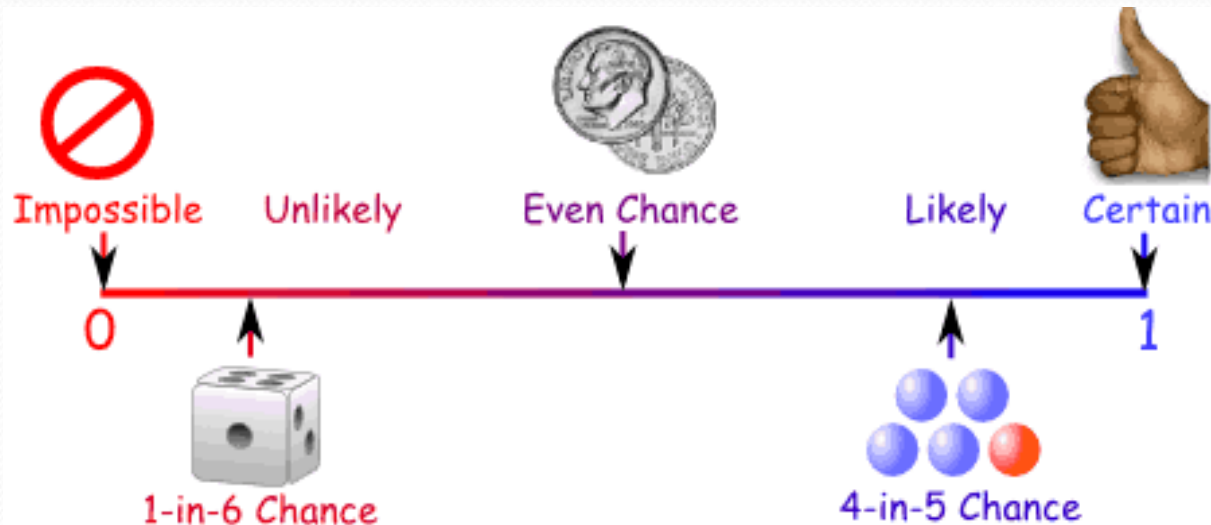


What is Probability?

Probability is the chance that something will happen - how likely it is that some event will happen.

Sometimes you can measure a probability with a number like "10% chance of rain", or you can use words such as impossible, unlikely, possible, even chance, likely and certain.

Example: "It is unlikely to rain tomorrow".



• Exhaustive Events:

The total number of all possible elementary outcomes in a random experiment is known as '*exhaustive events*'. In other words, a set is said to be exhaustive, when no other possibilities exist.

• Favorable Events:

The elementary outcomes which entail or favor the happening of an event is known as 'favorable events' i.e., the outcomes which help in the occurrence of that event.

• Mutually Exclusive Events:

Events are said to be '*mutually exclusive*' if the occurrence of an event totally prevents occurrence of all other events in a trial. In other words, two events A and B cannot occur simultaneously.

• Equally likely or Equi-probable Events:

Outcomes are said to be '*equally likely*' if there is no reason to expect one outcome to occur in preference to another. i.e., among all exhaustive outcomes, each of them has equal chance of occurrence.

• Complementary Events:

Let E denote occurrence of event. The complement of E denotes the non occurrence of event E . Complement of E is denoted by ' \bar{E} '.

• Independent Events:

Two or more events are said to be 'independent', in a series of a trials if the outcome of one event is does not affect the outcome of the other event or vice versa.

In other words, several events are said to be 'dependents' if the occurrence of an event is affected by the occurrence of any number of remaining events, in a series of trials.

Measurement of Probability:

There are three approaches to construct a measure of probability of occurrence of an event. They are:

- Classical Approach,
- Frequency Approach and
- Axiomatic Approach.

Classical or Mathematical Approach:

In this approach we assume that an experiment or trial results in any one of many possible outcomes, each outcome being Equi-probable or equally-likely.

Definition: If a trial results in 'n' exhaustive, mutually exclusive, equally likely and independent outcomes, and if 'm' of them are favorable for the happening of the event E, then the probability 'P' of occurrence of the event 'E' is given by-

$$P(E) = \frac{\text{Number of outcomes favourable to event E}}{\text{Exhaustive number of outcomes}} = \frac{m}{n}$$

Empirical or Statistical Approach:

This approach is also called the 'frequency' approach to probability. Here the probability is obtained by actually performing the experiment large number of times. As the number of trials n increases, we get more accurate result.

Definition: Consider a random experiment which is repeated large number of times under essentially homogeneous and identical conditions. If 'n' denotes the number of trials and 'm' denotes the number of times an event A has occurred, then, probability of event A is the limiting value of the relative frequency $\frac{m}{n}$.

$$\frac{m}{n}$$

Axiomatic Approach:

This approach was proposed by Russian Mathematician A.N.Kolmogorov in 1933.

'Axioms' are statements which are reasonably true and are accepted as such, without seeking any proof.

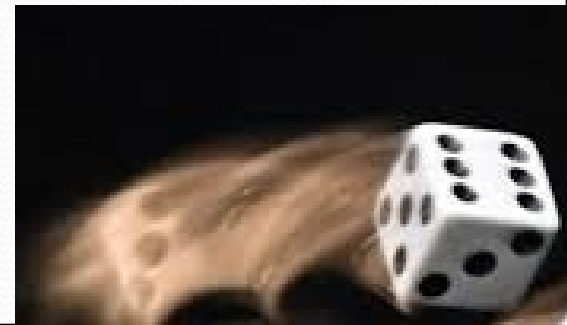
Definition: Let S be the sample space associated with a random experiment. Let A be any event in S . then $P(A)$ is the probability of occurrence of A if the following axioms are satisfied.

1. $P(A) > 0$, where A is any event.
2. $P(S) = 1$.
3. $P(A \cup B) = P(A) + P(B)$, when event A and B are mutually exclusive.

Three types of Probability

1. Theoretical probability:

For theoretical reasons, we assume that all n possible outcomes of a particular experiment are equally likely, and we assign a probability of $\frac{1}{n}$ to each possible outcome. Example: The theoretical probability of rolling a 3 on a regular 6 sided die is $\frac{1}{6}$



2. Relative frequency interpretation of probability:

$$\text{The probability of event A} = \frac{\text{How many times A occurs}}{\text{How many trials}}$$

Relative Frequency is based on observation or actual measurements.

Example: A die is rolled 100 times. The number 3 is rolled 12 times. The relative frequency of rolling a 3 is 12/100.

3. Personal or subjective probability:

These are values (between 0 and 1 or 0 and 100%) assigned by individuals based on how likely they think events are to occur. Example: The probability of my being asked on a date for this weekend is 10%.

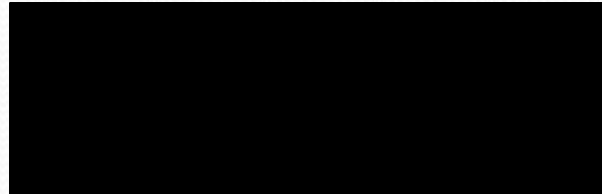
Probability Rules With Examples

1. The probability of an event is between 0 and 1. A probability of 1 is equivalent to 100% certainty. Probabilities can be expressed at fractions, decimals, or percents.

$$0 \leq pr(A) \leq 1$$

2. The sum of the probabilities of all possible outcomes is 1 or 100%. If A, B, and C are the only possible outcomes, then $pr(A) + pr(B) + pr(C) = 1$

Example: A bag contains 5 red marbles, 3 blue marbles, and 2 green marbles. $pr(\text{red}) + pr(\text{blue}) + pr(\text{green}) = 1$



3. The sum of the probability of an event occurring and it not occurring is 1. $pr(A) + pr(\text{not } A) = 1$ or $pr(\text{not } A) = 1 - pr(A)$

Example: A bag contains 5 red marbles, 3 blue marbles, and 2 green marbles.

$$pr(\text{red}) + pr(\text{not red}) = 1$$

$$\frac{3}{10} + pr(\text{not red}) = 1 \quad pr(\text{not red}) = \frac{7}{10}$$

4. If two events A and B are independent (this means that the occurrence of A has no impact at all on whether B occurs and vice versa), then the probability of A and B occurring is the product of their individual probabilities. **$pr(A \text{ and } B) = pr(A) \cdot pr(B)$**

Example: roll a die and flip a coin. **$pr(\text{heads and roll a } 3) = pr(H) \text{ and } pr(3)$**

$$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

5. If two events A and B are mutually exclusive (meaning A cannot occur at the same time as B occurs), then the probability of either A or B occurring is the sum of their individual probabilities. **$Pr(A \text{ or } B) = pr(A) + pr(B)$**
Example: A bag contains 5 red marbles, 3 blue marbles, and 2 green marbles.

$$pr(\text{red or green}) = pr(\text{red}) + pr(\text{green}) \quad \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

6. If two events A and B are not mutually exclusive (meaning it is possible that A and B occur at the same time), then the probability of either A or B occurring is the sum of their individual probabilities minus the probability of both A and B occurring. **$Pr(A \text{ or } B) = pr(A) + pr(B) - pr(A \text{ and } B)$**

Example: There are 20 people in the room: 12 girls (5 with blond hair and 7 with brown hair) and 8 boys (4 with blond hair and 4 with brown hair). There are a total of 9 blonds and 11 with brown hair. One person from the group is chosen randomly.

$$pr(\text{girl or blond}) = pr(\text{girl}) + pr(\text{blond}) - pr(\text{girl and blond})$$

$$\frac{12}{20} + \frac{9}{20} - \frac{5}{20} = \frac{16}{20}$$

7. The probability of at least one event occurring out of multiple events is equal to one minus the probability of none of the events occurring. $pr(\text{at least one}) = 1 - pr(\text{none})$

Example: roll a coin 4 times. What is the probability of getting at least one head on the 4 rolls.

$$pr(\text{at least one } H) = 1 - pr(\text{no } H) = 1 - pr(TTTT) = 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

8. If event B is a subset of event A , then the probability of B is less than or equal to the probability of A . $pr(B) \leq pr(A)$

Example: There are 20 people in the room: 12 girls (5 with blond hair and 7 with brown hair) and 8 boys (4 with blond hair and 4 with brown hair). $pr(\text{girl with brown hair}) \leq pr(\text{girl})$

$$\frac{7}{20} \leq \frac{12}{20}$$



THANK

YOU