Chapter # 09:-

Sinusoids and Phasors

{Examples and Practice Problems}

Example # 9.1:-

find amplitude, phase, period & frequency of sinusoid: $v(t) = 12\cos(50t + 10^\circ)$

$$v(t) = 12\cos(50t + 10^\circ) = V_m \angle \theta$$

so; V_m = amplitude = 12. $\because \omega = 50$

θ = phase = 10°

T = period = $\frac{2\pi}{\omega}$

$$T = \frac{2 \times 3.14}{50} = 0.1256\,s$$

$$f = \frac{1}{T} = \frac{1}{0.1256} = 7.958\,Hz$$

Practice Problem 9.1:-

find amplitude, phase, angular frequency, period and frequency of: $v(t) = 30\sin(4\pi t - 75^\circ)$

$$v(t) = 30\sin(4\pi t - 75^\circ)$$

$V_m = 30$

$\theta = -75^\circ$

$\omega = 4\pi = 12.56$ rad/s

$$T = \frac{2 \times 3.14}{12.56} = 0.5\,s$$

$$f = \frac{1}{T} = \frac{1}{0.5} = 2\,Hz$$

Example #9.2:-

Calculate phase angle between V_1 & V_2. State which sinusoid is leading.

$$V_1 = -10\cos(\omega t + 50^\circ)$$
$$V_2 = 12\sin(\omega t - 10^\circ)$$

$$V_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50 \pm 180^\circ)$$

So, $V_1 = 10\cos(\omega t + 230^\circ)$ or $V_1 = 10\cos(\omega t - 130^\circ)$ — ①

$$V_2 = 12\cos(\omega t - 10^\circ - 90^\circ)$$
$$V_2 = 12\cos(\omega t - 100^\circ) \text{ — ②}$$

from ① & ②; we can see phase difference is 30°. We can write V_2 as:

$V_2 = 12\cos(\omega t - 130^\circ + 30^\circ)$ or $V_2 = 12\cos(\omega t + 260^\circ)$ — ③

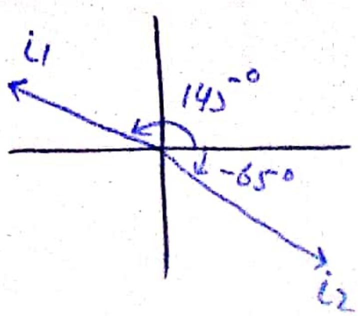
comparing ① & ③; we can say that V_2 is leading V_1, by 30°.

Practice problem #9.2:-

Find phase angle between $i_1 = -4\sin(377t + 55^\circ)$ and $i_2 = 5\cos(377t - 65^\circ)$. Which leads or lags?

$$i_1 = -4\sin(377t + 55^\circ) = 4\cos(377t + 55 + 90^\circ)$$
$$i_1 = 4\cos(377t + 145^\circ) \text{ — ①}$$
$$i_2 = 5\cos(377t - 65^\circ) \text{ — ②}$$

comparing ① & ②; we can say i_1 leads i_2.



Scanned with CamScanner

Example #9.3:-

Evaluate these complex numbers.

i) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

$= \sqrt{(40\angle 50^\circ + 20\angle -30^\circ)}$

$40\angle 50^\circ = 40[\cos 50^\circ + j\sin 50^\circ] = 25.71 + j30.64$

$20\angle -30^\circ = 20[\cos(-30^\circ) + j\sin(-30^\circ)] = 17.32 - j10.$

Then adding:-

$(25.71 + j30.64) + (17.32 - 10j) = 43.02 + 20.64j$

$r = \sqrt{(43.02)^2 + (20.64)^2}$; $\theta = \tan^{-1}\left(\frac{20.64}{43.02}\right)$

$r = \sqrt{2276.72}$; $\theta = \tan^{-1}(0.479)$

$\boxed{r = 47.71}$; $\boxed{\theta = 25.63^\circ}$

$\rightarrow \quad = (47.71\angle 25.63^\circ)^{1/2}$

$= \sqrt{47.71}\angle 25.63/2$

$= \boxed{6.91\angle 12.81^\circ}$ Ans.

ii) $\dfrac{(10\angle -30) + (3 - j4)}{(2 + j4)(3 - j5)^*}$

$= \dfrac{(8.66 - j5) + (3 - j4)}{(2 + j4)(3 + j5)}$

$= \dfrac{11.66 - j9}{-14 + j22} = \dfrac{14.73\angle -37.66^\circ}{26.08\angle 122.47^\circ}$

$= \boxed{0.565\angle -160.13^\circ}$ Ans:

Practice Problem # 9.3:-

Evaluate following complex numbers.

a) $[(5+j2)(-1+j4) - 5\angle 60^\circ]^*$

$= [(5+j2)(-1+j4) - (2.5+j4.3)]^*$

$= [(-13+18j) - (2.5+j4.3)]^*$

$= [-15.5+j13.7]^*$

$= \boxed{-15.5 - j13.7}$

b) $\dfrac{10+j5+3\angle 40^\circ}{-3+j4} + 10\angle 30^\circ + j5$

$= \dfrac{(10+j5)+(2.29+1.9j)}{-3+j4} + (8.6+5j)+j5$

$= (-0.37-2.79j) + (8.66+j10)$

$= \boxed{0.28+7.2j}$

Example # 9.4:- Transform to phasors.

a) $i = 6\cos(50t - 40^\circ)\,A$

$i = 6\cos(50t - 40^\circ) = I_m \angle \theta$

$\boxed{\mathbf{I} = 6\angle -40^\circ A}$

b) $v = -4\sin(30t+50^\circ)\,V$

$v = -4\sin(30t+50^\circ) = 4\cos(30t+50^\circ+90^\circ)$

$v = 4\cos(30t+140^\circ) = \boxed{4\angle 140^\circ\ V}$

Practice Problem #9.4:-

Express sinusoids as phasors.

a) $v = 7\cos(2t + 40)$ V

$V = 7\cos(2t + 40^\circ) = V_m \angle \theta$

$\boxed{V = 7\angle 40^\circ V}$

b) $i = -4\sin(10t + 10^\circ)$ A

$i = 4\cos(10t + 90^\circ + 10^\circ)$

$i = 4\cos(10t + 100^\circ)$

$\boxed{I = 4\angle 100^\circ}$

Example #9.5:-

Find sinusoides from phasors.

a) $I = (-3 + j4)$ A

$r = \sqrt{(-3)^2 + (4)^2}$; $\theta = \tan^{-1}(4/-3)$

$r = \sqrt{25} = 5$; $\theta = -53.13^\circ$

So, $\boxed{i(t) = 5\cos(\omega t - 53.13^\circ)}$

b) $V = j8e^{-j20^\circ}$ V

since: $j = 1\angle 90^\circ$

$V = j8\angle -20^\circ$

$= (1\angle 90^\circ)(8\angle -20^\circ)$

$= 8\angle 90^\circ - 20^\circ$

$V(t) = 8\angle 70^\circ$

So, $\boxed{v(t) = 8\cos(\omega t + 70^\circ) V}$

Practice Problem #9.5:-

find sinusoids from phasors.

a) $V = -10\angle 30^\circ\ V$

$v(t) = -10\cos(\omega t + 30^\circ) = 10\cos(\omega t + 30^\circ + 180^\circ)$

$\boxed{v(t) = 10\cos(\omega t + 210^\circ)\ V}$

b) $I = j(5-12j)\ A$

$I = 5j - 12j^2$

$I = 5j - 12(-1)$

$I = 12 + 5j$

or: $r = \sqrt{(12)^2 + (5)^2}$; $\theta = \tan^{-1}(5/12)$

$r = 13$ $\qquad$ $\theta = 67.3^\circ$

$\boxed{i(t) = 13\cos(\omega t + 67.3^\circ)\ A}$ Ans:

Example #9.6:-

given: $i_1 = 4\cos(\omega t + 30^\circ)\ A$

$i_2 = 5\sin(\omega t - 20^\circ)\ A$; find sum.

$i_1 = 4\cos(\omega t + 30^\circ) = 4\angle 30^\circ$

$i_2 = 5\sin(\omega t - 20^\circ)\ \cancel{= 5\sin}$

$= 5\cos(\omega t - 110^\circ) = 5\angle -110^\circ$

$i_1 = 4[\cos 30^\circ + j\sin 30^\circ] = 3.46 + 2j$

$i_2 = 5[\cos(-110^\circ) + j\sin(-110^\circ)] = -1.71 - 4.698j$

$i = i_1 + i_2 = (3.46 + 2j) + (-1.71 - 4.698j)$

$= 1.75 - 2.698j$

or $\boxed{i = 3.218\angle -57.03^\circ\ A}$ Ans:

Practice problem # 9.6:-

If: $V_2 = +20\cos(\omega t + 45^\circ)$
$V_1 = -10\sin(\omega t - 30^\circ)$; find sum.

$V_1 = -10\sin(\omega t - 30^\circ) = 10\cos(\omega t + 60^\circ)$

$V_1 = 10\angle 60^\circ$

$V_2 = 20\cos(\omega t + 45^\circ) = 20\angle 45^\circ$

$V = V_1 + V_2 = (10\angle 60^\circ) + (20\angle 45^\circ)$

$= 10[\cos 60^\circ + j\sin 60^\circ] + 20[\cos 45^\circ + j\sin 45^\circ]$

$= (5 + j8.66) + (14.14 + j14.14)$

$V = 19.14 + j22.8$

or $V = 29.77\angle 49.98^\circ$

$\boxed{v(t) = 29.77\cos(\omega t + 49.98^\circ)\,V}$ Ans:

Example # 9.7:-

Using phasor approach; find $i(t)$:

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$$

Here; $\omega = 2$

Transforming in phasors:-

$$4I + 8\cdot\frac{I}{j\omega} - 3Ij\omega = 50\angle 75^\circ$$

$$4I + \cancel{8}^{4}\cdot\frac{I}{j(\cancel{2})} - 3Ij\times 2 = 50\angle 75^\circ$$

$$4I + \frac{4I}{j} - 6jI = 50\angle 75^\circ$$

$$4I + j\frac{4I}{j^2} - 6jI = 50\angle 75^\circ$$

$$4I - j4I - 6jI = 50\angle 75^\circ$$

$$4I - 10jI = 50\angle 75^\circ.$$

$$I = \frac{50\angle 75^\circ}{(4-j10)}$$

$$I = \frac{50\angle 75^\circ}{10.77\angle -68.2^\circ}$$

$$I = 4.6.4\ \angle 143.2^\circ\ A$$

or $\boxed{i(t) = 4.64\cos(2t + 143.2^\circ)\ A}$ Ans:

Practice Problem #9.7:-

find $v(t) = ?$

$$2\frac{dv}{dt} + 5v + 10\int v\,dt = 50\cos(5t - 30^\circ)$$

$$2 \cdot j\omega V + 5V + 10 \cdot \frac{V}{j\omega} = 50\angle -30^\circ$$

$\because \omega = 5$

$$2j(5)V + 5V + 10\frac{V}{j\times 5} = 50\angle -30^\circ$$

$$10jV + 5V + \frac{\cancel{10}^{2}V}{8j} = 50\angle -30^\circ$$

$$8jV + 5V = 50\angle -30^\circ$$

$$V = \frac{50\angle -30^\circ}{5 + j8}$$

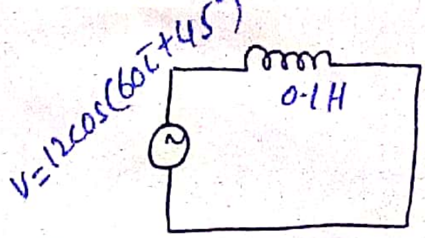
$$V = \frac{50\angle -30^\circ}{9.43\angle 57.9^\circ}$$

$$V = 5.302\angle -87.9^\circ$$

or $\boxed{v(t) = 5.302\cos(5t - 87.9^\circ)\ V}$ Ans:

Example # 9.8:-

Voltage $v = 12\cos(60t + 45°)$ is applied to 0.1H inductor. Find steady state current through inductor.



$$V = L\frac{di}{dt} \Rightarrow V_m = Lj\omega I$$

$$I = \frac{V}{j\omega L}$$

$$V = 12\cos(60t + 45°) = 12\angle 45°$$

$$I = \frac{12\angle 45°}{(1\angle 90°)(60)(0.1)}$$ $\because \omega = 60.$ $\because j = 1\angle 90°.$

$$I = \frac{12\angle 45°}{1\angle 90° \times 0.6}$$

$$I = 2\angle -45° A$$

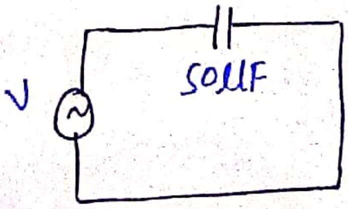
or $\boxed{i(t) = 2\cos(60t - 45°)A}$ Ans:

Practice Problem # 9.8:-

If voltage $v = 10\cos(100t + 30°)$ is applied to $50\mu F$ capacitor. Calculate $i(t)$.

$$V = 10\cos(100t + 30°) = 10\angle 30°.$$

$$\omega = 100 \; ; \quad C = 50 \times 10^{-6} F.$$



$$I = Cj\omega V$$

$$I = (50\times 10^{-6})(1\angle 90°)(100)(10\angle 30°)$$

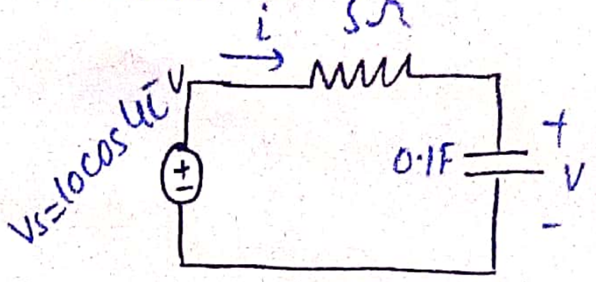
$$I = (50\times 10^{-6})(100)(10\angle 120°)$$

$$I = 50\angle 120° mA.$$

or $\boxed{i(t) = 50\cos(100t + 120°) mA.}$ Ans:

Example # 9.9:-

find $v(t)$ and $i(t)$ in circuit.



$V_s = 10\cos 4t = 10\angle 0°\ V$

impedence is:

$$Z = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4\times 0.1}$$

$$\boxed{Z = 5 - j2.5\ \Omega}$$

So, current;

$$I = \frac{V_s}{Z} = \frac{10\angle 0°}{(5-j2.5)} = \frac{10(5+j2.5)}{5^2+2.5^2}$$

$$I = 1.6 + j0.8$$

or $\boxed{I = 1.789\angle 26.57°\ A}$

Voltage across capacitor is:-

$$V = I Z_c = \frac{I}{j\omega C}$$

$$V = \frac{1.789\angle 26.57°}{j4\times 0.1} \qquad \because j = 1\angle 90°$$

$$V = \frac{1.789\angle 26.57°}{0.4\angle 90°}$$

$$\boxed{V = 4.47\angle -63.43°\ V}$$

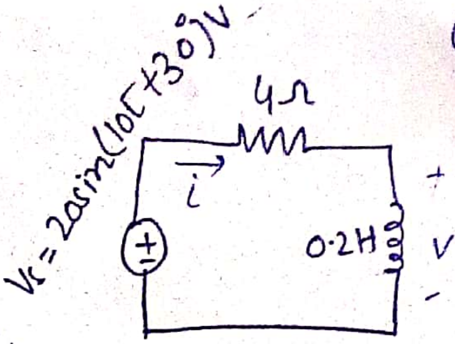
or

$i(t) = 1.789\cos(4t + 26.57°)A$

$v(t) = 4.47\cos(4t - 63.43°)V$

ractice Problem #9.9:-

Determine v(t) and i(t);



$V_s = 20\sin(10t+30^\circ)V$

$V_s = 20\cos(10t-60^\circ)V = 20\angle-60^\circ$

i) $I = \frac{V_s}{Z} = ?$

$Z = R + j\omega L = 4 + j(10)(0.2)$

$Z = 4 + 2j$

So, $I = \frac{20\angle-60^\circ}{4+j2} = \frac{20\angle-60^\circ}{4.47\angle26.56^\circ}$

$I = 4.47\angle-86.56^\circ A$

$\boxed{i(t) = 4.47\cos(10t - 86.56^\circ)A}$

ii) $V_L = I Z_L = I \times j\omega L$ $\quad \because j = 1\angle90^\circ$

$V_L = (4.47\angle-86.56^\circ)\, j(10)(0.2)$

$V_L = (4.47\angle-86.56^\circ)(1\angle90^\circ)(2)$

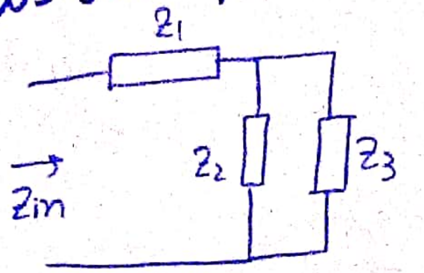
$V_L = 8.94\angle3.43^\circ$

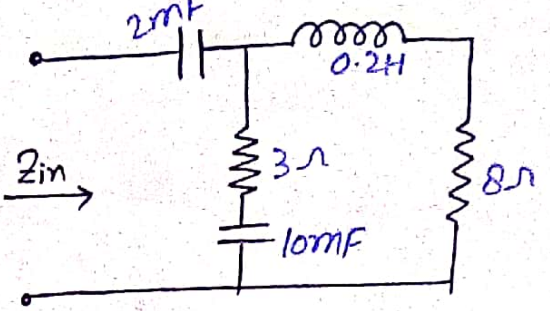
or $V_L = 8.94\cos(10t+3.43^\circ)$

or $\boxed{V_L = 8.94\sin(10t+93.43^\circ)V}$

Example #9.10:-

Find input impedence of circuit; while $\omega = 50\,rad/s$.





$$Z_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\,\Omega$$

$$Z_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\,\Omega$$

$$Z_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\,\Omega$$

The input impedence is:-

$$Z_{in} = Z_1 + Z_2 \parallel Z_3$$

$$= (-j10) + \frac{(3-j2)(8+j10)}{(3-j2)+(8+j10)}$$

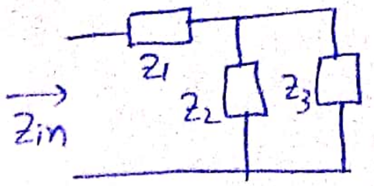
$$Z_{in} = -j10 + 3.22 - j1.07\,\Omega$$

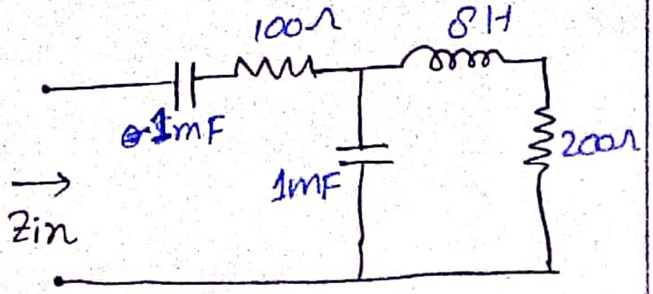
So,

$$\boxed{Z_{in} = 3.22 - j11.07\,\Omega}$$ Ans:

Practice Problem #9.10:-

$Z_{in} = ?$; $\omega = 10\,rad/s$.





$$Z_{in} = Z_1 + Z_2 \parallel Z_3$$

$$Z_1 = R + \frac{1}{j\omega C} = 100 + \frac{1}{j(10)(1 \times 10^{-3})}$$

$$Z_1 = 100 - j100\,\Omega$$

$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j \times 10 \times 1 \times 10^{-3}} = -j100\,\Omega$$

$$Z_3 = 200 + j(10)(8) = 200 + j80\,\Omega$$

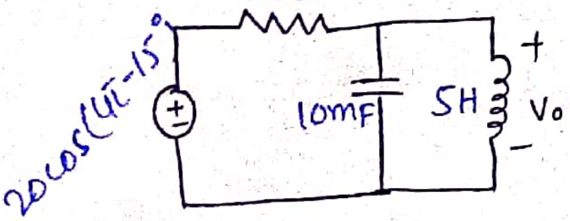
So,

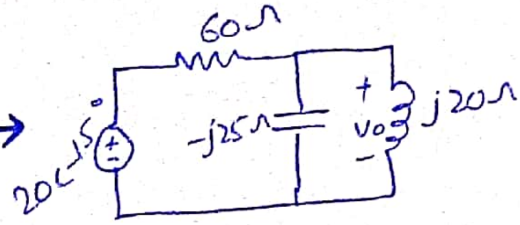
$$Z_{in} = (100 - j100) + \frac{(-j100)(200+j80)}{(-j100)+(200+j80)}$$

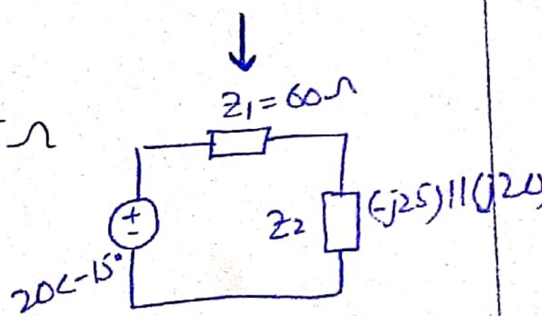
$$\boxed{Z_{in} = 149.5 - j195\,\Omega}$$ **Ans:**

Example # 9.11:-

Determine $v_o(t)$ in circuit.







$$V_s = 20\cos(4t - 15^\circ) = 20\angle -15^\circ$$

$$10mF \Rightarrow \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\,\Omega$$

$$5H \Rightarrow j\omega L = j4 \times 5 = j20\,\Omega$$

Hence;

$$Z_1 = 60\,\Omega$$

$$Z_2 = -j25 \,||\, j20 = \frac{-j25 \times j20}{-j25 + j20} = +j100\,\Omega$$

So,

$$V_o = \frac{Z_2}{Z_1 + Z_2} V_s = \frac{j100}{60 + j100} \times (20\angle -15^\circ)$$

$$V_o = (0.8575\angle 30.96^\circ)(20\angle -15^\circ)$$

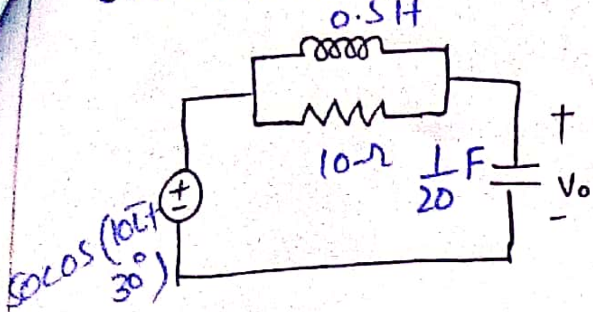
$$V_o = 17.15\angle 15.96^\circ\ V$$

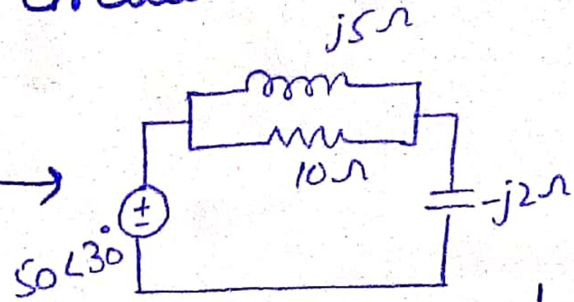
or converting to time domain:-

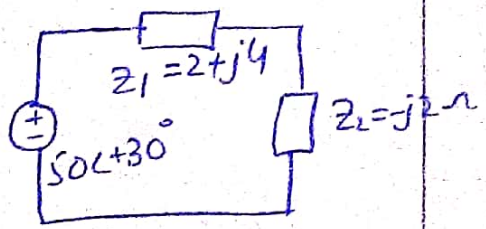
$$\boxed{v_o(t) = 17.15\cos(4t + 15.96^\circ)\,V}$$ **Ans:**

practice Problem # 9.11:-

calculate V_o in circuit:







$$V_s = 5\cos(10t + 30°) = 5\angle 30°$$

$$0.5H \Rightarrow j\omega L = j10(0.5) = j5\,\Omega$$

$$\frac{1}{20}F \Rightarrow \frac{1}{j\omega C} = \frac{1}{j10 \times \frac{1}{20}} = -j2\,\Omega$$

$$Z_1 = j5 \,||\, 10 = \frac{(j5)(10)}{j5 + 10} = 2 + j4$$

So,

$$V_{Z_2} = \frac{Z_2}{Z_1 + Z_2}(V_s)$$

$$Z_1 = 2 + j4 \;;\; Z_2 = -j2$$

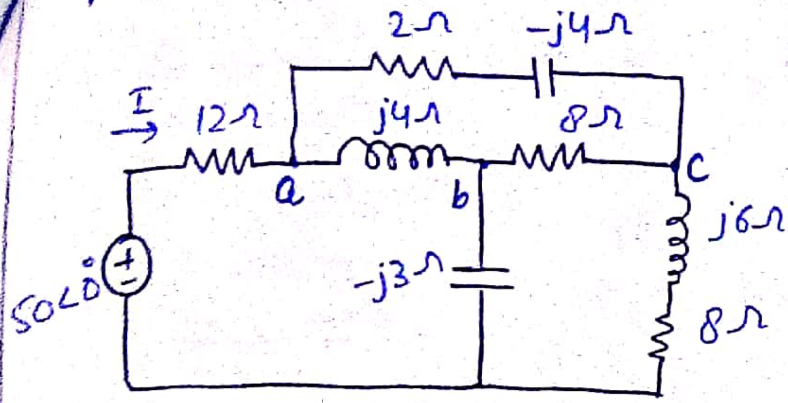
$$V_{Z_2} = \frac{-j2}{(2+j4)+(-j2)}(5\angle 30°)$$

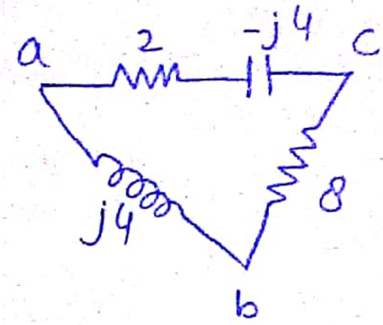
$$V_{Z_2} = 35.36\angle -105°$$

or $\boxed{V_o(t) = 35.36\cos(10t - 105°)\,V}$ Ans:

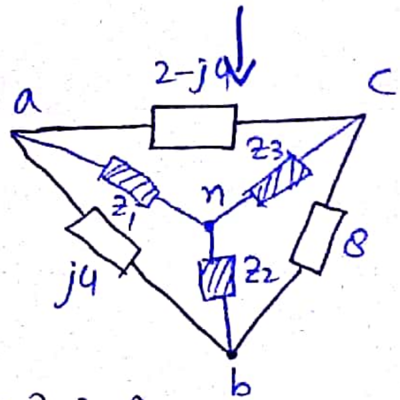
Example # 9.12 :-

Find current I in circuit using Δ-Y.





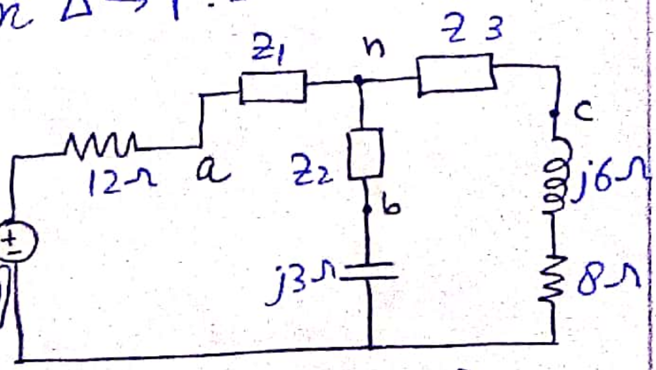
$$Z_{an} = Z_1 = \frac{(j4)(2-j4)}{(j4)+(2-j4)+(8)} = 1.6 + j0.8\,\Omega$$



$$Z_{bn} = Z_2 = \frac{(j4)(8)}{(j4)+(2-j4)+(8)} = j3.2\,\Omega$$

$$Z_{cn} = Z_3 = \frac{(2-j4)(8)}{(2-j4)+(8)+(j4)} = 1.6 - j3.2\,\Omega$$

Now, transforming from $\Delta \to Y$:-



$$Z_{eq} = 12 + Z_1 + (Z_2 \parallel Z_3)$$

$$Z_{eq} = 12 + Z_{an} + [(Z_{bn} - j3) \parallel (Z_{cn} + j6 + 8)]$$

$$Z_{eq} = 12 + (1.6 + j0.8) + [(j3.2 - j3) \parallel (1.6 - j3.2 + j6 + 8)]$$

$$Z_{eq} = (13.6 + j0.8) + \left[\frac{(j0.2)(9.6 + j2.8)}{(j0.2) + (9.6 + j2.8)}\right]$$

$$Z_{eq} = (13.6 + j0.8) + (3.79 + 0.19j) = 13.6 + 0.99j$$

$$\boxed{Z_{eq} = 13.64 \angle 4.2^\circ\,\Omega}$$

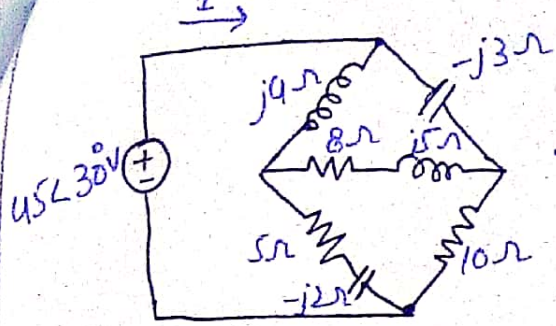
So,

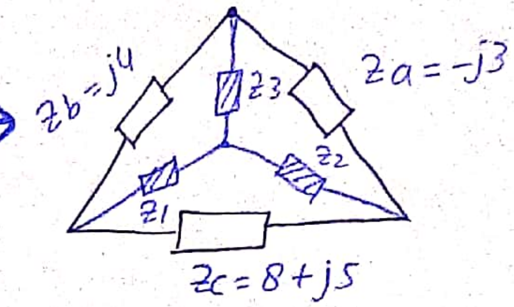
$$I = \frac{V}{Z_{eq}} = \frac{50\angle 0^\circ}{13.64\angle 4.2^\circ}$$

$$\boxed{I = 3.66\angle -4.2^\circ\ A}$$ Ans:

Practice Problem #9.12:-

Find I in circuit:



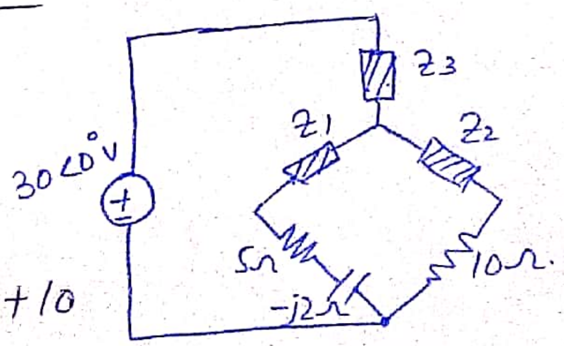


$$Z_1 = \frac{Z_c Z_b}{Z_a + Z_b + Z_c} = \frac{(8+j5)(j4)}{(j4)+(8+j5)+(-j3)} = 0.32 + j3.76\,\Omega$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} = \frac{(-j3)(8+j5)}{(-j3)+(8+j5)+(j4)} = 0.24 - 2.8j\,\Omega$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} = \frac{(-j3)(j4)}{(-j3)+(j4)+(8+j5)} = 0.96 - 0.72j\,\Omega.$$

Then circuit will be:-



$x = Z_1 + 5\Omega + (-j2)$

$x = (0.32 + 3.76j) + 5 + (-j2)$

$\boxed{x = 5.32 + 1.76j}$

$y = Z_2 + 10 = (-0.24 - 2.82j) + 10$

$\boxed{y = 9.76 - j2.82}$

$\boxed{z = Z_3 = 0.96 - j0.72}$

$Z_{eq} = x \parallel y \parallel z$

$$= \frac{(5.32 + 1.76j)(9.76 - 2.82j)(0.96 - 0.72j)}{(5.32 + j1.76) + (9.76 - j2.82) + (0.96 - j0.72)}$$

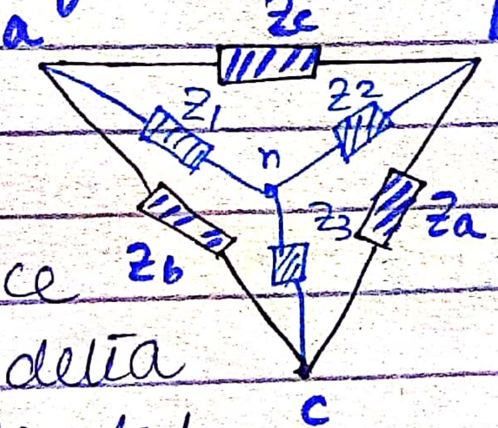
$\boxed{Z_{eq} = 4.24\angle -31^\circ\,\Omega}$

So, $I = \frac{V}{Z_{eq}} = \frac{30\angle 0^\circ}{4.24\angle -31^\circ}$

$\boxed{I = 7.07\angle 31^\circ\,A}$ Ans:

Δ-Y & Y-Δ Transformation:-

• Δ→Y :-



"The impedence in Y-network is the product of impedence of two adjacent delta branches and divided by sum of all / three delta branches"

• $Z_1 = \dfrac{Z_b Z_c}{Z_a + Z_b + Z_c}$

• $Z_2 = \dfrac{Z_a Z_c}{Z_a + Z_b + Z_c}$

• $Z_3 = \dfrac{Z_a Z_b}{Z_a + Z_b + Z_c}$

when $Z_a = Z_b = Z_c = Z$:-

$$Z_1 = Z_Y = \frac{Z^{\cancel{2}}}{3\cancel{Z}} = \frac{Z}{3} = \frac{1}{3} Z_\Delta \rightarrow \boxed{1:3}$$

• $Y \longrightarrow \Delta$:-

"Each impedence in delta(Δ) network is sum of all possible products of Y-impedences taken two at a time and divided by opposite impedences":-

$$\bullet Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$\bullet Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$\bullet Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

when $Z_1 = Z_2 = Z_3 = Z$:-

$$Z_a = \frac{Z^2 + Z^2 + Z^2}{Z} = \frac{3Z^{\cancel{2}}}{\cancel{Z}} = 3Z \rightarrow \boxed{3:1}$$