Random Experiment :- The experiment whose result are decided by the law of chance. We do not have personal control on the happening of the result. There is always variety in the result and number of the possible result are known. The only uncertain is that which of the possible result will happen.

Such experiment are Random. Thus the random experiment have the following properties in common.

i, The result of such experiment are decided by the law of chance

ii, The number of results will be two or more than two.

iii, The happening of particular result is uncertain and thus we can measure the degree of uncertainity for the particular result. This measure of uncertainty is known as probability.

Sample Space :- The sample space result from the random experiment. The set of all possible outcomes of a random experiment is known as the sample space. It is denoted by (S). For example

i, If we toss one coin

$$S = \{H, T\}$$

ii If we toss two coins

$$S = \{HH, HT, TH, TT\}$$

iii, If we toss three coins

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

(2)

(iv) If one die is thrown
$S = \{1, 2, 3, 4, 5, 6\}$

Event:- The outcomes of random experiment or the sample points of the sample space are known as the events. The most proper concept is as" The subsets of the sample space (S) are known as the events. If the subset contain single element we call them sample events and if the elements or two or more we call them Composit events.

Exhaustive Events:- The all possible results of a random experiment are known as exhustive events e.g If coin is thrown then head and tail are the two exhustive events. Likewise if die is thrown we have 6 exhustive events.

Mutually Exclusive Events:- If from a random experiment only one result could happen and other are excluded then such events are mutually exclusive. Such events only belong to a single random experiment. For such event the intersections of the events do not exist. Then A and B shall be mutually exclusive if $A \cap B = \phi$. If coin is thrown then head and tail are mutually exclusive.

Non Mutually Exclusive Events:-
The events are called not mutually exclusive if they have at least one outcome common between them. If A and B are not mutually exclusive events then $A \cap B \neq \phi$

For example

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

Let A = Prime number $= \{2, 3, 5, 7, 11\}$

B = odd face $= \{1, 3, 5, 7, 9, 11\}$

$A \cap B = \{3, 5, 7, 11\}$

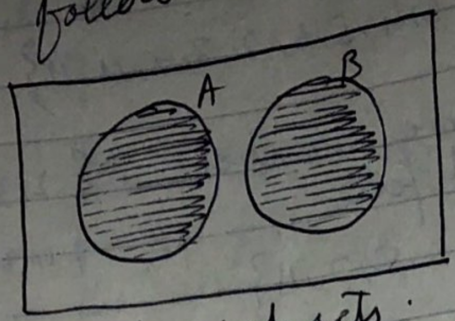
Independent Events:- When in some random experiment the events could happen together and they do not effect their respective chance of happening they are known as independent events. Therefore all the cases in the intersection are independent. e.g If two coins are thrown then head on 1st coin and tail on the second coin are independent. Likewise if two cards are drawn from the pack together then these cards are independent.

DEPENDENT EVENT:- The dependent events happen one after another. They so happen that the chance of happening of one effect that of other e.g If one ball is drawn from the bag and put aside to draw another ball then 1st ball will effect the chance of second ball.

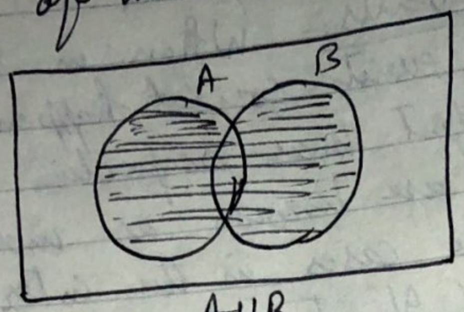
VENN DIAGRAM:- Venn diagram consist in a rectangle which indicate the universal set or sample space (S) and the circular regions, which represent the sub set of sample space (S). This diagram is meant for the stating the relation between the sub set. If (S) is the sample space and (A) (B) are the subsets then the venn diagram can be de

(4)

drawn as follow

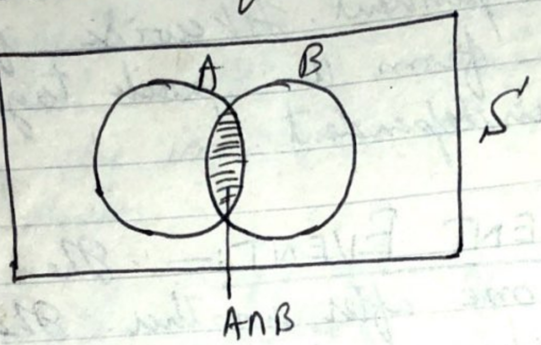


i, Union of the subsets.



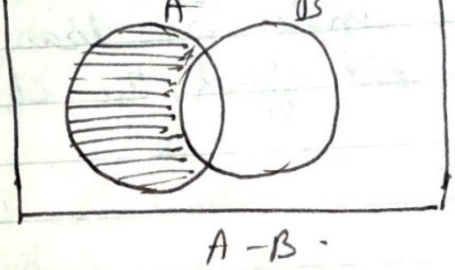
A∪B

ii, Intersection of subsets



A∩B

iii, A − B



A − B.

Rule of Permutation:- A permutation is any ordered subset from a set of n distinct objects. The numbers of permutation of r objects selected in a definite order from n distinct objects is denoted by the symbol ^nP_r.

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

e.g $n = 4, \quad r = 2$

$${}^{4}P_{2} = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4\times3\times\cancel{2\times1}}{\cancel{2\times1}}$$

$$= 12$$

The number of permutation of n objects selected all at a time when n objects consist of n_1 of one kind, n_2 of a second kind —— n_k of a kth kind

$$P = \frac{n!}{n_1!\, n_2! \text{——} n_k!}$$

Find Permutation of word.

e.g <u>STATISTICS</u>.

$n = 10$

$n_1 = S = 3$

$n_2 = T = 3$

$n_3 = I = 2$

$n_4 = A = 1$

$n_5 = C = 1$

$$P = \frac{10!}{3!\,3!\,2!\,1!\,1!}$$

$P =$

<u>Rule of Combination</u>. A combination is any subset of r objects selected without regard to their order, from a set of n distinct objects. The total number of such combination is denoted by the symbol

$${}^{n}C_{r} \text{ or } \binom{n}{r} = \frac{n!}{r!\,n-r!}$$

e.g $$\binom{10}{5} = \frac{10!}{5!\,5!} = \frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\cdot5!}$$

DEFINITIONS OF PROBABILITY.

(1) THE CLASSICAL OR A PRIORI DEFINITION OF PROBABILITY.

Let an experiment has (n) equally likely, mutually exclusive total cases. Let event (A) has (m) cases out of (n) ~~values~~ when m<n. Let the Prob of happening of event (A) is defined as the ratio of the cases of event (A) ~~is defined~~ as to the total cases.

$$p(A) = m/n = \frac{\text{Favourable cases}}{\text{Total cases}}.$$

This definition is helpful when all the cases are equally likely and the no. of cases of the experiment are finit.

However this definition has some defects which are as follows.

① The condition of equally likely is not suitable for every experiment. By equally likely mean equally probability which shows that we have prior idea of prob before the experiment.

② Here (n) is fixed which means that we can only conduct small experiment on its basis.

RELATIVE FREQUENCY DEFINITION.

If an experiment is repeated very large number of time under possible the similar conditions so that the total (n) cases of such experiment are very large and out of the (m) cases favours and event (A) when $m < n$ then the probability of event (A) is defined as the relative frequency of the favourable to the total cases.

$$p(A) = \lim_{n \to \infty} \frac{m}{n}$$

i, This definition is useful when we wish to conduct experiment at the large scale

ii, This definition also cover such cases whose condition of equally likely could not be maintained.

However there is one deffect in the definition of the similar condition for the experiment could not be maintained.

3- THE AXIOMATIC DEFINITION OF PROBABILITY.

Let S be a sample space with the sample points E_1, E_2 ---- E_n. To each sample point we assign a real number denoted by symbol $P(E_i)$, and called the Probability of E_i that must satisfy the following basic axioms.

AXIOM i, $0 \leq P(E_i) \leq 1$

AXIOM ii, $P(S) = 1$

AXIOM iii, $P(A \cup B) = P(A) + P(B)$.

4- SUBJECTIVE OR PERSONALISTIC PROBABILITY

The subjective or personalistic Prob is a measure of the strength of a person's belief regarding the occurrence of an event.

This definition may be applied to those real world situation where neither an equally likely nor a relative frequency approach is possible.

THEOREM:- If $\bar{A}$ is the complement of an event A relative to the sample space S

$$P(\bar{A}) = 1 - P(A)$$

Proof:- Since ~~Bt~~ the event A & $\bar{A}$ are mutually exclusive

$$A \cup \bar{A} = S$$

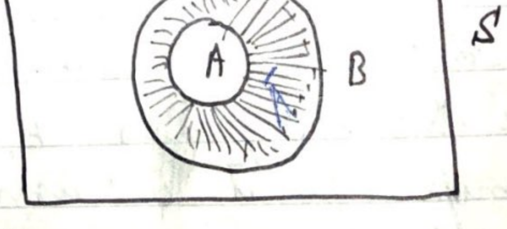
$$P(A \cup \bar{A}) = P(S)$$

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A).$$

THEOREM. If A & B are two events such that $A \subset B$ then $P(A) \leq P(B)$

Proof



$A \subset B$, the event B may be written as the union of two mutually exclusive event $B \cap A$ and $B \cap \bar{A}$

ie $B = (B \cap A) \cup (B \cap \bar{A})$

But $B \cap A = A$

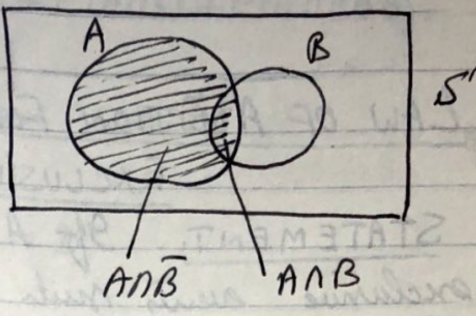
~~$B = (B \cap A)$~~

$B = P(A) + P(B \cap \bar{A})$

But $P(B \cap \bar{A}) \geq 0$

Hence $P(A) \leq P(B)$.

THEOREM:- If A and B are any two events defined in a sample space S then $P(A \cap \bar{B}) = P(A) - P(A \cap B)$



Proof:- The events $A \cap B$ and $A \cap \bar{B}$ are mutually exclusive and their union is A

$$A = (A \cap \bar{B}) \cup (A \cap B)$$

$$P(A) = P(A \cap \bar{B}) + P(A \cap B)$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

LAW OF ADDITION FOR MUTUALLY EXCLUSIVE EVENTS.

STATEMENT:- If two events A and B are mutually exclusive then the Prob that A or B happen is given by the sum of the Probability of A & B.

PROOF:- Let us consider that an experiment has (n) equally likely cases. Let (m_1) cases favoures an event (A). Let (m_2) cases favours an event (B). When (A) & (B) are mutually exclusive

$$\therefore \quad P(A) = \frac{m_1}{n} \quad \& \quad P(B) = \frac{m_2}{n}$$

Since $A \cap B = \phi$. Therefore for probability of A or B the cases will come in union as $(m_1 + m_2)$

$$P(A \cup B) = \frac{m_1 + m_2}{n} = \frac{m_1}{n} + \frac{m_2}{n}$$

$$P(A \cup B) = P(A) + P(B)$$

(10)

This formula can be projected to any number of event. Thus if $A_1, A_2 \text{ —— } A_n$ are events! then

$$P(A_1 \cup A_2 \cup A_3 \text{ —— } \cup A_n) = P(A_1) + P(A_2) + \text{ —— } P(A_n)$$

$$= \sum_{i=1}^{n} P(A_i) = 1$$

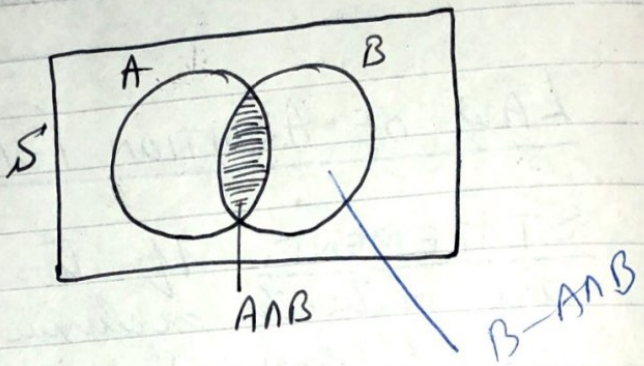
LAW OF ADDITION FOR NON MUTUALLY EXCLUSIVE EVENTS:-

STATEMENT:- If A and B are non-mutually exclusive events such that at least one of them happen then Prob is given as

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof

From this venn diagram we can write union (A) and (B) as follows



$P(A \cup B) = A \cup B - A \cap B$

Intersection does not exist between (A) and $(B - A \cap B)$. operating probability both sides

$$p(A \cup B) = P[A \cup B - A \cap B]$$
$$= P(A) \cup P(B - A \cap B) \quad \text{—— (1)}$$

Now (B) is union of $(A \cap B)$ and $(B - A \cap B)$

$$B = (A \cap B) \cup (B - A \cap B)$$

operating Prob both sides

$$P(B) = P(A \cap B) \cup P(B - A \cap B)$$

$$P(B) = P(A \cap B) + P(B - A \cap B)$$

$$P(B - A \cap B) = P(B) - P(A \cap B) \quad \text{—— (2)}$$

Putting (2) in (1) we get.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

THEOREM:- Prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

Proof

By law we can write

$$A \cup B \cup C = A \cup (B \cup C)$$

Let $B \cup C = D$

$$A \cup B \cup C = A \cup D$$

$$P(A \cup B \cup C) = P(A \cup D)$$

$$= P(A) + P(D) - P(A \cap D)$$

Putting $D = B \cup C$

$$A \cap D = A \cap B \cup C$$

$$P(A \cup B \cup C) = P(A) + P(B \cup C) - P(A \cap B \cup C) \quad \text{--- (1)}$$

By law of distribution we know that

$$A \cap B \cup C = (A \cap B) \cup (A \cap C)$$

$$P(A \cap B \cup C) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \quad \text{--- (2)}$$

Putting (2) in (1)

$$P(A \cup B \cup C) = P(A) + P(B \cup C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

LAW OF MULTIPLICATION
(FOR INDEPENDENT EVENTS)

STATEMENT:- When two or more events are independent then the Prob that both will happen together is given by the Product of their respective ~~expected~~ Probabilities.

$$P(A \cap B) = P(A) \cdot P(B)$$

Proof Let us suppose that event (A) has (n) favourable cases when an experiment of (N) total cases is done. Let us suppose that event (B) has (m) favourable cases when another experiment of (M) total cases is conducted. Let us also suppose that (A) and (B) events are independent.

$$\therefore \quad P(A) = n/N, \quad P(B) = m/M$$

Since the events are independent therefor each case of (A) & (B) will combine with each other. Therefore for joint happening of (A) & (B) the total cases will be $N \times M$. & favourable cases will be $n \times m$

$$P(A \cap B) = \frac{n \times m}{N \times M}$$

$$= \frac{n}{N} \times \frac{m}{M}$$

$$P(A \cap B) = P(A) \times P(B)$$

THEOREM:- If A and B are two independent events in a sample space S then

(i) A and $\bar{B}$ are independent

(ii) $\bar{A}$ and B are independent

(iii) $\bar{A}$ and $\bar{B}$ are independent

Proof.-

(i) The event $A \cap B$ and $A \cap \bar{B}$ are mutually exclusive and their union is A

$$A = (A \cap B) \cup (A \cap \bar{B})$$

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$= P(A) - P(A) \cdot P(B)$$

$$= P(A)[1 - P(B)]$$

$$P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})$$

A & $\bar{B}$ are independent

(ii) $P(B) = P(B \cap A) + P(B \cap \bar{A})$

$$P(B \cap \bar{A}) = P(B) - P(B \cap A)$$

$$= P(B) - P(A) \cdot P(B)$$

$$= P(B)[1 - P(A)]$$

$$P(B \cap \bar{A}) = P(B) \cdot P(\bar{A})$$

$\bar{A}$ & B are independent.

(iii) Using De Morgan's law $\bar{A} \cap \bar{B} = \overline{A \cup B}$

$$P(\bar{A} \cap \bar{B}) = \overline{P(A \cup B)}$$

$$= 1 - P(A \cup B)$$

$$= 1 - P(A) + P(B) + P(A \cap B)$$

$$= 1 - P(A) - P(B) + P(A) \cdot P(B)$$

$$= [1 - P(A)] - P(B)[1 - P(A)]$$

$$= [1 - P(A)][1 - P(B)]$$

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$$

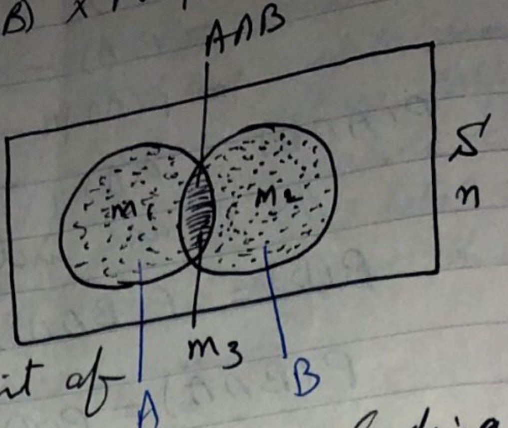
$\bar{A}$ & $\bar{B}$ are independent.

LAW OF MULTIPLICATION.
(FOR DEPENDENT EVENTS)

STATEMENT:- If two event A and B are dependent such (A) happen when (B) has already happened or (B) happen when (A) has already happened then the Prob is given as

$P(A \cap B) = P(A) \times P(B/A)$

$P(A \cap B) = P(B) \times P(A/B)$



Proof

Let (S) contain (n) sample points
Let (A) contain (m_1) points (excluding point of intersection)
Let (B) contain (m_2) points (excluding points of intersection)
Let the subset ($A \cap B$) contain (m_3) points

By def $P(A) = m_1/n$

$P(B) = m_2/n$

$P(A \cap B) = m_3/n$

multiplying and dividing (R.H.S) by m_1

$$P(A \cap B) = \frac{m_3}{n} \cdot \frac{m_1}{m_1}$$

$$= \frac{m_1}{n} \cdot \frac{m_3}{m_1}$$

But $\frac{m_1}{n} = P(A)$ therefore $\frac{m_3}{m_1}$ be the Prob that (B) happened and (A) has already happened.

$$P(A \cap B) = P(A) \cdot P(B/A).$$

Likewise

$$P(A \cap B) = \frac{m_3}{n} \cdot \frac{m_2}{m_2}$$

$$= \frac{m_2}{n} \cdot \frac{m_3}{m_2}$$

$$P(A \cap B) = P(B) \cdot P(A/B)$$

CONDITIONAL PROBABILITY :-

If A & B are two events in a sample space S and if P(B) is not equal to zero then the conditional probability of the event A given that event B has occured written as P(A/B)

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \qquad P(B) > 0$$

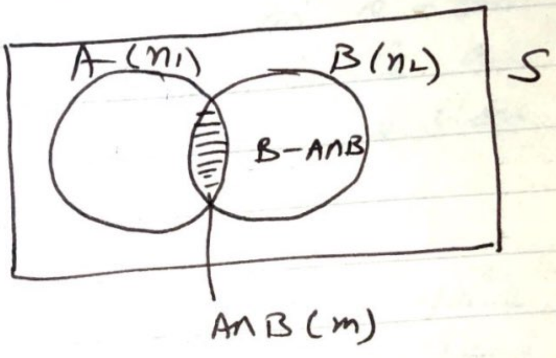
Similarly

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \qquad P(A) > 0$$

Addition Law For Not Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof



suppose there is a sample space S containing N Points which are equally likely. The two events A & B belong to S and contain n_1 & n_2 Points resp.
The events $A \cap B$ has m Points.
$A \cup B$ will occure if any Point out of $n_1 + n_2$ occure. $A \cup B$ can be written as union of two disjoint events which are

(i) A

(ii) $(B - A \cap B)$

$A \cup B = A \cup (B - A \cap B)$
A contain n_1 Points & $(B - A \cap B)$ contain $(n_2 - m)$ Points

(16)

Thus $A \cup B$ has $n_1 + (n_2 - m)$ points

$$P(A \cup B) = \frac{n_1 + n_2 - m}{N}$$

$$= \frac{n_1}{N} + \frac{n_2}{N} - \frac{m}{N}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Questions

Example No. 6.4 to

Example No. 6.31