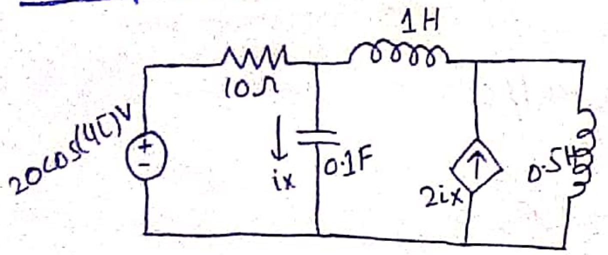
Chapter#10:-

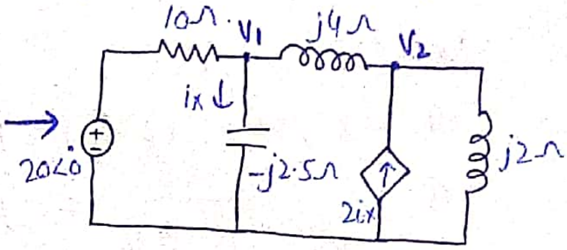
Sinusoidal Steady State Analysis

→ Nodal Analysis:-

Example # 10.1:- find i_x using nodal analysis.



→



1) $V_s = 20\cos 4t = 20\angle 0°\,V$; $\omega = 4$ rad/s.

- $0.1F \Rightarrow \frac{1}{j\omega C} = \frac{1}{j\times 4\times 0.1} = -j2.5$
- $1H \Rightarrow j\omega L = j\times 4\times 1 = j4$
- $0.5H \Rightarrow j\omega L = j\times 4\times 0.5 = j2$

2) **Nodal:-**

at V_1: $\frac{V_1 - 20}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4}$; $\because I_x = \frac{V_1}{-j2.5}$

$$\boxed{(1+j1.5)V_1 + j2.5V_2 = 20} \text{ — } ①$$

at V_2: $2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2}$

$$\boxed{11V_1 + 15V_2 = 0} \text{ — } ②$$

Cramer's rule:-

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300$$

$$\Delta_2 = \begin{vmatrix} 1+j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15-j5} = 18.9\angle 18.4°$$

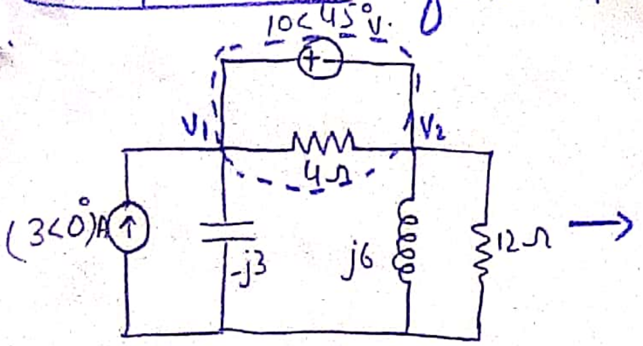
So,

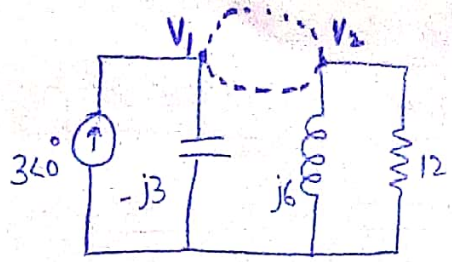
$$I_x = \frac{V_1}{-j2.5} = \frac{18.9\angle 18.4°}{-j2.5}$$

$$I_x = 7.59\angle 108.4°\,A$$

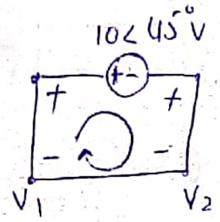
$$\boxed{I_x = 7.5\cos(4t + 108.4°)\,A}$$

Example#10·2:- find V_1 & V_2 by nodal (super-node).





super-node eqn:-



$$V_1 - V_2 = 10\angle 45^\circ$$

$$\boxed{V_1 = V_2 + 10\angle 45^\circ} \text{ — } 1$$

$$\rightarrow \quad 3 = \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12}$$

$$\boxed{36 = j4V_1 + (1-j2)V_2} \text{ — } 2$$

putting 1 in 2 :-

$$36 - j4(V_2 + 10\angle 45^\circ) = (1-j2)V_2$$

$$36 - 40\angle 135^\circ = (1+j2)V_2$$

$$V_2 = \frac{36 - 40\angle 135^\circ}{1+j2}$$

$$\boxed{V_2 = 31{\cdot}40\angle -87{\cdot}18^\circ V} = 1{\cdot}54 - j31{\cdot}36 V$$

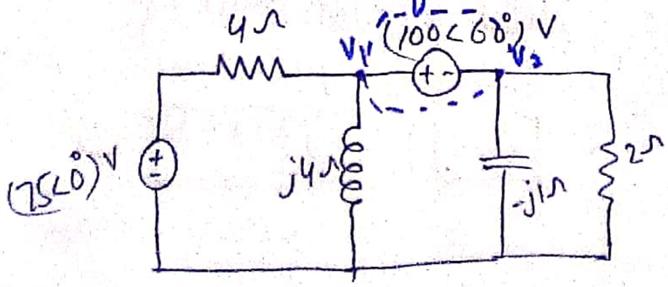
• put V_2 in 1 :-

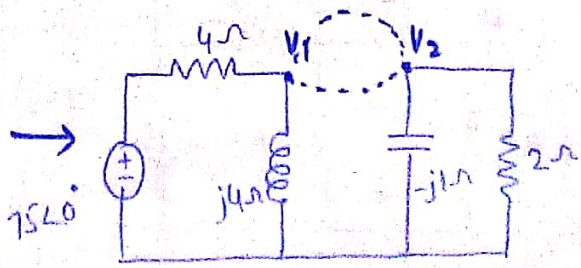
$$V_1 = (31{\cdot}40\angle -87{\cdot}18^\circ) + (10\angle 45^\circ)$$

$$\boxed{V_1 = 25{\cdot}78\angle -70{\cdot}48^\circ V}$$

Practice Problem #10·2:-

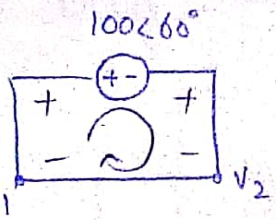
find V_1 & V_2.





KVL

Super-node eqn :-



$V_1 - V_2 = 100\angle 60^\circ$

$$\boxed{V_1 = V_2 + 100\angle 60^\circ} \text{ — ①}$$

→ $\dfrac{V_1 - 75}{4} = \dfrac{V_1}{j4} + \dfrac{V_2}{-j1} + \dfrac{V_2}{2}$

$$\boxed{(1-j)V_1 + (2+j4)V_2 = 75} \text{ — ②}$$

put ① in ② :-

$(1-j)(V_2 + 100\angle 60^\circ) + (2+j4)V_2 = 75$

$V_2 - V_2 j + 2V_2 + 4jV_2 = 75 - (1-j)(50 + j86.66)$

$3V_2 + j3V_2 = 75 - (136.6 + j36.6)$

$V_2 = \dfrac{75 - (136.6 + j36.6)}{(3+3j)}$

$V_2 = \dfrac{-61.60 - j36.60}{3+j3}$

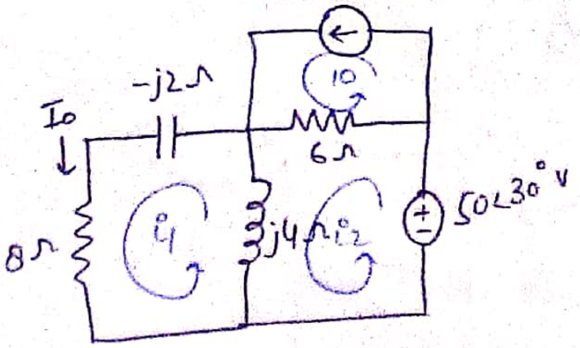
$$\boxed{V_2 = 16.88\angle 165.72^\circ V}$$

① ⇒ $V_1 = (16.88\angle 165.72^\circ) + (100\angle 60^\circ)$

$$\boxed{V_1 = 96.8\angle 69^\circ V}$$

→ Mesh Analysis :-

P.P
Example # 10.3 :- Find I_o by Mesh Analysis.



KVL at i_1 :-

$j4(i_1 - i_2) - j2i_1 + 8i_1 = 0.$

$8i_1 + j2i_1 - j4i_2 = 0.$

÷ by 2 $\boxed{(4+j)i_1 - j2i_2 = 0}$ — ①

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KVL at i_2:-

$$j4(i_2-i_1)+6(i_2+10)+50\angle 30^\circ=0.$$
$$-j4i_1+j4i_2+6i_2+60+(50\angle 30^\circ)=0$$
$$\boxed{j4i_1-(6+j4)i_2=103.3+j25} \quad (2)$$

Cramer's rule:-

$$\begin{bmatrix} 4+j & -j2 \\ j4 & -(6+j4) \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 103.3+j25 \end{bmatrix}$$

$$\Delta=\begin{vmatrix} 4+j & -j2 \\ j4 & -(6+j4) \end{vmatrix} = -28-j22$$

$$\Delta_1=\begin{vmatrix} 0 & -j2 \\ 103.3+j25 & -(6+j4) \end{vmatrix} = -50+j206.6$$

$$i_1=\frac{\Delta_1}{\Delta}=\frac{-50+j206.6}{-28-j22}$$

$$\boxed{i_1=5.96\angle -164.5^\circ A}$$

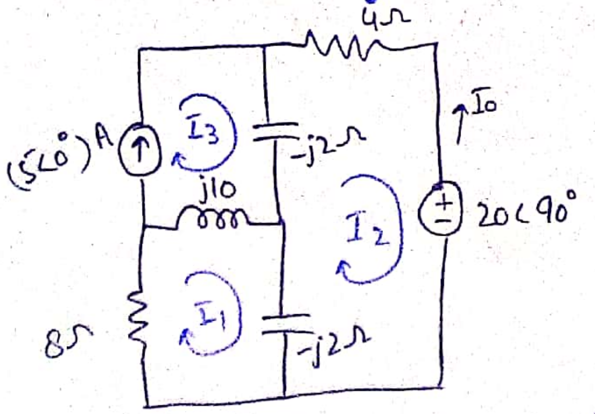
but : $I_o=-i_1=-5.96\cos(\omega t-114.5^\circ)A$

so, $I_o=5.96\cos(\omega t-114.5^\circ+180^\circ)$

$$\boxed{I_o=5.96\angle 65.45^\circ A}$$

Example # 10.3:-

find I_o by Mesh analysis.



$\because I_3=5A$

KVL at I_1:-

$$8I_1+j10(I_1-I_3)-j2(I_1-I_2)=0$$
$$\boxed{(8+j8)I_1+j2I_2=j50} \quad (1)$$

$\because j=1\angle 90^\circ$

KVL at I_2:-

$$-j2(I_2-I_3)+4I_2+20\angle 90^\circ-j2(I_2-I_1)=0$$
$$\boxed{j2I_1+(4-j4)I_2=-j30} \quad (2)$$

using cramer's rule:-

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$\because I_o = -I_2$

$$\Delta = \begin{vmatrix} 8+j8 & j8 \\ j2 & 4-j4 \end{vmatrix} = 68$$

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340-j240$$

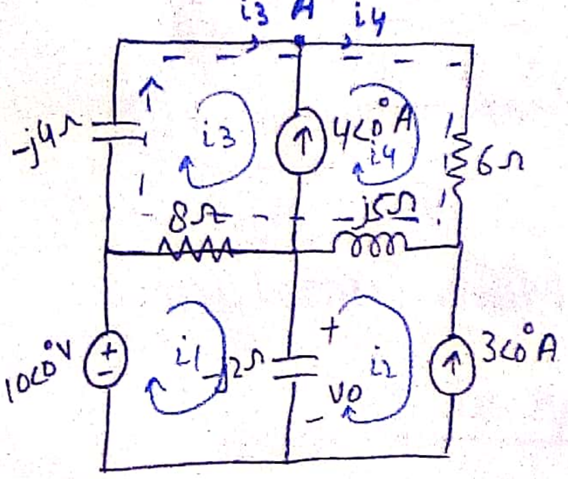
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{340-j240}{68} = 6.2\angle -35.21^\circ \, A$$

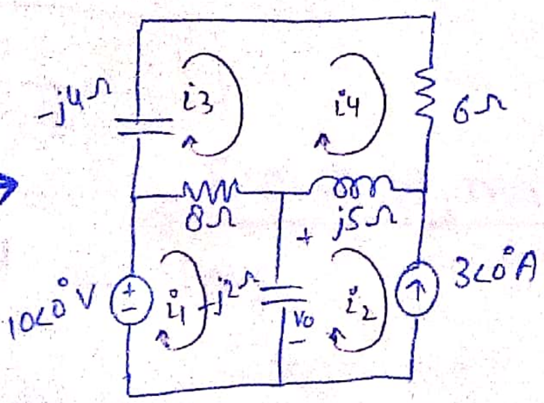
Then:- $I_o = -I_2 = 6.2\cos(\omega t - 35.21 + 180^\circ)A$

$$\boxed{I_o = 6.2\cos(\omega t + 144.78^\circ)A}$$ **Ans:**

Example #10.4:-

Find Vo by super-mesh.





KCL at A:-

$$4 + i_3 = i_4$$

$$\boxed{i_3 = i_4 - 4} \quad \text{—(a)}$$

KVL at mesh-2:-

$$\boxed{i_2 = -3A} \quad \text{—(b)}$$

Mesh # 1:-

$$-10\angle 0^\circ + 8(i_1 - i_3) - j2(i_1 - i_2) = 0 \qquad \because i_3 = i_4 - 4$$

$$8i_1 - 8i_3 - j2i_1 - j6 = 10 \qquad i_2 = -3A$$

$$(8-j2)i_1 - 8i_3 - j6 = 10.$$

$$\boxed{(8-j2)\,i_1 - 8\,i_3 = 10 + j6} \quad -(1)$$

Super-mesh:-

$$-j4i_3 + 6i_4 + j5(i_4 - \underset{3A}{i_2}) + 8(i_3 - i_1) = 0.$$

$$-j4i_3 + 6i_4 + j5i_4 + j15 + 8i_3 - 8i_1 = 0.$$

$$-j4i_3 + 6(i_3+4) + j5(i_3+4) + 8i_3 - 8i_1 = -j15$$

$$-8i_1 + 8i_3 + 6i_3 - j4i_3 + j5i_3 + 24 + 20j = -15j$$

$$\boxed{-8i_1 + (14+j)\,i_3 = -24 - j35} \quad -(2)$$

Cramer's rule:-

$$\Delta = \begin{vmatrix} (8-j2) & -8 \\ -8 & (14+j) \end{vmatrix} = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10+6j & -8 \\ -24-35j & 14+j \end{vmatrix} = -58 - j186$$

$$\Delta_3 = \begin{vmatrix} 8-j2 & (10+6j) \\ -8 & (-24-j35) \end{vmatrix} = -182 - j184.$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{-58-j186}{50-j20} = 3.61\angle \cancel{-85.54}\ 274^\circ\ A$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{-182-j184}{50-j20} = 4.8\angle -112^\circ A$$

So,

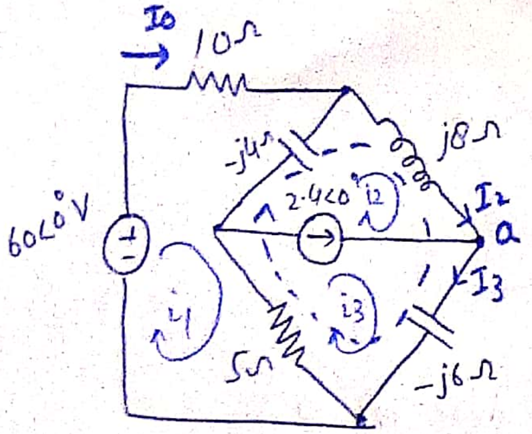
$$V_o = -2j\,(i_1 - i_2)$$

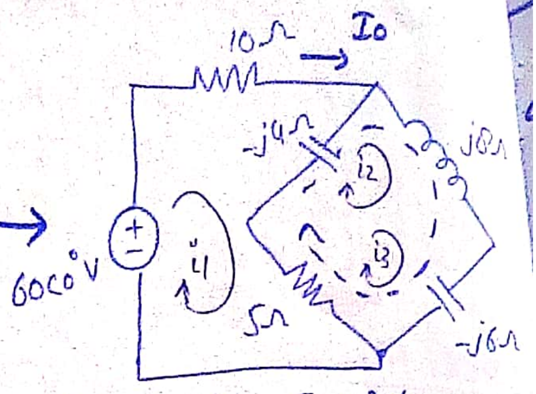
$$= -2j\,(3.61\angle \cancel{-85.51}\ 274^\circ + 3\angle 0^\circ)$$

$$\cancel{V_o = 9.74\angle -137.6^\circ V}$$

$$\boxed{V_o = 9.746\angle 222.2^\circ V} \quad \text{Ans:}$$

Practice Problem # 10.4 :- find I_o by mesh.





node a $\therefore$ $I_3 = I_2 + 2.4$

Mesh # 1 :-

$$-60\angle 0° + 10i_1 - j4(i_1 - i_2) + 5(i_1 - i_3) = 0$$

$$10i_1 - j4i_1 + j4i_2 + 5i_1 - 5i_3 = 60$$

$$(15 - j4)i_1 + j4i_2 - 5i_3 = 60$$

putting $I_3 = I_2 + 2.4$:-

$$\boxed{(15 - j4)i_1 + (-5 + j4)i_2 = 72} \quad (1)$$

Super-mesh :-

$$-j4(i_2 - i_1) + j8i_2 - j6i_3 + 5(i_3 - i_1) = 0$$

$$-j4i_2 + j4i_1 + j8i_2 - j6(i_2 + 2.4) + 5(i_2 + 2.4) - 5i_1 = 0$$

$$(-5 + j4)i_1 + (-j4 + j8 - j6 + 5)i_2 = -12 + j14.4$$

$$\boxed{(-5 + j4)i_1 + (5 - j2)i_2 = -12 + j14.4} \quad (2)$$

Cramer's rule :-

$$\begin{bmatrix} 15 - j4 & -5 + j4 \\ -05 + j4 & 5 - j2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 72 \\ -12 + j14.4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 15 - j4 & -5 + j4 \\ -5 + j4 & 5 - j2 \end{vmatrix} = 58 - j10$$

$$\Delta_1 = \begin{vmatrix} 72 & -5 + j4 \\ -12 + j14.4 & 5 - j2 \end{vmatrix} = 357.6 - j24$$

$$I_o = I_1 = \frac{\Delta_1}{\Delta} = \frac{357.6 - j24}{58 - j10} = \boxed{6.08\angle 5.94° A} \text{ Ans:}$$

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Super-position Theorem:- (S.P.T)

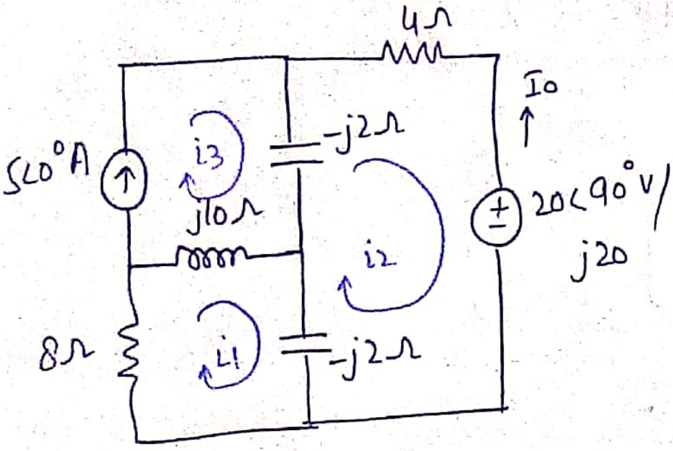
→ When a circuit has more than one source (V & I) Then, we have to solve all The sources independently. This principle is called S.P.T.

→ If a circuit has two or more independent source, one way to determine value of specific variable (V & I) is to use nodal, mesh analysis.

→ Another way to determine contribution of each independent source to variable & Then add Them up. (steps from notes)

→ Super-position Theorem:-

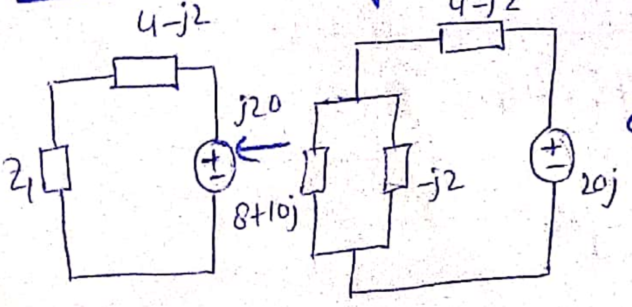
Example #10.5:- find Io.

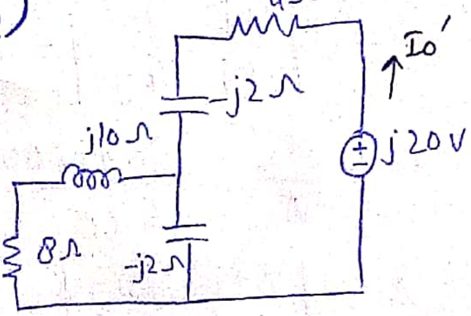


let $\boxed{I_o = I_o' + I_o''}$

→ I_o' is due to voltage source & I_o'' is due to current source.

Due to voltage source:- (I_o')





$$Z_1 = (8+j10) \parallel (-j2)$$

$$Z_1 = 2.6\angle 83.65^\circ$$

So, $Z_{eq} = (4-j2) + (2.6\angle 83.65^\circ)$

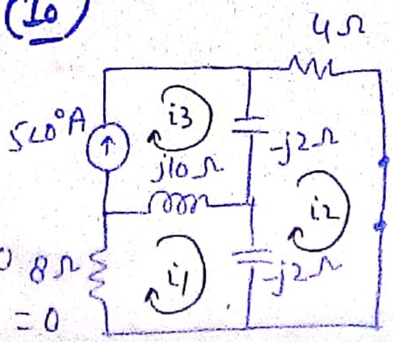
$$\boxed{Z_{eq} = 6.01\angle -45^\circ\,\Omega}$$

So, $I_o' = \dfrac{V}{Z_{eq}} = \dfrac{20\angle 90^\circ}{6.01\angle -45^\circ}$

$$\boxed{I_o' = 3.32\angle 135^\circ\,A}$$

Due to current source:- (I_o'')

$\boxed{i_3 = 5A}$



Mesh#1:-

$$8i_1 + j10(i_1 - 5) - j2(i_1 - i_2) = 0$$

$$8i_1 + j10i_1 - j50 - j2i_1 + j2i_2 = 0$$

$$\boxed{(8+j8)i_1 + j2i_2 = j50}$$

Mesh #2:-

$$-j2(i_2-5)+4i_2-j2(i_2-i_1)=0$$

$$-j2i_2+4i_2-j2i_2+j2i_1=-j10$$

$$\boxed{j2i_1+(4-j4)i_2=-j10}$$

Cramer's rule:-

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} j50 \\ -j10 \end{bmatrix}$$

$$\Delta=\begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix}=68$$

$$\Delta_1=\begin{vmatrix} 50j & j2 \\ -10j & 4-j4 \end{vmatrix}=180+j200$$

$$i_1=\frac{\Delta_1}{\Delta}=\frac{180+j200}{68}=3.95\angle 48.01^\circ A$$

$$\Delta_2=\begin{vmatrix} 8+j8 & j50 \\ j2 & -j10 \end{vmatrix}=180-j80$$

$$i_2=\frac{\Delta_2}{\Delta}=\frac{180-j80}{68}=2.89\angle-23.96^\circ A=2.64-j1.176$$

but: $I_o''=-i_2=-2.64+1.176j\ A.$

so,

$$I_o=I_o'+I_o''$$

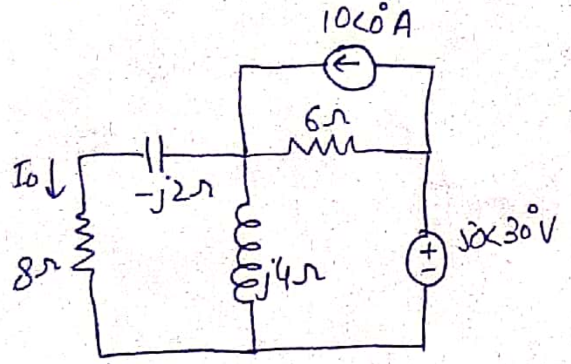
$$=(3.32\angle 135^\circ)+(-2.64+j1.176)$$

$$=(-2.34+2.34j)+(-2.64+j1.176)$$

$$=-4.9+j3.5=\boxed{6.12\angle 144.7^\circ A}$$ Ans:-

Practice Problem #10.5:-

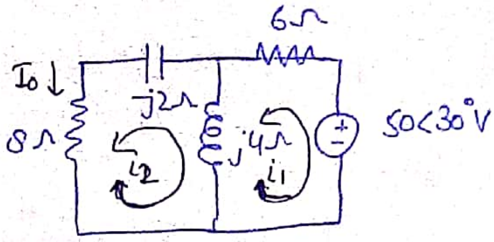
Find I_o using super-position Theorem.



let $\boxed{I_o = I_o' + I_o''}$

where I_o' is due to voltage source and I_o'' is due to current source.

Due to voltage source:- (I_o')



$\rightarrow 6i_1 + j4(i_1 - i_2) = 50\angle 30°$

$\boxed{(6+j4)i_1 - j4i_2 = 50\angle 30°}$ — (1)

$\rightarrow 8i_2 - j2i_2 + j4(i_2 - i_1) = 0$

$-j4i_1 + (8+j2)i_2 = 0.$

or ÷ by 2 $\boxed{-j2i_1 + (4+j)i_2 = 0}$ — (2)

By Crammer's rule:-

$$\begin{bmatrix} (6+j4) & -j4 \\ -j2 & (4+j) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50\angle 30° \\ 0 \end{bmatrix}$$

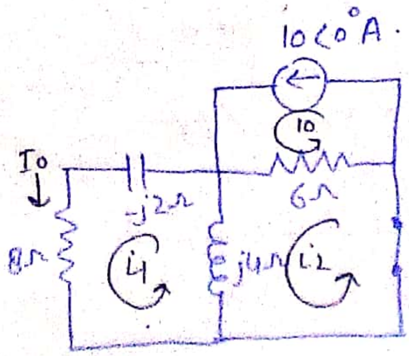
$$\Delta = \begin{vmatrix} 6+j4 & -j4 \\ -j2 & 4+j \end{vmatrix} = (6+j4)(4+j) - (-j2)(-j4)$$

$\boxed{\Delta = 28 + j22}$

$$\Delta_2 = \begin{vmatrix} 6+j4 & 43.3+j25 \\ -j2 & 0 \end{vmatrix} = -50 + j86.6.$$

$$I_o' = i_2 = \frac{\Delta_2}{\Delta} = \frac{-50+j86.6}{28+j22} = \boxed{2.8\angle 81° A.}$$

Due to current source:-



$\rightarrow -j2i_1 + 8i_1 + j4(i_1 - i_2) = 0.$

$\boxed{(8+j2)i_1 - j4i_2 = 0}$ — (1)

$$\rightarrow j4(i_2 - i_1) + j6(i_2 + 10) = 0.$$

$$\boxed{-j4i_1 + (6+j4)\,i_2 = 60} \quad \text{--- (2)}$$

Crammer's rule:-

$$\Delta = \begin{vmatrix} (8+j2) & -j4 \\ -j4 & (6+j4) \end{vmatrix} = 56 + j44$$

$$\Delta_1 = \begin{vmatrix} 0 & -j4 \\ 60 & (6+j4) \end{vmatrix} = 0 - (-j4)(60) = j240$$

$$i_1 = I_o'' = \frac{\Delta_1}{\Delta} = 2.08 + j2.64 = \boxed{3.36\angle 51.84^\circ A}$$

So,

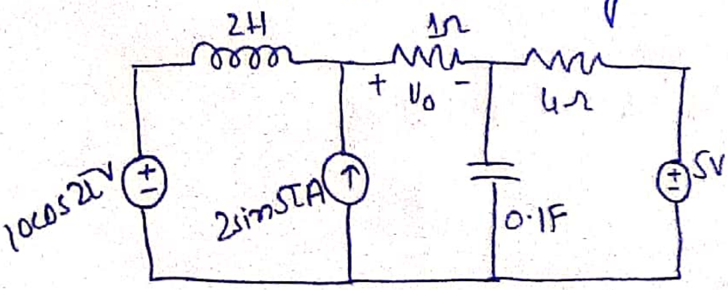
$$I_o = I_o' + I_o''$$

$$= (2.8\angle 81^\circ A) + (3.36\angle 51.84^\circ A)$$

$$\boxed{I_o = 5.97\angle 65^\circ A}$$

Example #10.6:-

Find V_o using S.P.T.



Here;

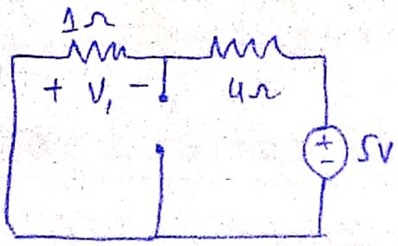
$$\boxed{V_o = V_1 + V_2 + V_3}$$

V_1 is due to 5V;

V_2 is due to 10cos2t V;

V_3 is due to 2sin5t A.

For V_1:-



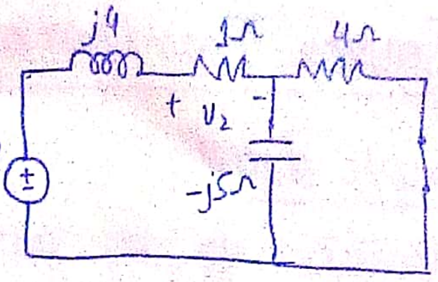
$$-V_1 = \frac{1}{1+4}(5V)$$

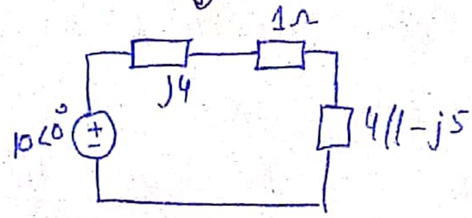
$$-V_1 = \cancel{5}/\cancel{5}$$

$$\boxed{V_1 = -1V}$$

For V_2:-

- $10\cos 2t V \Rightarrow 10\angle 0°;\ \omega = 2$
- $2H \Rightarrow j \times 2 \times 2 = 4j$
- $0.1F \Rightarrow \frac{1}{j \times 2 \times 0.1} = -j5\Omega$





$$Z = 4 \| -j5 = \frac{(4)(-j5)}{4+(-j5)}$$

$$Z = 2.439 - j1.95\ \Omega$$

V.D.R:-

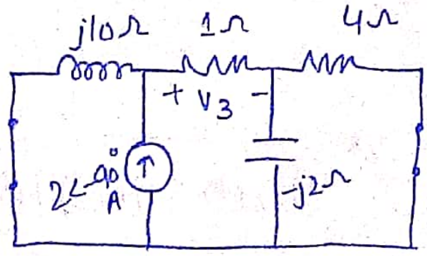
$$V_2 = \frac{1}{1+j4+Z}(10\angle 0°) = \frac{1}{1+j4+2.439-j1.95}(10\angle 0°)$$

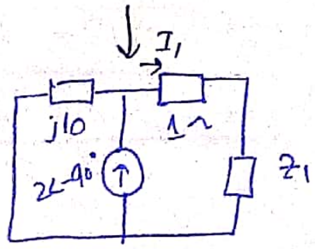
$$V_2 = 2.498\angle -30.79° V$$

or $\boxed{V_2 = 2.498\cos(2t - 30.79°)V}$

For V_3 :-

- $2\sin 5t V \Rightarrow 2\angle -90° V;\ \omega = 5$
- $2H \Rightarrow j \times 5 \times 2 = j10$
- $0.1F \Rightarrow \frac{1}{j \times 5 \times 0.1} = -j2\Omega$





$$Z_1 = (-j2) \| 4 = \frac{(-j2)(4)}{-j2+4} = 0.8 - j1.6$$

$$I_1 = \frac{j10}{j10+1+Z_1}(2\angle -90°)$$

So, $V_3 = I_1 R = \frac{j10}{(1+j10)+0.8-1.6j}(-j2) \times (1\Omega)$

$$V_3 = 0.48 - j2.27V = \boxed{2.328\angle -78° V} = 2.328\cos(5t - 78°)$$

or $\boxed{V_3 = 2.328\sin(5t + 12°)V}$

$V_0 = V_1 + V_2 + V_3$

$= -1 + 2.49\cos(2t - 30.79°) + 2.38\sin(5t + 12°)$ **Ans:**

Source Transformation:-

A source Transformation is The process of replacing a voltage source (V_s) with an impedence (Z) / Resistan(R) by a current source (I_s) in parallel with an impedence or vice versa.

$$V_s = Z_s I_s \implies I_s = \frac{V_s}{Z_s}$$

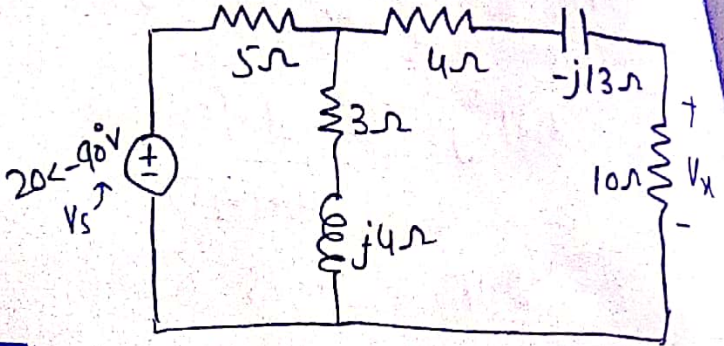
Steps:-

1) Replace all voltage sources which ~~current~~ is in series with That sourse to The current source That contain ||al impedence.
2) Add all impedences.
3) Solve cct.

→ Source Transformation :-

Example #10.7 :-

Calculate V_x by source transform.



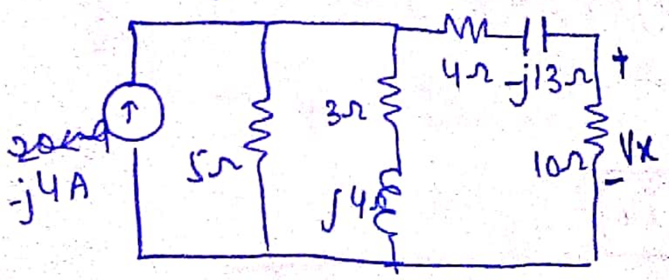
$V_s = 20\angle -90^\circ V$

$Z_s = 5\Omega$

So, $I_s = \dfrac{V_s}{Z_s} = \dfrac{20\angle -90^\circ}{5}$

$\boxed{I_s = 4\angle -90^\circ A = -j4A}$

↓



→

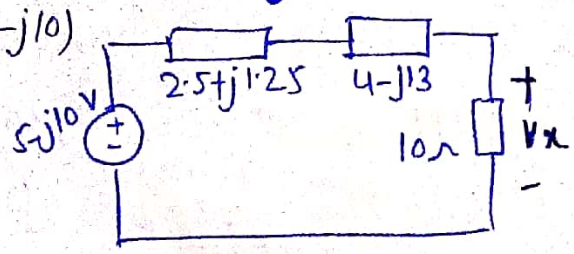
$Z_{eq} = \dfrac{(5)(3+j4)}{5+3+j4} = 2.5+j1.25$

$V_s = I_s Z_{eq}$

$V_s = (-j4)(2.5+j1.25)$

$\boxed{V_s = 5-j10V}$

↓



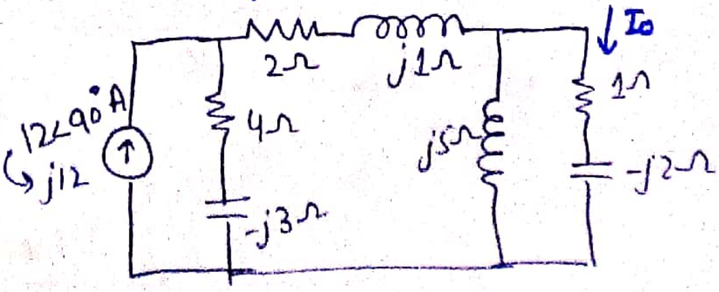
$V_x = \dfrac{10}{(10)+(2.5+j1.25)+(4-j13)} \times (5-j10)$

$V_x = 5.519\angle -27.9^\circ V.$

or $\boxed{V_x = 5.51\angle -28^\circ V}$ Ans:-

Practice Problem # 10.7 :-

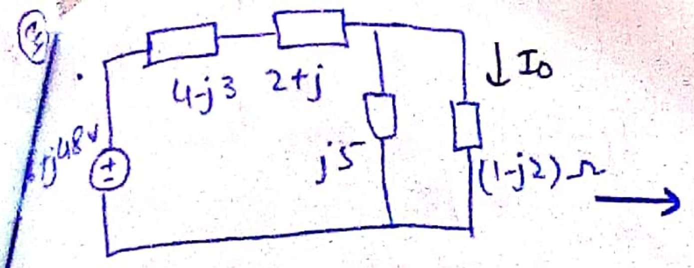
Find I_o using source Transformation.



$V_s = I_s Z_s$

$V_s = (j12)(4-j3)$

$\boxed{V_s = 36+j48V}$



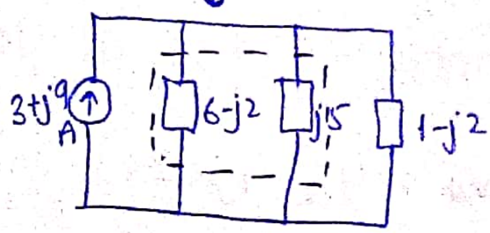
$$I_s = \frac{V_s}{Z_s}$$

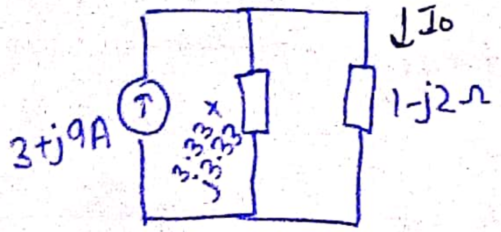
$\because Z_s = 4-j3+2+j$

$Z_s = 6-2j$

$$I_s = \frac{36+j48}{6-j2}$$

$$\boxed{I_s = 3+j9A}$$





C.D.R:-

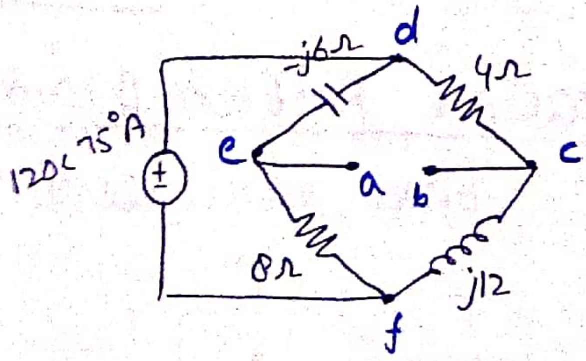
$$I_o = \frac{3.33+j3.33}{3.33+j3.33+(1-j2)} \times (3+j9)$$

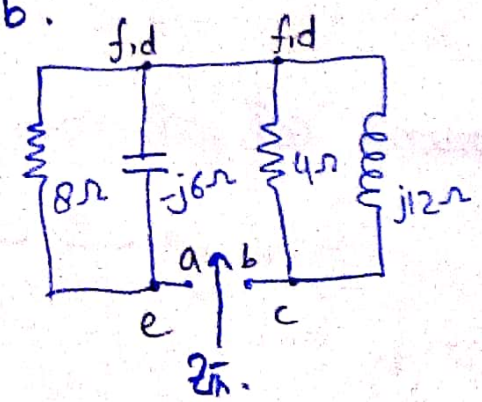
$$\boxed{I_o = 9.86 \angle 99.49^\circ A}$$ Ans:-

→ Thevenin's Theorem:-

Example # 10.8:-

Obtain Thevenin's equivalent cct. at terminal a-b.



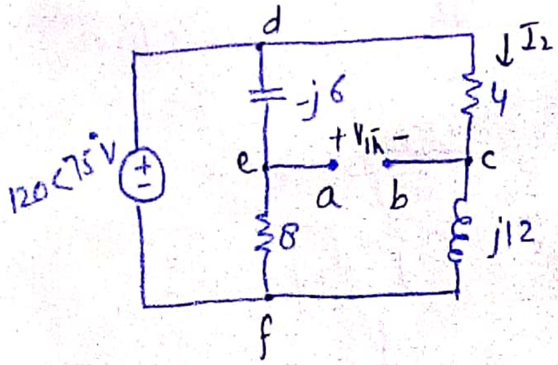


$Z_1 = (8) \| (-j6) = 2.88 - j3.84 \Omega$

$Z_2 = (4) \| (j12) = 3.6 + j1.2 \Omega$

$Z_{th} = Z_1 + Z_2 = 6.4 - j2.64 \Omega$.

if cct. is re-arranged:-



$$I_1 = \frac{120\angle 75^\circ}{8-j6}$$

$$I_2 = \frac{120\angle 75^\circ}{4+j12}$$

applying KVL at loop bcdeab:-

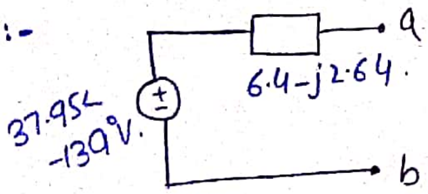
$$V_{th} = 4I_2 + j6I_1$$

$$= 4\left[\frac{120\angle 75^\circ}{4+j12}\right] + j6\left[\frac{120\angle 75^\circ}{8-j6}\right]$$

$$V_{th} = 37.94\angle 3.43^\circ + 72\angle -158^\circ$$

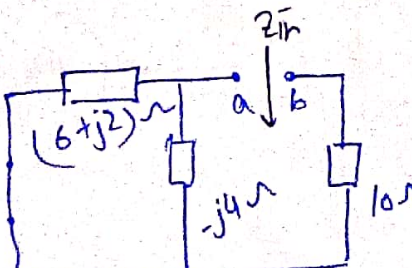
$$\boxed{V_{th} = 37.95\angle -139^\circ V}$$ Ans.

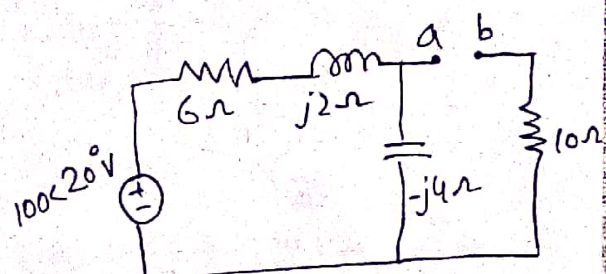
eq. cct:-



Practice Problem #10.8:-

Find equivalent Thevenin's cct at a-b.





$$Z_{12} = (6+j2) \parallel (-j4)$$

$$Z_{12} = 2.4 - j3.2\Omega.$$

So,

$$Z_{th} = (10) + (2.4 - j3.2\Omega)$$

$$\boxed{Z_{th} = 12.4 - j3.2\Omega.}$$

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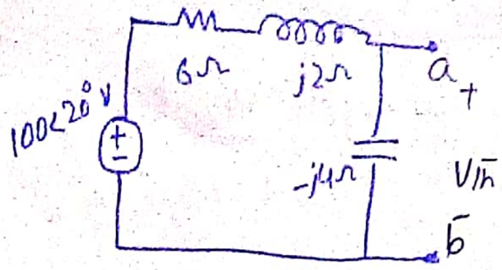
Vth:-

voltage across $-j4 = V_{th}$

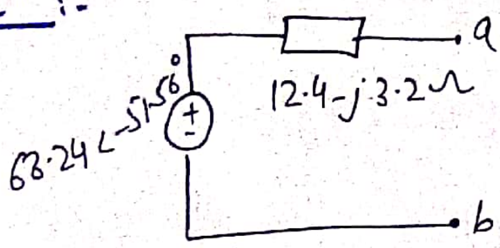
by VDR:-



$$V_{th} = \frac{(-j4)}{(-j4)+(6+j2)}\,(100\angle 20^\circ)$$

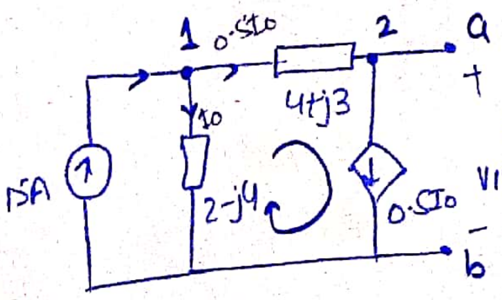
$$\boxed{V_{th} = 63.24\angle -51.56^\circ V}$$ Ans:-

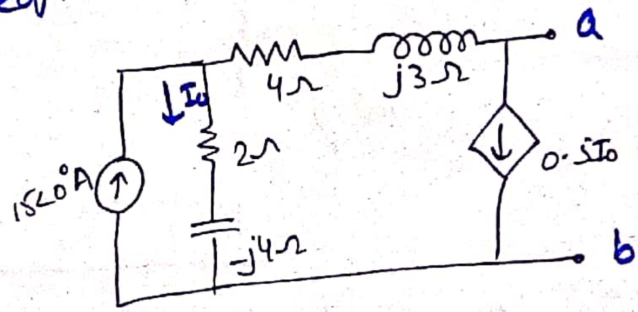
equv. cct.:-



Example #10.9:-

Find Thevenin's eqv. cct. at terminal a-b.





KCL at node-1:-

$$15 = I_o + 0.5I_o$$

$$\boxed{I_o = 10A}$$

KVL at loop:-

$$-I_o(2-j4) + 0.5I_o(4+j3) + V_{th} = 0.$$

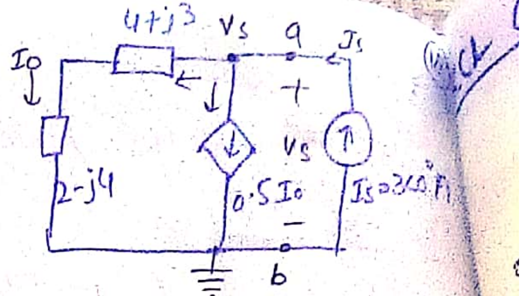
$$V_{th} = 10(2-j4) - 5(4+j3)$$

$$V_{th} = -j55V$$

or

$$\boxed{V_{th} = 55\angle -90^\circ V}$$

For Z_{th}:-



at node Vs; KCL gives:-

$\rightarrow \quad 3 = I_o + 0.5 I_o$

$$\boxed{I_o = 2A}$$

Applying KVL to outer loop:-

$$V_s = I_o(4+j3+2-j4)$$

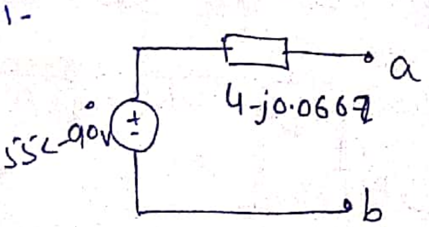
$$V_s = 2\cdot(6-j)$$

Thevenin's impedence (Z_{th}) is:-

$$Z_{th} = \frac{V_s}{I_s} = \frac{2(6-j)}{3}$$

$$\boxed{Z_{th} = 4-j0.0667\,\Omega}$$

eq. cct.:-

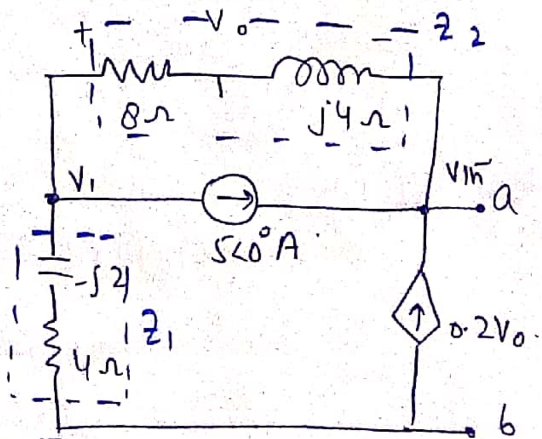


Practice Prob. # 10.9:-

Determine Thevenin's eq. cct. at a-b.

$Z_1 = (4-j2)\,\Omega$

$Z_2 = (8+j4)\,\Omega$



KCL at V_1:-

$$\frac{V_1}{4-j2} + \frac{V_1 - V_{th}}{8+j4} + 5\angle 0^\circ = 0$$

$$V_1\left(\frac{1}{4-j2}\right) + V_1\left(\frac{1}{8+j4}\right) - V_{th}\left(\frac{1}{8+j4}\right) = -5$$

$$\boxed{(0.3+j0.05)V_1 - (0.1-j0.05)V_{th} = -5} \quad \text{—} \; ①$$

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(b) KCL at node V_{th} :-

$$\frac{V_1 - V_{th}}{8+j4} + 0.2V_o + 5 = 0$$

or

$$\frac{+V_{th} - V_1}{8+j4} - 0.2V_o - 5 = 0. \qquad \because V_o = V_1 - V_{th}$$

$$V_{th}\left(\frac{1}{8+j4}\right) - V_1\left(\frac{1}{8+j4}\right) - 0.2(V_1 - V_{th}) - 5 = 0$$

$$(0.1 - 0.05j)V_{th} - (0.1 - j0.05)V_1 - 0.2V_1 + 0.2V_{th} = 5.$$

or

$$\boxed{(-0.3 + j0.05)V_1 + (0.3 - j0.05)V_{th} = 5} \text{ — (2)}$$

from eq (1):-

$$(0.3 + j0.05)V_1 = -5 + [(0.1) - j0.05]V_{th}$$

$$V_1 = \left[\frac{0.1 - j0.05}{0.3 + j0.05}\right]V_{th} - \frac{5}{0.3 + j0.05}$$

$$\boxed{V_1 = (0.297 - j0.216)V_{th} - (16.216 - j2.7)}$$

put in (2)

$$(2) \Rightarrow (-0.3 + j0.05)\left[(0.297 - j0.216)V_{th} - (16.216 - j2.7)\right] + (0.3 - j0.05)V_{th} = 5$$

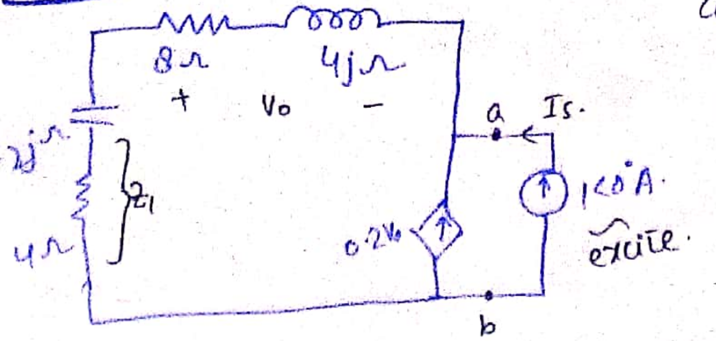
$$(0.2217 + j0.02965)V_{th} = 5 - 4.7298 + j1.62$$

$$V_{th} = \frac{5 - 4.7298 + j1.62}{0.2217 + j0.02965}$$

$$V_{th} = 2.156 + j7.01V$$

or $\boxed{V_{th} = 7.34\angle 72.92^\circ \text{ V}}$

for Z_{th}:-



as: $Z_1 = 4 - j2$

$Z_2 = 8 + j4.$

So,

$$V_o = -V_s\left(\frac{Z_2}{Z_1 + Z_2}\right)$$

KCL at Vs:-

$$\frac{V_s}{Z_1+Z_2} - 0.2\left[V_o\right] - 1 = 0.$$

put value of V_o:-

$$\frac{V_s}{Z_1+Z_2} - 0.2\left[-V_s\left(\frac{Z_2}{Z_1+Z_2}\right)\right] = 1$$

$$V_s\left[1+0.2\,Z_2\right] = Z_1+Z_2$$

$$V_s = \frac{12+j2}{2.6+j0.8}$$

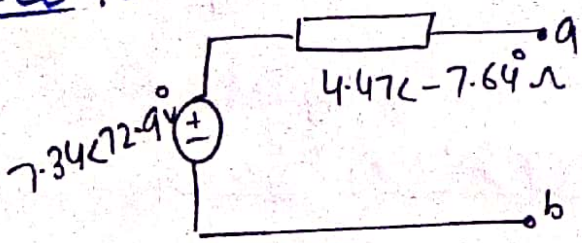
$$\boxed{V_s = 4.473\angle -7.64^\circ V}$$

so, $Z_{in} = \frac{V_s}{I_s}$

$$Z_{in} = \frac{4.473\angle -7.64^\circ V}{1\angle 0^\circ A}$$

$$\boxed{Z_{in} = 4.473\angle -7.64^\circ \Omega}$$

eq. cct. :-



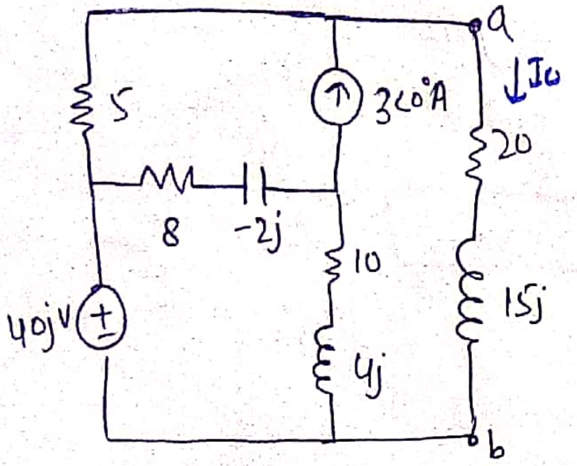
Thevenin's Theorem:-

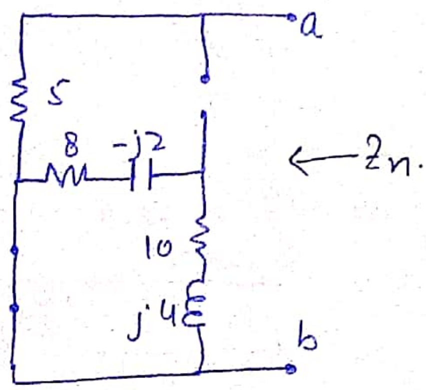
1) find R_{Th} / Z_{Th} by turn off all sources (V → short, I → open)
2) for V_{Th} ; find V_{Th} using any C.D.R, V.D.R etc.
3) Remove Load / R_L for V_{Th}.
4) Excite any of arbitrary source if there is any independent source.

Norton's Theorem:-

Example # 10.10:-

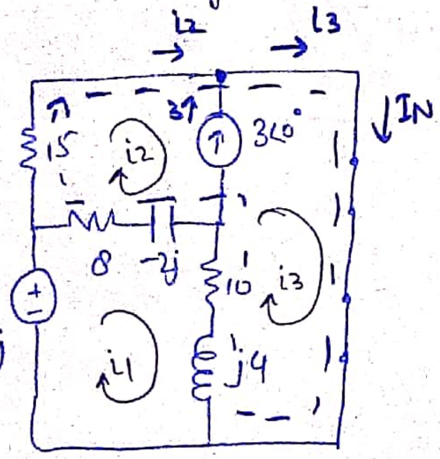
Find I_o using Norton's Theorem.





current pass through short-wired / easiest path through 5Ω so; $\boxed{Z_N = 5\Omega}$

for V_N:-



KCL at node:-

$$\boxed{i_3 = i_2 + 3}$$

Mesh # 1:-

$$-40j + (8-j2)(i_1 - i_2) + (10+j4)(i_1 - i_3) = 0$$

$$(8-j2)i_1 + (10+j4)i_1 + (8-j2)(-i_2) + (10+j4)(i_2+3) = 40j$$

$$\boxed{(18+j2)i_1 + (-18-j2)i_2 = 30+j52} \text{ — (1)}$$

Super-mesh:-

$$(10+4j)(\underbrace{i_3}_{i_2+3} - i_1) + (8-j2)(i_2 - i_1) + 5i_2 = 0$$

$$10i_2 + 30 - 10i_1 + 4j i_2 + 12j - 4j i_1 + 8i_2 - 8i_1 - j2i_2 + j2i_1 + 5i_2 = 0$$

$$\boxed{(-18-j2)i_1 + (23+j2)i_2 = -30-j12} \text{ — (2)}$$

adding ① & ② :-

$$\cancel{(18+j2)}\,i_1 + \cancel{(-18-j2)}\,i_2 = \cancel{30} + j52$$

$$+\quad \cancel{(-18-j2)}\,i_1 + (23-\cancel{j2})\,i_2 = \cancel{-30} - j12$$

$$5 i_2 = j40$$

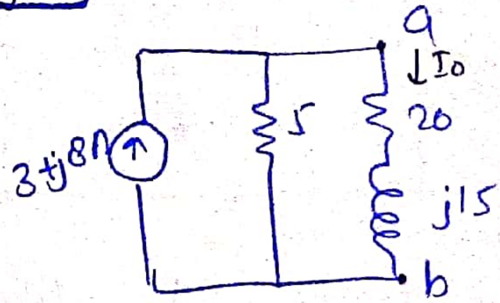
$$\boxed{i_2 = j8}$$

so,

$$i_3 = i_2 + 3 \Rightarrow \boxed{i_3 = 3 + j8}$$

&

$$\boxed{I_N = I_3 = (3+j8)\,A}$$



C.D.R :-

$$I_o = \frac{5}{5+(20+j15)}\,(3+j8)$$

$$\boxed{I_o = 1.465\angle 38.48^\circ A}$$ Ans:-