I_2 + j3I_2 = 0$

series (w.r.t I_1)



$\boxed{(5+j2)I_1 - j3I_2 = 100\angle 60}$ — ①

$(L = L_1 + L_2 - 2M)$

Mesh #2:-

$j6(I_2 - I_1) - j4I_2 + j3I_1 = 0$

$-j3I_1 + j2I_2 = 0$

$\cancel{-j}3I_1 = \cancel{-j}2I_2$

$I_2 = 3/2\, I_1$ or $\boxed{I_2 = 1.5I_1}$ → put in ①

$(5+j2)I_1 - j3(1.5I_1) = 100\angle 60^\circ$

$5I_1 + j2I_1 - j4.5I_1 = 100\angle 60^\circ$

$$I_1 = \frac{100\angle 60^\circ}{5 - j2.5}$$

$\boxed{I_1 = 17.88\angle 86.56^\circ A}$

$I_2 = (1.5)(17.88\angle 86.56^\circ)$

$\boxed{I_2 = 26.83\angle 86.57^\circ A}$

Q-factor:- (Quality factor)

"It is ratio of inductor 'L' to Resistor 'R' of a coil at given frequency & defines efficiency of an inductor."

→ The higher Q-factor of inductor, close inductor to ideal without losses.

Formula:- $\boxed{Q = \frac{\omega L}{R} = \frac{2\pi f L}{R}}$ $\because \omega = 2\pi f$

Q bandwidth $\propto \frac{1}{Q}$; $\uparrow L \propto Q\uparrow$; $\downarrow R \propto \frac{1}{Q\uparrow}$

Normally / generally: $0 \le Q \le 100$.

Q-factor of Series Resonating cct (RLC) :-

↳ in resonating cct, Q-factor is ratio of voltage across inductor(V_L) / (V_C)capacitor to applied voltages.

i.e :- $Q = \frac{V_L}{V_{applied}}$ / $Q = \frac{V_C}{V_{applied}}$

at f_r

$X_C = X_L$

$\frac{1}{\omega C} = \omega L$

$\omega = \frac{1}{\sqrt{LC}}$

for inductor :-

$$Q = \frac{\cancel{I} X_L}{\cancel{I} R}$$

$$Q = \frac{X_L}{R} = \boxed{\frac{\omega L}{R}}$$

for capacitor :-

$$Q = \frac{\cancel{I} X_C}{\cancel{I} R}$$

$$Q = \frac{X_C}{R} = \boxed{\frac{\frac{1}{\omega C}}{R}} = \frac{1}{\omega CR}$$

put $\omega = \frac{1}{\sqrt{LC}}$ in both :-

$$Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{\sqrt{\cancel{L}} \times \sqrt{L}}{\sqrt{\cancel{L}}\sqrt{C} \cdot R}$$

$$\boxed{Q = \sqrt{\frac{L}{C}} \cdot \frac{1}{R}}$$

$$Q = \frac{1}{\frac{1}{\sqrt{LC}} \cdot CR} = \frac{\sqrt{L} \times \sqrt{\cancel{C}}}{\sqrt{\cancel{C}} \times \sqrt{C} \times R}$$

$$\boxed{Q = \sqrt{\frac{L}{C}} \cdot \frac{1}{R}}$$ → same for both L & C.

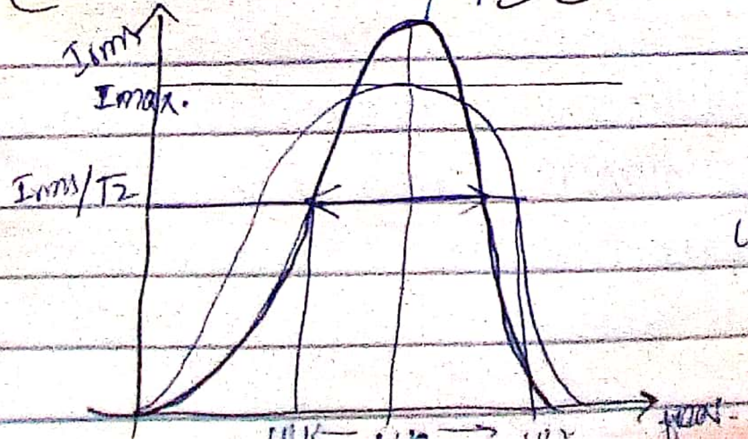
$$Q = \frac{\omega}{B.W \text{ (bandwidth)}}$$ → less bandwidth, better quality-factor

$\boxed{b.w = f_c = \text{higher freq.} - \text{Lower freq.} / \omega_2 - \omega_1}$

→ Sound Quality (sharpness of tuning) :-

↳ Q-factor is physical quantity which tells us better sound quality / at which value of current sound will better / power will better. (I↑, P↑, sound quality↑)

→



$\because P_{max} = I^2 R$

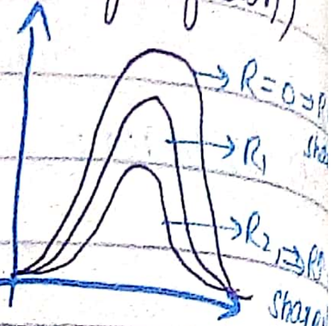
ω_0 = bandwidth = B·W

B·W ↓ , sound ↑ (better).

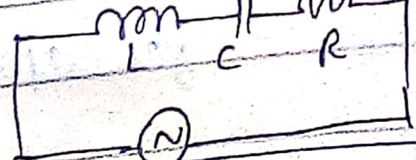
if ω_0 ~~<~~ I↓, P↓ ; if ω_0 > I, if I↑, P↑.

→ Q-factor decides how sharp resonance curve / Determine sharpness of resonance curve

→ If R↓, sharpness↑. (0→100 (value of Q-factor))



RLC-cct:-



When I is max. in cct,
This condition is called **resonance**.

$$\because Z = \sqrt{R^2 + (X_L - X_C)^2}$$

at $X_L = X_C$; $\boxed{Z = R}$ → Purely resistive cct.

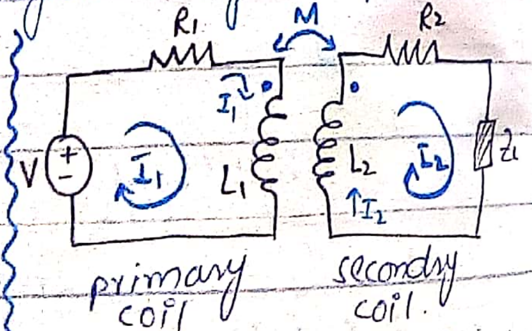
Wednesday
08-04-2020

13.4 Linear Transformers:- / Air-Core T/F:-

"A transformer is generally a four terminal device comprising two magnetically coupled coils."

→ In fig, coil L_1, is directly coupled to voltage source, is called "primary winding/coil":-

→ The coil connected to load is called "secondry coil"



(A linear T/F)

→ Resistances R_1 & R_2 are included to account for losses (power dissipation) in coil.

→ Transformer is said to be linear; if coils are wound on magnetically linear material.

→ A material for which magnetic permeability (μ) is constt e.g:- air, plastic, wood etc.

→ linear magnetic materials are characterized by a linear relation b/w magnetization & magnetic field.

→ Permeability is measure of reactance-material against formation of magnetic field.
→ Linear transformers are sometimes called Air-core transformers. They are used in radio & TV.
⇒ We would like to obtain input impedence Z_{in} as seen from source, bcz Z_{in} control behaviour of primary coil.

Applying KVL to both meshes (in fig) :-

Mesh #1 :- $-V + I_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$

or :

$$-V + I_1R_1 + j\omega L_1 I_1 - j\omega M I_2 = 0$$

$$\boxed{(R_1 + j\omega L_1)I_1 - j\omega M I_2 = V} \quad — (1)$$

Mesh #2 :- $I_2R_2 + I_2Z_L + j\omega L_2 I_2 - j\omega M I_1 = 0$

$$\boxed{(R_2 + Z_L + j\omega L_2)I_2 - j\omega M I_1 = 0} \quad — (2)$$

or

$$I_2 = \frac{j\omega M I_1}{R_2 + Z_L + j\omega L_2} \quad (3) \rightarrow \text{put in } (1)$$

(1) ⇒ $(R_1 + j\omega L_1)I_1 - j\omega M\left[\frac{j\omega M I_1}{R_2 + Z_L + j\omega L_2}\right] = V$

$\because j^2 = -1$

$$I_1\left[(R_1 + j\omega L_1) - \frac{j^2\omega^2M^2}{R_2 + Z_L + j\omega L_2}\right] = V$$

$\because \frac{V}{I_1} = Z_{in}$

or

$$R_1 + j\omega L_1 + \frac{\omega^2M^2}{R_2 + Z_L + j\omega L_2} = \frac{V}{I_1} = Z_{in}$$

So,

$$\boxed{Z_{in} = \underbrace{R_1 + j\omega L_1}_{\text{1st term is primary impedence}} + \underbrace{\frac{\omega^2M^2}{R_2 + Z_L + j\omega L_2}}_{\text{2nd term due to coupling b/w pri. \& sec. coils}}} \quad — (4)$$

→ Notice Z_{in} consists of 2-Terms- However, This impedence is reflected to primary, Thus it is known as Reflected Impedence (Z_R) or coupled impedence (Z_C).

i.e:- $Z_R = \dfrac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$ — ⑤

→ It should be noted from results in ④ & ⑤; They are not affected by location of dots on T/F, bcz same result is produced in ④ & ⑤ if M is replaced with (−M) → bcz of square of M (M^2)

Determining the equivalent cct. of a linear T/F:-

For Analysis:- It is sometime convinient to replace a magnetically coupled cct. by an equivalent cct. with no magnetic coupling. We want to replace linear T/F in 13·21 by an T(Y) or π(Δ) cct. which would have no mutual inductance.

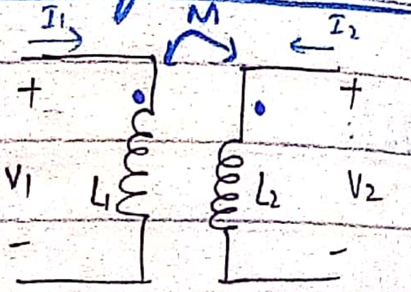


fig # 13·21

→ Voltage-current relationship for primary & sec. coil given by applying **KVL on** both loops for **fig # 13·21.**

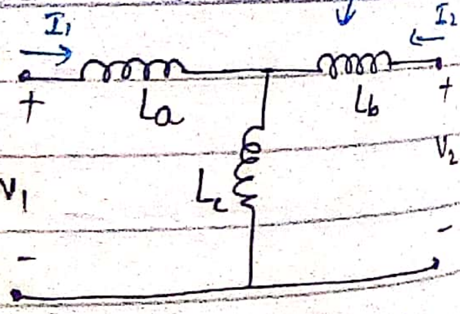


fig # 13·22 (T-eq. cct.)

$\boxed{\text{loop \#1}}$:-

$$-V_1 + j\omega L_1 I_1 + j\omega M I_2 = 0.$$

$$\boxed{j\omega L_1 I_1 + j\omega M I_2 = V_1} \quad — ①$$

$\boxed{\text{loop \#2}}$:-

$$-V_2 + j\omega L_2 I_2 + j\omega M I_1 = 0.$$

$$j\omega L_2 I_2 + j\omega M I_1 = V_2 \Rightarrow \boxed{j\omega M I_1 + j\omega L_2 I_2 = V_2}$$

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Putting these 2 eqn's in matrix form:-

$$\underbrace{\begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{B} \quad \text{—— ⑥}$$

$AX = B$

$X = A^{-1}B$

for A^{-1}, we change sign of ↗ & position of ↘ (diagonals) divided by det

By matrix inversion:-

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} j\omega L_2 & -j\omega M \\ -j\omega M & j\omega L_1 \end{bmatrix}}{j^2\omega^2 L_1 L_2 - j^2\omega^2 M^2} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{j^2\omega^2 L_1 L_2 - j^2\omega^2 M^2} \cdot \begin{bmatrix} j\omega L_2 & -j\omega M \\ -j\omega M & j\omega L_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow \text{simplify}$$

Taking $j^2\omega^2$ common.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{j\omega L_2}{j^2\omega^2 (L_1 L_2 - M^2)} & \frac{-j\omega M}{j^2\omega^2 (L_1 L_2 - M^2)} \\ \frac{-j\omega M}{j^2\omega^2 (L_1 L_2 - M^2)} & \frac{j\omega L_1}{j^2\omega^2 (L_1 L_2 - M^2)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{L_2}{j\omega (L_1 L_2 - M^2)} & \frac{-M}{j\omega (L_1 L_2 - M^2)} \\ \frac{-M}{j\omega (L_1 L_2 - M^2)} & \frac{L_1}{j\omega (L_1 L_2 - M^2)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{—— ⑦}$$

→ our main objective is to match eqn ⑥ & ⑦ with corresponding eqn for T & π-networks -

loop#1:-

$-V_1 + j\omega L_a I_1 + j\omega L_c (I_1 + I_2) = 0$

⑧ $\boxed{j\omega (L_a + L_c) I_1 + j\omega L_c I_2 = V_1}$

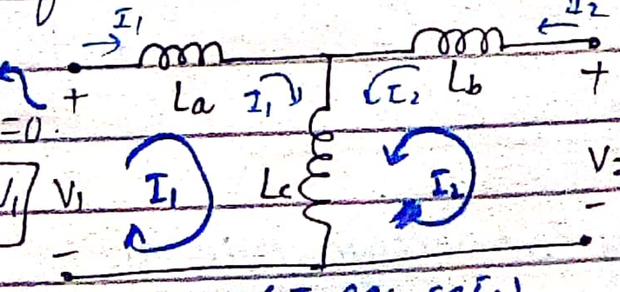


fig 13.22 (T-eq-cct.)

loop #2:-

$-V_2 + j\omega L_b I_2 + j\omega L_c (I_2 + I_1) = 0$

$j\omega (L_b + L_c) I_2 + j\omega L_c I_1 = V_2$

or $\boxed{j\omega L_c I_1 + j\omega (L_b + L_c) I_2 = V_2}$ —— ⑨

→ For T/Y network of above fig.; mesh analysis provides terminal equations as under in matrix; for (considering eq. (8) & (9)):-

$$\begin{bmatrix} j\omega(L_a+L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b+L_c) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (10)$$

If circuits in fig# 13.21 & 13.22 are equivalent in value, then eq. (6) & (10) should identical.

(6) ⟹ $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

(10) ⟹ $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a+L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b+L_c) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

→ Consider/Equating terms in impedence matrices of eq. (6) & (10):-

$\cancel{j\omega}L_1 = \cancel{j\omega}(L_a+L_c)$ & $\cancel{j\omega}M = \cancel{j\omega}L_c$

$L_1 = L_a + L_c \xleftarrow{\text{put in}} \boxed{M = L_c}$

so, $L_1 = L_a + M \rightarrow \boxed{L_a = L_1 - M}$

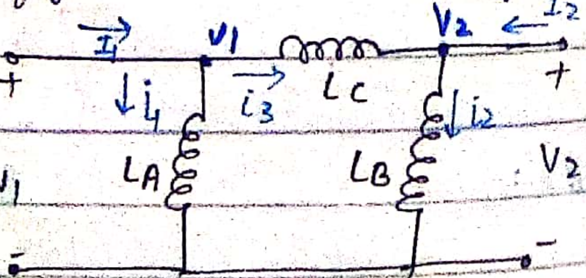
Similarly:-

$j\omega M = j\omega L_c$ & $\cancel{j\omega}L_2 = \cancel{j\omega}(L_b + L_c)$

$\boxed{M = L_c} \xrightarrow{\text{Put in}}$ & $L_2 = L_b + L_c$

$L_2 = L_b + M \rightarrow \boxed{L_b = L_2 - M}$ & $\boxed{L_c = M}$

→ For π/Δ network in fig# 13.23; we apply nodal analysis:-



Fig# 13.23 (eq. π cct.)

By inspection method:-

at node: V_1:-

$I_1 = i_1 + i_3$

$I_1 = \dfrac{V_1 - 0}{X_A} + \dfrac{V_1 - V_2}{X_C}$

$I_1 = \dfrac{X_C V_1 + X_A(V_1 - V_2)}{X_A X_C}$

$$I_1 = \frac{V_1(X_C + X_A) - X_A V_2}{X_A X_C} = V_1\left(\frac{\cancel{X_C}}{X_A \cancel{X_C}} + \frac{\cancel{X_A}}{\cancel{X_A} X_C}\right) - \frac{\cancel{X_A} V_2}{\cancel{X_A} X_C}$$

$$I_1 = \left(\frac{1}{X_A} + \frac{1}{X_C}\right) V_1 - \frac{1}{X_C} \cdot V_2 \quad \text{—— (a)}$$

or

$$I_1 = \left(\frac{1}{j\omega L_A} + \frac{1}{j\omega L_C}\right) V_1 - \frac{1}{j\omega L_C} \cdot V_2 \quad \text{—— (b)}$$

KCL at node: V_2:- $\quad I_2 + i_3 = i_2$

$$I_2 = i_2 - i_3$$

$$I_2 = \left(\frac{V_2 - 0}{X_B}\right) - \left(\frac{V_1 - V_2}{X_C}\right)$$

or

$$I_2 = \frac{X_C V_2 - X_B (V_1 - V_2)}{X_B X_C} = \frac{X_C V_2 - X_B V_1 + X_B V_2}{X_B X_C}$$

$$I_2 = \left(\frac{\cancel{X_C}}{X_B \cancel{X_C}} + \frac{\cancel{X_B}}{\cancel{X_B} X_C}\right) V_2 - \frac{\cancel{X_B} V_1}{\cancel{X_B} X_C}$$

$$I_2 = \left(\frac{1}{X_B} + \frac{1}{X_C}\right) V_2 - \frac{V_1}{X_C} = \left(\frac{1}{j\omega L_B} + \frac{1}{j\omega L_C}\right) V_2 - \frac{1}{j\omega L_C} V_1$$

or

$$I_2 = -\frac{1}{j\omega L_C} V_1 + \left(\frac{1}{j\omega L_B} + \frac{1}{j\omega L_C}\right) V_2 \quad \text{—— (c)}$$

re-arrange eqn. (b) & (c) in matrix-form.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{—— (11)}$$

Equating Terms in Admittance matrices of (7) & (11) :-

eqn (7) becomes

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{L_2}{j\omega(L_1 L_2 - M^2)} & \frac{-M}{j\omega(L_1 L_2 - M^2)} \\ \frac{-M}{j\omega(L_1 L_2 - M^2)} & \frac{L_1}{j\omega(L_1 L_2 - M^2)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{—— (7)}$$

Equating Terms :-

$$\frac{L_2}{j\omega(L_1L_2-M^2)} = \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} \quad \& \quad \frac{\cancel{+}M}{j\omega(L_1L_2-M^2)} = \frac{\cancel{+}1}{j\omega L_C}$$

$$\frac{L_2}{\cancel{j\omega}(L_1L_2-M^2)} = \frac{1}{\cancel{j\omega}}\left(\frac{1}{L_A}+\frac{1}{L_C}\right) \qquad \frac{M}{L_1L_2-M^2} = \frac{1}{L_C} \text{ — (13) put in (12).}$$

$$\frac{L_2}{L_1L_2-M^2} - \frac{1}{L_C} = \frac{1}{L_A} \text{ — (12)}$$

$$\frac{L_2}{L_1L_2-M^2} - \frac{M}{L_1L_2-M^2} = \frac{1}{L_A} \Rightarrow \frac{L_2-M}{L_1L_2-M^2} = \frac{1}{L_A}.$$

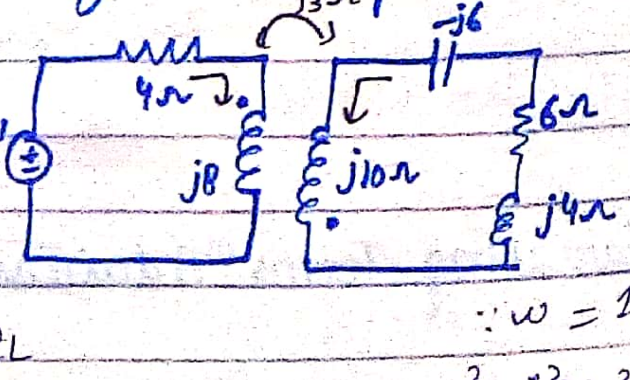
$$\boxed{L_A = \frac{L_1L_2-M^2}{L_2-M}} \quad \& \quad \boxed{L_C = \frac{L_1L_2-M^2}{M}}$$

Similarly :- $\boxed{L_B = \frac{L_1L_2-M^2}{L_1-M}}$

Analysis :- Note in fig# 13.22 & 13.23; The inductors are not magnetically coupled & also note that changing location of dots in (13.21) can cause M to become −M.

→ Therefore, eq. model is still mathematically valid.

PP#13.4 :- find Zin & I from voltage source.



$$Z_{in} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

$$Z_{in} = R_1 + X_{L_1} + \frac{X_m^2}{R_2 + j\omega L_2 + Z_L} \qquad \because \omega = 1\,rad/s$$

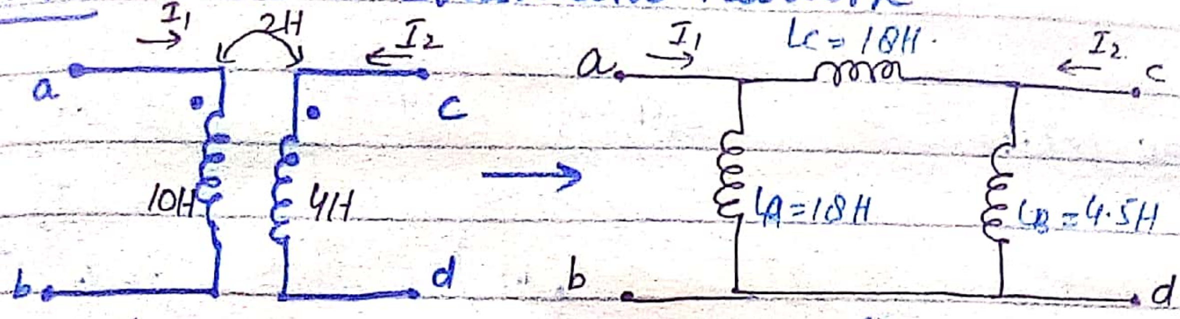
$$X_m^2 = j^2(3)^2$$

$$= 4 + j8 + \frac{(3)^2}{j10 - j6 + 6 + j4}$$

$$Z_{in} = 4 + j8 + 0.54 - j0.72 = 4.57 + j7.28$$

$$\boxed{Z_{in} = 8.58\angle 58.05^\circ\,\Omega} \rightarrow I = \frac{40\angle 0^\circ}{8.58\angle 58.05} = \boxed{4.66\angle -58^\circ}$$

PP#13.5:- Find π equivalent network.



(π-eq. cct)

Here, $L_1 = 10H$, $L_2 = 4H$, $M = 2H$.

$$\text{so, } L_A = \frac{L_1L_2 - M^2}{L_2 - M} = \frac{(10)(4)-(2)^2}{4-2} = 18H$$

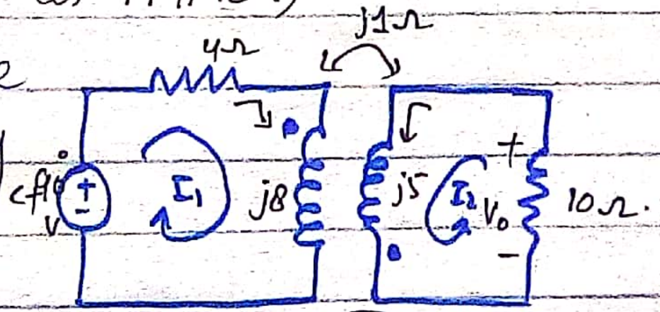
$$L_B = \frac{L_1L_2 - M^2}{L_1 - M} = \frac{(10)(4)-(2)^2}{10-2} = 4.5H$$

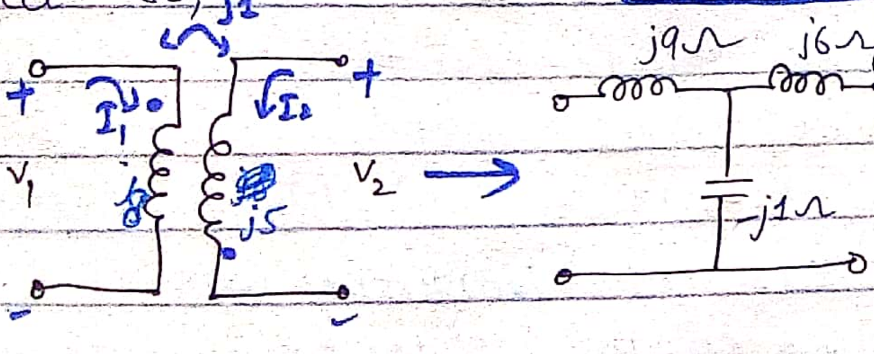
$$L_C = \frac{L_1L_2 - M^2}{M} = \frac{(10)(4)-(2)^2}{2} = 18H.$$

for T-eq. cct:-
$L_a = L_1 - M$
$L_b = L_2 - M$
$L_c = M.$

Ex# 13.6:- imp Solve for I_1, I_2, V_0 using T-eq. cct. for linear T/F. (Same as PP#13.1)

for T-eq. cct, we have to change magnetically coupled coil with T-cct. So,





I_1 enters & I_2 leaves so, M is -ve.

$$L_a = L_1 - M = j8 - (-j1) = j9$$

$$L_b = L_2 - M = j5 - (-j1) = j6$$

$$L_c = M = -j1.$$

for capacitor's capacitance, so we placed capacitor in T-model

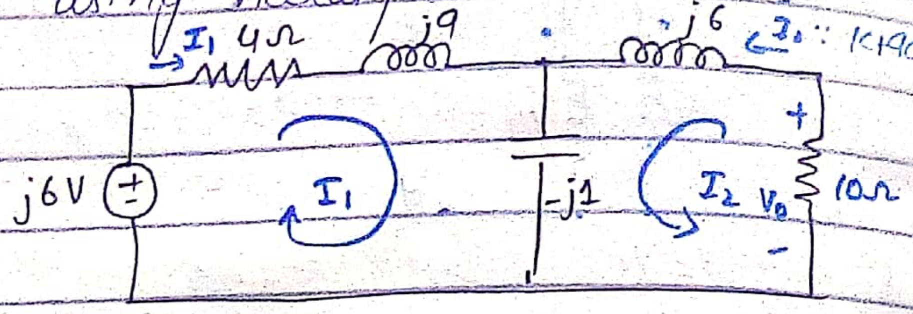
$$X_c = \frac{1}{j\omega c} = -j\omega c$$

→ Inserting T-eq cct in fig 13-29 (given cct.)
Then solving using nodal / Mesh.

$\because j6 = 6\angle 90°$

$\because 1\angle 90 = j$

Now, cct :-



Mesh Analysis :-

Loop#1 :-

$$-j6 + 4I_1 + j9I_1 - j1(I_1 + I_2) = 0$$

$$\boxed{(4 + j8)I_1 + (-j1)I_2 = j6} \quad —①$$

loop#2 :-

$$10I_2 + j6I_2 - j1(I_2 + I_1) = 0.$$

$$\boxed{(10 + j5)I_2 + (-j1)I_1 = 0} \quad —②$$

$$\cancel{-}j1I_1 = \cancel{-}(10 + j5)I_2$$

$$I_1 = \frac{(10 + j5)I_2}{j} \times \frac{+j}{j}$$

$$\boxed{I_1 = (5 - j10)I_2} \quad ③$$

Put ③ in ① :-

$$j6 = (4 + j8)(5 - j10)I_2 - j1I_2$$

$$(20 - j40 + j40 - j^2 80)I_2 - j1I_2 = j6.$$

$$(20 + 80)I_2 - j1I_2 = j6.$$

$$\boxed{(100 - j1)I_2 = j6} \quad — ⓐ$$

100 → real part ; -1 → imaginary part.

So, $(100 >>> 1) \Longrightarrow$ so $100 - j \cong 100$

$$100I_2 = j6$$

$$I_2 = j6/100 = \frac{6\angle 90°}{100}$$

$$\boxed{I_2 = 0.06\angle 90° A}$$

putting value of I_2 in eq③ :-

$I_1 = (5-j10)(0.06\angle 90^\circ)$

$I_1 = (5-j10)(j0.06)$

$\boxed{I_1 = 0.6+j0.3\,A}$

→ $V_0 = I_2 R_{10\Omega}$

$V_0 = -10\,I_2$

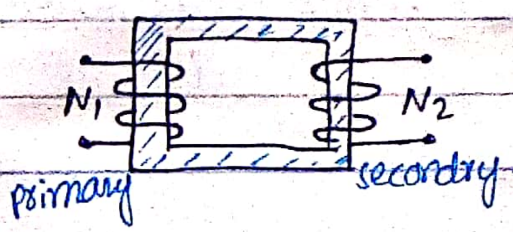
$V_0 = (-10)(j0.06) = -j0.6$

$\boxed{V_0 = 0.6\angle -90^\circ\,V}$

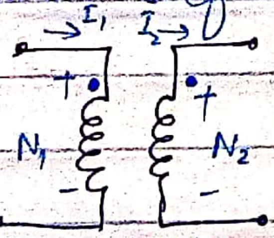
(I_2 → opposite direction so -ve.)

13.5 Ideal Transformer:-

"An ideal transformer is a unity-coupled, lossless transformer, in which primary & secondary coils have infinite self-inductance."



ideal T/F



$N_1 : N_2$

cct symbol.

(AC only, $\boxed{n=1}$)

→ if $V_2 > V_1$ → step-up T/F

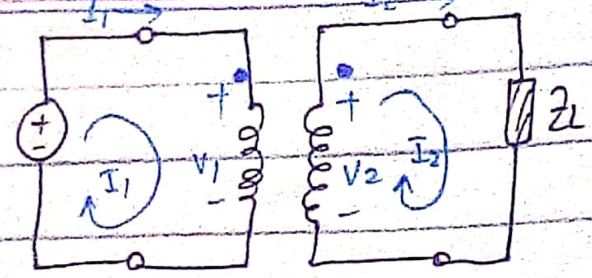
→ if $V_2 < V_1$ → step-down T/F

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = n \text{ (turn-ratio)}$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{1}{n}$$

Application:-

* T/F as an isolation device to isolate dc b/w two amplifier stages (ac-only, $n=1$)
* T/F as matching device using ideal T/F to match speaker to amplifier.



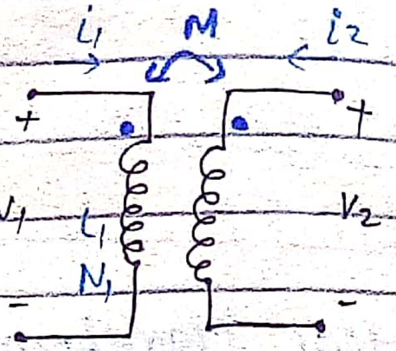
• $K = \dfrac{M}{\sqrt{L_1 L_2}}$

• $M = K\sqrt{L_1 L_2}$

$$\left\{\begin{array}{l} K < 0.5 \\ K = 1 \\ K > 0.5 \end{array}\right\}$$

fig(13.31) Relating Pri & sec coils quantities is ideal T/F

By KVL:-



$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \quad \text{—13.49(a)}$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \rightarrow \text{13.49 (b)}$$

$$V_1 - j\omega M I_2 = j\omega L_1 I_1$$

$$I_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1} \quad \text{—13.49(C)}$$

$\because V_L = L\dfrac{di}{dt}$

$V_L = j\omega L_1 I_1$

→ Put I_1 in 13.49(b):-

$$V_2 = \cancel{j\omega} M \left(\frac{V_1 - j\omega M I_2}{\cancel{j\omega} L_1}\right) + j\omega L_2 I_2$$

$\because V_M = M\dfrac{di_2}{dt} = j\omega M I_2$

$$V_2 = \frac{MV_1}{L_1} - \frac{j\omega M^2 I_2}{L_1} + j\omega L_2 I_2$$

re-arrange:-

$$V_2 = j\omega L_2 I_2 + \frac{MV_1}{L_1} - \frac{j\omega M^2 I_2}{L_1}$$

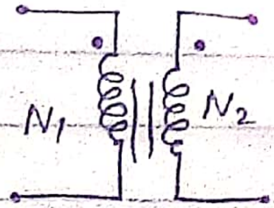
But $M = \sqrt{L_1 L_2}$ for perfect coupling ($\because k=1$);

$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2} \times V_1}{\sqrt{L_1}\cdot\sqrt{L_1}} - j\omega \frac{(\sqrt{L_1 L_2})^2 \cdot I_2}{L_1}$$

$$V_2 = \cancel{j\omega L_2 I_2} + \sqrt{\frac{L_2}{L_1}}\, V_1 - \cancel{j\omega L_2 I_2}$$

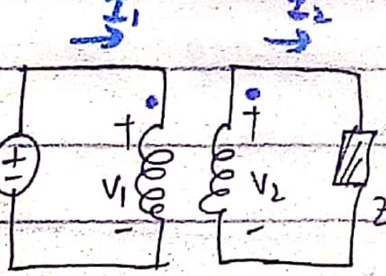
$$\boxed{V_2 = \sqrt{\frac{L_2}{L_1}}\, V_1}$$

→ Vertical lines b/w coils indicates an iron core as distinct from air core used in linear transformer.



- Primary winding has N_1-turns.
- Secondary winding has N_2-turns.

→ **in fig# 13.31:-**



When sinusoidal voltage is applied to primary winding, same flux(ϕ) goes through both winding.

* Acc. to Faraday's law; voltage across primary:

$$V_1 = N_1 \frac{d\phi_1}{dt} \quad — 13.50(a)$$

$\because \phi_1 = \phi_2 = \phi$.

* Voltage across secondary:-

$$V_2 = N_2 \frac{d\phi_2}{dt} \quad — 13.50(b)$$

13.50 (b) ÷ by 13.50(a):-

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \cdot \frac{\cancel{d\phi/dt}}{\cancel{d\phi/dt}}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \quad — 13.52.$$

$\because n$ is turns ratio / Transformer ratio.

→ For power conservation, energy supplied to primary must equal to secondary's absorbed power since; no losses in ideal T/F:-

$$P_1 = P_2$$

$\because \frac{V_2}{V_1} = n = \frac{N_2}{N_1}$.

$$V_1 I_1 = V_2 I_2 \; / \; V_1 i_1 = V_2 i_2 \quad — 13.53$$

$$\boxed{\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = n.} \quad — 13.54$$

→ Similarly, pri & sec currents are related to turn ratio, in inverse manner of voltage

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n} \quad -13.55$$

Note:-

* when $n > 1$, we have <u>step-up T/F</u>, as in step-up T/F, sec V > pri V ($V_2 > V_1$) / $N_2 > N_1$
* when $n=1$, T/F is generally named as <u>isolation T/F</u>.
* when $n<1$, transformer is <u>step-down</u> ($V_2 < V_1$) ($N_2 < N_1$)

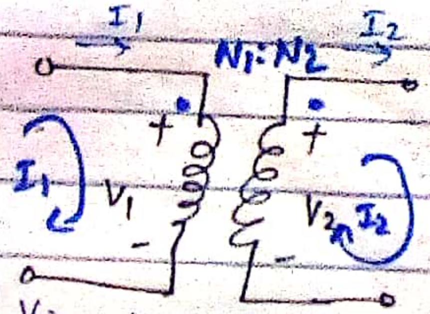
↳ Ratings of T/F are usually specified as V_1/V_2

→ **Proper polarity of voltage & current Direction:-**

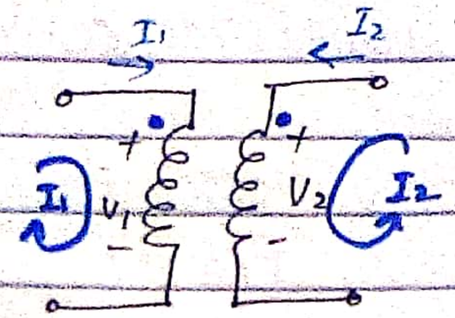
Rules:-

1) If V_1 & V_2 both are +ve / both -ve at dotted end, use $n \to +ve$, other-wise (for different conventions of V_1 & V_2) use $n \to -ve$.
2) If I_1 & I_2 both enter into / both leave from dotted ends, use $n \to -ve$, otherwise (for same convention of I_1 & I_2, both enter dot / leave dot) use $n \to +ve$.

e.g:-

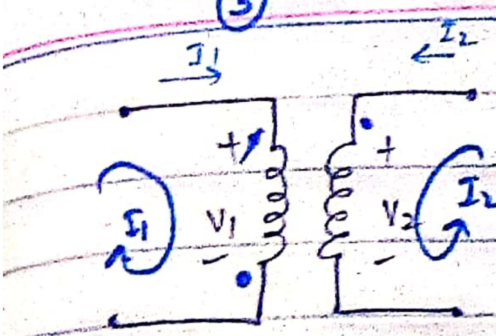


$$\frac{V_2}{V_1} = \frac{N_2}{N_1}, \quad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$



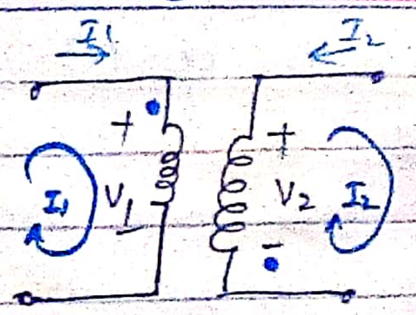
$$\frac{V_2}{V_1} = \frac{N_2}{N_1}, \quad \frac{I_2}{I_1} = -\frac{N_1}{N_2}$$

(3)



$\frac{V_2}{V_1} = -\frac{N_2}{N_1}$, $\frac{I_2}{I_1} = \frac{N_1}{N_2}$

(4)



$\frac{V_2}{V_1} = -\frac{N_2}{N_1}$, $\frac{I_2}{I_1} = -\frac{N_1}{N_2}$

$\rightarrow \frac{V_2}{V_1} = n$

$V_2 = nV_1$, $\boxed{V_1 = \frac{1}{n} \cdot V_2}$ 13.56

$\rightarrow \frac{I_2}{I_1} = \frac{1}{n}$

$\boxed{I_2 = \frac{I_1}{n} \; ; \; I_1 = n I_2}$ 13.57

→ complex power in Primary winding :-

$S_1 = V_1 I_1$ $\quad \because V_1 = \frac{V_2}{n}$ & $I_1 = nI_2$

So, $S_1 = \frac{V_2}{\cancel{n}} \times \cancel{n} I_2$

$S_1 = V_2 \times I_2$

$\boxed{S_1 = S_2}$ → show complex power supplied to primary is delivered to sec. without any loss. T/F (ideal) absorbs no-power.

$\because V_1 = V_2/n$

$I_1 = nI_2$

$\rightarrow Z_{in} = \frac{V_1}{I_1}$

$\because \frac{V_2}{I_2} = Z_L$

So, $Z_{in} = \frac{V_2}{n} \cdot \frac{1}{nI_2}$

$\boxed{Z_{in} = \frac{Z_L}{n^2}}$ → input / reflected impedence.

→ This ability of T/F to tranform given impedence into another impedence means us "impedence matching."