Lecture 2:- Group of Tranformation:

We now introduce the group of transformation to transform the sys. of partial differential eq. into ODE.

$$\left.\begin{aligned} \psi &= \sqrt{\nu x U_\infty}\, f(\eta) \\ \eta &= y\sqrt{\frac{U_\infty}{\nu x}} \end{aligned}\right\} \quad (4)$$

where $f(\eta)$ denotes the dimensionless stream function and $\eta$ is similarity variable. The velocity components

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

$$u = \frac{\partial}{\partial y} \sqrt{\nu x U_\infty}\, f(\eta)$$

$$= \sqrt{\nu x U_\infty}\, \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial y}$$

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$\frac{1}{2}(\nu x U_\infty)^{1/2} \cdot \frac{1}{2} x^{-\frac{1}{2}}$ $\qquad$ $\frac{\partial}{\partial x}\left(y\sqrt{\frac{U_\infty}{\nu x}}\right)$ $\qquad$ $y\sqrt{\frac{U_\infty}{\nu}} \cdot \frac{-1}{2} x^{-3/2}$



$$\frac{\partial \eta}{\partial y} = \sqrt{\frac{U_\infty}{\nu x}}$$

$$u = \sqrt{\nu x U_\infty}\, f' \cdot \sqrt{\frac{U_\infty}{\nu x}}$$

$$u = U_\infty f' \quad -(5)$$

$$v = -\frac{\partial}{\partial x}\left(\sqrt{\nu x U_\infty}\, f(\eta)\right)$$

$$= -\left[(\nu x U_\infty)^{1/2}\frac{\partial f}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} + \frac{\partial}{\partial x}(\nu x U_\infty)^{1/2} f\right]$$

$$= -\left[(\nu x U_\infty)^{1/2} f'\left(-\frac{1}{2} y\sqrt{\frac{U_\infty}{\nu x}} \cdot \frac{1}{x}\right) + \frac{1}{2}(\nu x U_\infty)^{1/2} \cdot x^{-1}\right]$$

$$= -\left((\nu x U_\infty)^{1/2} f'\left(-\frac{\eta}{2x}\right) + f \cdot \frac{1(\nu U_\infty)^{1/2}}{2x^{1/2}}\right)$$

$$= -\left((\nu x U_\infty)^{1/2} f'\left(-\frac{\eta}{2x}\right) + f\,\frac{(\nu x U_\infty)^{1/2}}{2x}\right)$$

$$= -(\nu x U_\infty)^{1/2}\left[\frac{-\eta f'}{2x} + \frac{f}{2x}\right]$$

$$v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left(\eta f' - f\right) \quad -(6)$$

$$u = U_\infty f'$$

$$\frac{\partial u}{\partial y} = U_\infty\left(\frac{\partial f'}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}\right)$$

$$= U_\infty f'' \cdot \sqrt{\frac{U_\infty}{\nu x}}$$

$$\frac{\partial u}{\partial y} = \frac{U_\infty^{3/2}}{\sqrt{\nu x}} f'' \quad -(7)$$

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$\frac{3}{2}+\frac{1}{2}$

(6)

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^{3/2}}{\sqrt{\nu x}} \cdot \frac{\partial f''}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}$$

$$= \frac{u_\infty^{3/2}}{\sqrt{\nu x}} f''' \cdot \sqrt{\frac{u_\infty}{\nu x}}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x} f''' \quad \text{—— } 8$$

$$u = u_\infty f'(\eta)$$

$$\frac{\partial u}{\partial x} = \frac{u_\infty}{x} \frac{\partial f'}{\partial \eta} \cdot \frac{\partial \eta}{\partial x}$$

$$= u_\infty \cdot f'' \cdot \left(-\frac{1}{2} y \sqrt{\frac{u_\infty}{\nu x}} \cdot \frac{1}{x}\right)$$

$$= -\frac{u_\infty}{2x} \eta \cdot f'' \quad \text{—— } 9$$

using 5 – 9 in 2

$$u\left(-\frac{u_\infty}{2x} \eta \cdot f''\right) + v\left(u_\infty f'' \sqrt{\frac{u_\infty}{\nu x}}\right) = \nu\left(\frac{u_\infty^2}{\nu x} f'''\right)$$

$$u_\infty f'\left(-\frac{u_\infty}{2x} \eta \cdot f''\right) + \frac{1}{2}\sqrt{\frac{\nu u_\infty}{x}}\left(\eta f' - f\right)\left(u_\infty f'' \sqrt{\frac{u_\infty}{\nu x}}\right)$$

$$= \cancel{\nu}\left(\frac{u_\infty^2}{\cancel{\nu} x} f'''\right)$$

$$-\frac{\cancel{u_\infty^2}}{2x} \eta \cdot f' f'' + \frac{\cancel{u_\infty^2}}{2x}(\eta f' - f) f'' = \frac{\cancel{u_\infty^2}}{x} f'''$$

$$-\cancel{\eta f' f''} + \cancel{\eta f' f''} - f f'' = 2 f'''$$

$$\boxed{f''' + \frac{1}{2} f f'' = 0}$$

$u = u_\infty$ at $y \to \infty$

$u_\infty f' = u_\infty$ at $y \to \infty$

$f'(\eta) = 1$

$2 \cdot \frac{1}{2} \int x^{-1/2} dx$



which is called Blausius's equation.

using B.C :- $u = 0$, $v = 0$ at $y = 0$

$u_\infty \cdot f' = 0$ , $v = 0$

$\Rightarrow f' = 0$ , $\frac{1}{2}\sqrt{\frac{\nu u_\infty}{x}}(\eta f' - f) = 0$

$\Rightarrow \boxed{f = 0}$

$u \to u_\infty$ as $y \to \infty$

$u_\infty f' \to u_\infty$ as $\eta \to \infty$ $\quad \boxed{f' \to 1 \text{ as } \eta \to \infty}$

Now the final model;

$$f''' + \frac{1}{2} f f'' = 0$$

$f = 0$, $f' = 0$ at $\eta = 0$ , $f' \to 1$ as $\eta \to \infty$

Skin friction :-

The skin friction can be easily determined

$$D = b \int_{x=0}^{l} \tau_0 \, dx$$

$$\tau_0(x) = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} f''(0)$$

$$= \alpha \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}}$$

where $f''(0) = \alpha = 0.332$ from table.

$$D = \alpha \mu b u_\infty \sqrt{\frac{u_\infty}{\nu}} \int_{x=0}^{l} \frac{dx}{\sqrt{x}} = 2\alpha \mu b u_\infty \sqrt{\frac{u_\infty}{\nu}} \left( x^{1/2} \Big|_0^l \right)$$

$$= 2\alpha b \mu u_\infty \sqrt{\frac{u_\infty l}{\nu}} .$$