7 Consolidation

A stress increase caused by the construction of foundations or other loads compresses the soil layers. The compression is caused by (a) deformation of soil particles, (b) relocations of soil particles, and (c) expulsion of water or air from the void spaces. In general, the soil settlement caused by load may be divided into three broad categories:

- **1.** *Immediate settlement*, which is caused by the elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content. Immediate settlement calculations are generally based on equations derived from the theory of elasticity.
- **2.** *Primary consolidation settlement*, which is the result of a volume change in saturated cohesive soils because of the expulsion of water that occupies the void spaces.
- **3.** *Secondary consolidation settlement*, which is observed in saturated cohesive soils and is the result of the plastic adjustment of soil fabrics. It follows the primary consolidation settlement under a constant effective stress.

This chapter presents the fundamental principles for estimating the consolidation settlement of soil layers under superimposed loadings.

7.1 Fundamentals of Consolidation

When a saturated soil layer is subjected to a stress increase, the pore water pressure suddenly increases. In sandy soils that are highly permeable, the drainage caused by the increase in the pore water pressure is completed immediately. Pore water drainage is accompanied by a reduction in the volume of the soil mass, resulting in settlement. Because of the rapid drainage of the pore water in sandy soils, immediate settlement and consolidation take place simultaneously. This is not the case, however, for clay soils, which have low hydraulic conductivity. The consolidation settlement is time dependent.

Keeping this in mind, we can analyze the strain of a saturated clay layer subjected to a stress increase (Figure 7.1a). A layer of saturated clay of thickness H



Figure 7.1 Variation of total stress, pore water pressure, and effective stress in a clay layer drained at top and bottom as the result of an added stress, $\Delta \sigma$

is confined between two layers of sand and is subjected to an instantaneous increase in *total stress* of $\Delta \sigma$. From Chapter 6, we know that

$$\Delta \sigma = \Delta \sigma' + \Delta u \tag{7.1}$$

where

 $\Delta \sigma' =$ increase in the effective stress

 Δu = increase in the pore water pressure

Since clay has very low hydraulic conductivity and water is incompressible compared with the soil skeleton, at time t = 0, the entire incremental stress, $\Delta \sigma$, will be carried by water ($\Delta \sigma = \Delta u$) at all depths (Figure 7.1b). None will be carried by the soil skeleton (that is, incremental effective stress, $\Delta \sigma' = 0$).

After the application of incremental stress, $\Delta\sigma$, to the clay layer, the water in the void spaces will begin to be squeezed out and will drain in both directions into the sand layers. By this process, the excess pore water pressure at any depth on the clay layer will gradually decrease, and the stress carried by the soil solids (effective stress) will increase. Thus, at time $0 < t < \infty$,

$$\Delta \sigma = \Delta \sigma' + \Delta u$$
 $(\Delta \sigma' > 0 \text{ and } \Delta u < \Delta \sigma)$

However, the magnitudes of $\Delta \sigma'$ and Δu at various depths will change (Figure 7.1c), depending on the minimum distance of the drainage path to either the top or bottom sand layer.

Theoretically, at time $t = \infty$, the entire excess pore water pressure would dissipate by drainage from all points of the clay layer, thus giving $\Delta u = 0$. Then the total stress increase, $\Delta \sigma$, would be carried by the soil structure (Figure 7.1d), so

$$\Delta \sigma = \Delta \sigma$$

This gradual process of drainage under the application of an additional load and the associated transfer of excess pore water pressure to effective stress causes the time-dependent settlement (consolidation) in the clay soil layer.

7.2

One-Dimensional Laboratory Consolidation Test

The one-dimensional consolidation testing procedure was first suggested by Terzaghi (1925). This test is performed in a consolidometer (sometimes referred to as an oedometer). Figure 7.2 is a schematic diagram of a consolidometer. The soil specimen is placed inside a metal ring with two porous stones, one at the top of the specimen and another at the bottom. The specimens are usually 63.5 mm in diameter and 25.4 mm thick. The load on the specimen is applied through a lever arm, and compression is measured by a micrometer dial gauge. The specimen is kept under water during the test. Each load is usually kept for 24 hours. After that, the load is usually doubled, thus doubling the pressure on the specimen, and the compression measurement is continued. At the end of the test, the dry weight of the test specimen is determined. Figure 7.3 shows a consolidation test in progress (right-hand side).



Figure 7.2 Consolidometer

The general shape of the plot of deformation of the specimen versus time for a given load increment is shown in Figure 7.4. From the plot, it can be observed that there are three distinct stages, which may be described as follows:

Stage I:Initial compression, which is mostly caused by preloading.Stage II:Primary consolidation, during which excess pore water pressure is
gradually transferred into effective stress by the expulsion of pore
water.



Figure 7.3 Consolidation test in progress (right-hand side) (Courtesy of Braja Das)



Time (log scale)



Stage III: Secondary consolidation, which occurs after complete dissipation of the excess pore water pressure, when some deformation of the specimen takes place because of the plastic readjustment of soil fabric.

7.3 Void Ratio–Pressure Plots

After the time-deformation plots for various loadings are obtained in the laboratory, it is necessary to study the change in the void ratio of the specimen with pressure. Following is a step-by-step procedure:

1. Calculate the height of solids, H_s , in the soil specimen (Figure 7.5):

$$H_s = \frac{W_s}{AG_s \gamma_w} \tag{7.2}$$

where

 $W_s = dry$ weight of the specimen

A = area of the specimen

 G_s = specific gravity of soil solids

 γ_w = unit weight of water



Figure 7.5 Change of height of specimen in one-dimensional consolidation test

2. Calculate the initial height of voids, H_{v} :

$$H_v = H - H_s \tag{7.3}$$

where H = initial height of the specimen.

3. Calculate the initial void ratio, e_0 , of the specimen:

$$e_{0} = \frac{V_{\nu}}{V_{s}} = \frac{H_{\nu}}{H_{s}}\frac{A}{A} = \frac{H_{\nu}}{H_{s}}$$
(7.4)

4. For the first incremental loading σ_1 (total load/unit area of specimen), which causes deformation ΔH_1 , calculate the change in the void ratio Δe_1 :

$$\Delta e_1 = \frac{\Delta H_1}{H_s} \tag{7.5}$$

 ΔH_1 is obtained from the initial and the final dial readings for the loading. At this time, the effective pressure on the specimen is $\sigma' = \sigma_1 = \sigma'_1$.

5. Calculate the new void ratio, e_1 , after consolidation caused by the pressure increment σ_1 :

$$e_1 = e_0 - \Delta e_1 \tag{7.6}$$

For the next loading, σ_2 (*note*: σ_2 equals the cumulative load per unit area of specimen), which causes additional deformation ΔH_2 , the void ratio e_2 at the end of consolidation can be calculated as

$$e_2 = e_1 - \frac{\Delta H_2}{H_s} \tag{7.7}$$

Note that, at this time, the effective pressure on the specimen is $\sigma' = \sigma_2 = \sigma'_2$.

Proceeding in a similar manner, we can obtain the void ratios at the end of the consolidation for all load increments.

The effective pressures ($\sigma = \sigma'$) and the corresponding void ratios (e) at the end of consolidation are plotted on semilogarithmic graph paper. The typical shape of such a plot is shown in Figure 7.6.



Figure 7.6 Typical plot of *e* versus $\log \sigma'$

7.4 Normally Consolidated and Overconsolidated Clays

Figure 7.6 showed that the upper part of the e-log σ' plot is somewhat curved with a flat slope, followed by a linear relationship for the void ratio, with log σ' having a steeper slope. This can be explained in the following manner.

A soil in the field at some depth has been subjected to a certain maximum effective past pressure in its geologic history. This maximum effective past pressure may be equal to or greater than the existing overburden pressure at the time of sampling. The reduction of pressure in the field may be caused by natural geologic processes or human processes. During the soil sampling, the existing effective overburden pressure is also released, resulting in some expansion. When this specimen is subjected to a consolidation test, a small amount of compression (that is, a small change in the void ratio) will occur when the total pressure applied is less than the maximum effective overburden pressure in the field to which the soil has been subjected in the past. When the total applied pressure on the specimen is greater than the maximum effective past pressure, the change in the void ratio is much larger, and the e-log σ' relationship is practically linear with a steeper slope.

This relationship can be verified in the laboratory by loading the specimen to exceed the maximum effective overburden pressure, and then unloading and reloading again. The e-log σ' plot for such cases is shown in Figure 7.7, in which cd represents unloading and dfg represents the reloading process.



This leads us to the two basic definitions of clay based on stress history:

- **1.** *Normally consolidated*: The present effective overburden pressure is the maximum pressure to which the soil has been subjected in the past.
- **2.** *Overconsolidated*: The present effective overburden pressure is less than that which the soil has experienced in the past. The maximum effective past pressure is called the *preconsolidation pressure*.

The past effective pressure cannot be determined explicitly because it is usually a function of geological processes and, consequently, it must be inferred from laboratory test results.

Casagrande (1936) suggested a simple graphic construction to determine the preconsolidation pressure, σ'_c , from the laboratory e-log σ' plot. The procedure follows (see Figure 7.8):

- 1. By visual observation, establish point *a* at which the $e-\log \sigma'$ plot has a minimum radius of curvature.
- **2.** Draw a horizontal line *ab*.
- 3. Draw the line *ac* tangent at *a*.
- 4. Draw the line *ad*, which is the bisector of the angle *bac*.
- 5. Project the straight-line portion gh of the $e \log \sigma'$ plot back to intersect ad at f. The abscissa of point f is the preconsolidation pressure, σ'_c .

The overconsolidation ratio (OCR) for a soil can now be defined as

$$OCR = \frac{\sigma'_c}{\sigma'}$$



Effective pressure, σ' (log scale)



where

 σ_c' = preconsolidation pressure of a specimen

 σ' = present effective vertical pressure

7.5

Effect of Disturbance on Void Ratio–Pressure Relationship

A soil specimen will be remolded when it is subjected to some degree of disturbance. This will affect the void ratio-pressure relationship for the soil. For a normally consolidated clayey soil of low to medium sensitivity (Figure 7.9) under an effective overburden pressure of σ'_o and with a void ratio of e_0 , the change in the void ratio with an increase of pressure in the field will be roughly as shown by curve 1. This is the *virgin compression curve*, which is approximately a straight line on a semilogarithmic plot. However, the laboratory consolidation curve for a fairly undisturbed specimen of the same soil (curve 2) will be located to the left of curve 1. If the soil is completely remolded and a consolidation test is conducted on it, the general position of the e-log σ' plot will be represented by curve 3. Curves 1, 2, and 3 will intersect approximately at a void ratio of $e = 0.4e_0$ (Terzaghi and Peck, 1967).

For an overconsolidated clayey soil of low to medium sensitivity that has been subjected to a preconsolidation pressure of σ'_c (Figure 7.10) and for which the present effective overburden pressure and the void ratio are σ'_o and e_0 , respectively, the field consolidation curve will take a path represented approximately by *cbd*. Note that *bd* is a part of the virgin compression curve. The laboratory consolidation test results on a specimen subjected to moderate disturbance will be represented by



Figure 7.10 Consolidation characteristics of overconsolidated clay of low to medium sensitivity

curve 2. Schmertmann (1953) concluded that the slope of line cb, which is the field recompression path, has approximately the same slope as the laboratory rebound curve fg.

7.6

Calculation of Settlement from One-Dimensional Primary Consolidation

With the knowledge gained from the analysis of consolidation test results, we can now proceed to calculate the probable settlement caused by primary consolidation in the field, assuming one-dimensional consolidation.

Let us consider a saturated clay layer of thickness H and cross-sectional area A under an existing average effective overburden pressure σ'_{o} . Because of an increase of pressure, $\Delta\sigma$, let the primary settlement be S_p . At the end of consolidation, $\Delta\sigma = \Delta\sigma'$. Thus, the change in volume (Figure 7.11) can be given by

$$\Delta V = V_0 - V_1 = HA - (H - S_p)A = S_pA \tag{7.8}$$

where V_0 and V_1 are the initial and final volumes, respectively. However, the change in the total volume is equal to the change in the volume of voids, ΔV_{ν} . Thus,

$$\Delta V = S_p A = V_{\nu 0} - V_{\nu 1} = \Delta V_{\nu}$$
(7.9)

where $V_{\nu 0}$ and $V_{\nu 1}$ are the initial and final void volumes, respectively. From the definition of the void ratio, we have

$$\Delta V_{\nu} = \Delta e V_s \tag{7.10}$$

where $\Delta e =$ change of void ratio. But

$$V_s = \frac{V_0}{1 + e_0} = \frac{AH}{1 + e_0} \tag{7.11}$$



Figure 7.11 Settlement caused by one-dimensional consolidation

where $e_0 =$ initial void ratio at volume V_0 . Thus, from Eqs. (7.8), (7.9), (7.10), and (7.11), we get

$$\Delta V = S_p A = \Delta e V_s = \frac{AH}{1 + e_0} \Delta e$$

or

$$S_p = H \frac{\Delta e}{1 + e_0} \tag{7.12}$$

For normally consolidated clays that exhibit a linear e-log σ' relationship (Figure 7.9) (*note*: $\Delta \sigma = \Delta \sigma'$ at the end of consolidation),

$$\Delta e = C_c[\log(\sigma'_o + \Delta \sigma') - \log \sigma'_o]$$
(7.13)

where $C_c =$ slope of the e-log σ'_o plot and is defined as the compression index. Substituting Eq. (7.13) into Eq. (7.12) gives

$$S_p = \frac{C_c H}{1 + e_0} \log\left(\frac{\sigma'_o + \Delta \sigma'}{\sigma'_o}\right)$$
(7.14)

For a thicker clay layer, a more accurate measurement of settlement can be made if the layer is divided into a number of sublayers and calculations are made for each sublayer. Thus, the total settlement for the entire layer can be given as

$$S_p = \sum \left[\frac{C_c H_i}{1 + e_0} \log \left(\frac{\sigma'_{o(i)} + \Delta \sigma'_{(i)}}{\sigma'_{o(i)}} \right) \right]$$

where

 H_i = thickness of sublayer *i*

 $\sigma'_{o(i)}$ = initial average effective overburden pressure for sublayer *i*

 $\Delta \sigma'_{(i)}$ = increase of vertical pressure for sublayer *i*

In overconsolidated clays (Figure 7.10), for $\sigma'_o + \Delta \sigma' \leq \sigma'_c$, field *e*-log σ' variation will be along the line *cb*, the slope of which will be approximately equal to the slope of the laboratory rebound curve. The slope of the rebound curve, C_s , is referred to as the *swell index*, so

$$\Delta e = C_s[\log(\sigma'_o + \Delta \sigma') - \log \sigma'_o]$$
(7.15)

From Eqs. (7.12) and (7.15), we have

$$S_p = \frac{C_s H}{1 + e_0} \log\left(\frac{\sigma'_o + \Delta \sigma'}{\sigma'_o}\right)$$
(7.16)

If $\sigma'_o + \Delta \sigma > \sigma'_c$, then

$$S_p = \frac{C_s H}{1 + e_0} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1 + e_0} \log \left(\frac{\sigma'_o + \Delta \sigma'}{\sigma'_c}\right)$$
(7.17)

However, if the $e-\log \sigma'$ curve is given, it is possible simply to pick Δe off the plot for the appropriate range of pressures. This value may be substituted into Eq. (7.12) for calculating the settlement, S_p .

7.7 Compression Index (C_c) and Swell Index (C_s)

We can determine the compression index for field settlement caused by consolidation by graphic construction (as shown in Figure 7.8) after obtaining laboratory test results for the void ratio and pressure.

Skempton (1944) suggested empirical expressions for the compression index. For undisturbed clays:

$$C_c = 0.009(LL - 10) \tag{7.18}$$

For remolded clays:

$$C_c = 0.007(LL - 10) \tag{7.19}$$

where LL = liquid limit (%). In the absence of laboratory consolidation data, Eq. (7.18) is often used for an approximate calculation of primary consolidation in the field. Several other correlations for the compression index are also available now. Several of those correlations have been compiled by Rendon-Herrero (1980), and these are given in Table 7.1

Table 7.1 Correlations for Compression Index, C_c (compiled from Rendon-Herrero, 1980)

Equation	Region of applicability
$C_c = 0.01 w_N$	Chicago clays
$C_c = 1.15(e_O - 0.27)$	All clays
$C_c = 0.30(e_O - 0.27)$	Inorganic cohesive soil: silt, silty clay, clay
$C_c = 0.0115 w_N$	Organic soils, peats, organic silt, and clay
$C_c = 0.0046(LL - 9)$	Brazilian clays
$C_c = 0.75(e_O - 0.5)$	Soils with low plasticity
$C_c = 0.208e_O + 0.0083$	Chicago clays
$C_c = 0.156e_O + 0.0107$	All clays

Note: $e_0 = in \, situ$ void ratio; $w_N = in \, situ$ water content.

Based on observations on several natural clays, Rendon-Herrero (1983) gave the relationship for the compression index in the form

$$C_c = 0.141 G_s^{1.2} \left(\frac{1+e_0}{G_s}\right)^{2.38}$$
(7.20)

More recently, Park and Koumoto (2004) expressed the compression index by the following relationship.

$$C_c = \frac{n_o}{371.747 - 4.275n_o} \tag{7.21}$$

where $n_o = in \, situ$ porosity of the soil

Based on the modified Cam clay model, Wroth and Wood (1978) have shown that

$$C_c \approx 0.5G_s \frac{[PI(\%)]}{100}$$
 (7.22)

where PI = plasticity index

If an average value of G_s is taken to be about 2.7 (Kulhawy and Mayne, 1990)

$$C_c \approx \frac{PI}{74} \tag{7.23}$$

The swell index is appreciably smaller in magnitude than the compression index and generally can be determined from laboratory tests. Typical values of the liquid limit, plastic limit, virgin compression index, and swell index for some natural soils are given in Table 7.2.

From Table 7.2, it can be seen that $C_s \approx 0.2$ to 0.3 C_c . Based on the modified Cam clay model, Kulhawy and Mayne (1990) have shown than

$$C_s \approx \frac{PI}{370} \tag{7.24}$$

Tak	ble	7.2	Compr	ession	and	Swell	of	Natural	Soils
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Soil	Liquid limit	Plastic limit	Compression index, <i>C_c</i>	Swell index, <i>C_s</i>	C _s /C _c
Boston blue clay	41	20	0.35	0.07	0.2
Chicago clay	60	20	0.4	0.07	0.175
Ft. Gordon clay, Georgia	51	26	0.12	0.04	0.33
New Orleans clay	80	25	0.3	0.05	0.17
Montana clay	60	28	0.21	0.05	0.24

Example 7.1

Following are the results of a laboratory consolidation test on a soil specimen obtained from the field. Dry mass of specimen = 128 g, height of specimen at the beginning of the test = 2.54 cm, $G_s = 2.75$, and area of the specimen = 30.68 cm².

Final height of specimen at the end of consolidation (cm)				
2.540				
2.488				
2.465				
2.431				
2.389				
2.324				
2.225				
2.115				

Make necessary calculations and draw an e vs. log σ' curve.

Solution

Calculation of H_s From Eq. (7.2)

$$H_s = \frac{m_s}{AG_s\rho_w} = \frac{128 \text{ g}}{(30.68 \text{ cm}^2)(2.75)(1 \text{ g/cm}^3)} = 1.52 \text{ cm}$$

Now the following table can be prepared.



Figure 7.12 Variation of void ratio with pressure

Pressure, σ' kN/m ²	Height at the end of consolidation, <i>H</i> (cm)	$m{H_{v}}=m{H}-m{H_{s}}$ (cm)	$e = H_v/H_s$
0	2.540	1.02	0.671
50	2.488	0.968	0.637
100	2.465	0.945	0.622
200	2.431	0.911	0.599
400	2.389	0.869	0.572
800	2.324	0.804	0.529
1600	2.225	0.705	0.464
3200	2.115	0.595	0.391

The *e* vs. log σ plot is shown in Figure 7.12.

Example 7.2

The laboratory consolidation data for an undisturbed clay sample are as follows:

 $e_1 = 1.1$ $\sigma'_1 = 95 \text{ kN/m}^2$ $e_2 = 0.9$ $\sigma'_2 = 475 \text{ kN/m}^2$

What will be the void ratio for a pressure of 600 kN/m²? (*Note:* $\sigma'_c < 95$ kN/m².)

Solution

From Figure 7.13

$$C_c = \frac{e_1 - e_2}{\log \sigma_2' - \log \sigma_1'} = \frac{1.1 - 0.9}{\log 475 - \log 95} = 0.286$$
$$e_1 - e_3 = C_c (\log 600 - \log 95)$$



$$e_3 = e_1 - C_c \log \frac{600}{95}$$
$$= 1.1 - 0.286 \log \frac{600}{95} = 0.87$$

Example 7.3

A soil profile is shown in Figure 7.14. If a uniformly distributed load $\Delta \sigma$ is applied at the ground surface, what will be the settlement of the clay layer caused by primary consolidation? We are given that σ'_c for the clay is 125 kN/m² and $C_s = \frac{1}{6}C_c$.

Solution

The average effective stress at the middle of the clay layer is

$$\sigma'_{o} = 2.5\gamma_{\rm dry(sand)} + (4.5)[\gamma_{\rm sat(sand)} - \gamma_{w}] + \left(\frac{5}{2}\right)[\gamma_{\rm sat(clay)} - \gamma_{w}]$$

or

$$\sigma'_{o} = (2.5)(16.5) + (4.5)(18.81 - 9.81) + (2.5)(19.24 - 9.81)$$
$$= 105.33 \text{ kN/m}^{2}$$
$$\sigma'_{c} = 125 \text{ kN/m}^{2} > 105.33 \text{ kN/m}^{2}$$
$$\sigma'_{o} + \Delta \sigma' = 105.33 + 50 = 155.33 \text{ kN/m}^{2} > \sigma'_{c}$$





(*Note*: $\Delta \sigma = \Delta \sigma'$ at the end of consolidation.) So we need to use Eq. (7.17):

$$S_p = \frac{C_s H}{1 + e_0} \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1 + e_0} \log\left(\frac{\sigma'_o + \Delta \sigma'}{\sigma'_c}\right)$$

We have H = 5 m and $e_0 = 0.9$. From Eq. (7.18),

$$C_c = 0.009(LL - 10) = 0.009(50 - 10) = 0.36$$

 $C_s = \frac{1}{6}C_c = \frac{0.36}{6} = 0.06$

Thus,

$$S_p = \frac{5}{1+0.9} \left[0.06 \log\left(\frac{125}{105.33}\right) + 0.36 \log\left(\frac{105.33+50}{125}\right) \right]$$

= 0.1011 m \approx 101 mm

7.8 Settlement from Secondary Consolidation

Section 7.2 showed that at the end of primary consolidation (that is, after complete dissipation of excess pore water pressure) some settlement is observed because of the plastic adjustment of soil fabrics, which is usually termed *creep*. This stage of consolidation is called *secondary consolidation*. During secondary consolidation, the plot of deformation versus the log of time is practically linear (Figure 7.4). The variation of the void ratio *e* with time *t* for a given load increment will be similar to that shown in Figure 7.4. This variation is illustrated in Figure 7.15.



Figure 7.15 Variation of *e* with log *t* under a given load increment, and definition of secondary compression index