SHEAR STRENGTH OF SOIL

Definition:

1. The shear strength of soil is generally regarded as the resistance to deformation by continuous shear displacement of soil particles along surface of rupture.
2. It can also be defined as the ability of soil to withstand shear stresses.

General Discussion:

Knowledge of shear strength of soil is necessary for the solution of a large number of soil mechanics problems such as:

a. Stability of slopes.
b. Ultimate B.C of a Soil.
c. Lateral pressure against retaining walls.
d. Friction developed by piles.

The shear strength of a soil is basically made up of:

a. Frictional resistance to sliding between solid particles.
b. Cohesion and adhesion between soil particles.
c. Interlocking and bridging of solid particles to resist deformation.

BASIC CONCEPT

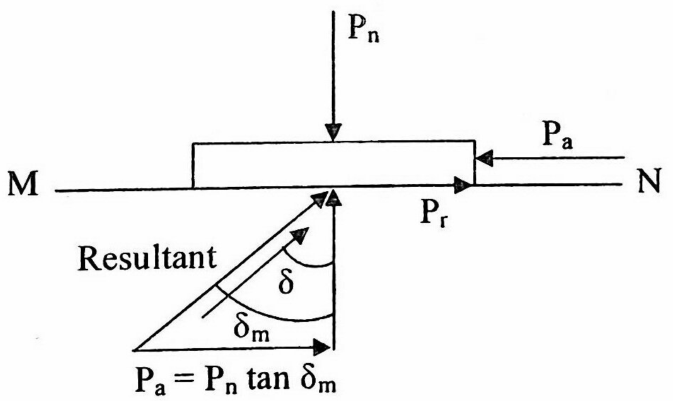


Fig. 1

Consider a metal block B resting on a plane surface MN as shown in Fig. 1. Block is acted upon by a force P_n normal to the plane surface and P_a is tangential to plane surface. The force P_a is resisted by an equal and opposite force P_r. The magnitude of force P_n is kept constant, where as P_a is gradually increased from zero to higher value such that the angle δ, which the resultant (of normal and tangential forces) makes with normal attains limiting value δ_m where sliding of the block occurs. The angle δ is known as "OBLIQUITY ANGLE" and

$\delta_m = \phi$ = angle of friction provided block and plane surface are of the same material

$\tan\phi$ = Coefficient of friction

A = Contact area of block with surface MN

Now

$$P_a = P_n \tan\phi \qquad \text{Eq-1}$$

Where

P_a = applied force = Shear force.

P_n = normal force.

Dividing both sides of eq. 1 by 'A' we get

$$P_a = P_n/A \tan\phi$$

$$\tau_f = \sigma_n \tan\phi \qquad \text{Eq-2}$$

Where

τ_f = Shearing stress at failure (Sliding)

σ_n = Normal stress.

If S = Maximum shearing stress at failure

= Maximum resistance which a material is capable of developing under applied system of stresses

Than

$$S = \sigma_n \tan\phi \qquad \text{Eq-3}$$

With the increase of normal stress σ_n, the shearing stress increases.

APPLICATION OF ABOVE INFORMATION TO SOILS:

For purely granular soils

$$S = \sigma_n \tan\phi$$

For soils other than purely granular soils

$$S = \sigma_n \tan\phi + C \qquad \text{Eq- 4}$$

Where

S = shearing strength

σ_n = normal stress.

C = Stress developed due to cohesive forces between the soil grains. Cohesion is the internal property of the material.

Cohesion is the shear strength of the material when applied normal stress is zero i.e.

$$S = C \qquad \text{Eq-5}$$

Equation 4 is called Coulomb's Equation or Coulomb's law. Based on Coulomb's law we have three types of soils

1. C- SOILS:

These are purely cohesive soils or pure clays. In saturated conditions these may have some value of ϕ otherwise $\phi = 0$.

i.e. $S = C$

Therefore c-soil is represented by a straight line (In σ_n Vs τ graph) having cohesion intercept with y-axis. See Fig.2a.

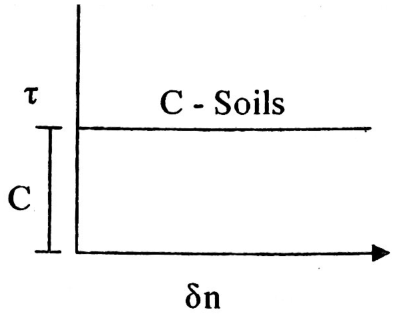


Fig. 2a.

2. C- ϕ SOILS:

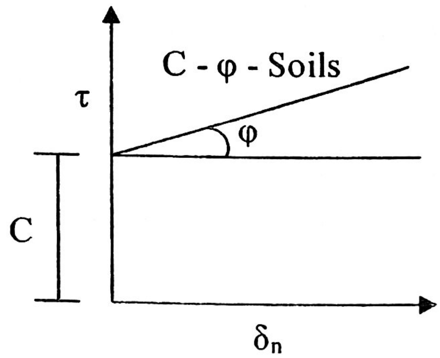


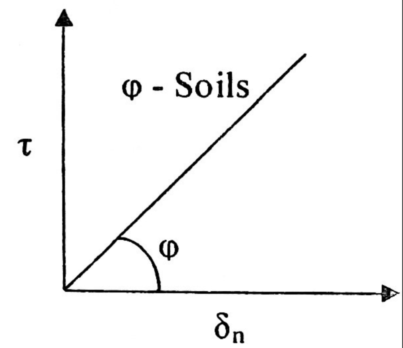
Fig. 2b.

These are silty soils and represent a line having cohesion intercept 'C', along with an angle of internal friction ϕ. The soil has properties intermediate between C and ϕ soils.

3. ϕ SOILS:

These are sandy soils having cohesion intercept equal to zero (Fig.2c)

$C = 0$ and $S = \sigma_n \tan\phi$.



For dense sands	ϕ	=	35°	to	46°
For loose sands	ϕ	=	28°	to	34°
For well graded	ϕ	=	up to	50° gravels	

ANGLE OF REPOSE:

This applies to loose sands. If dust free loose sand is poured on a horizontal surface from a small elevation, it will form a heap, as shown.

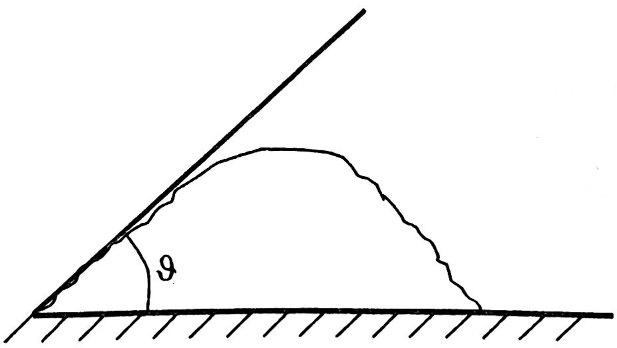


fig 3

It has been found by experiments that $\vartheta = \phi$ for loose sands ϕ can be found directly by heaping it. Where ϕ is the angle of internal friction.

PRINCIPAL PLANE:

A plane that is acted upon by a normal stress only is known as a principal plane. There is no tangential, or shear stress present.

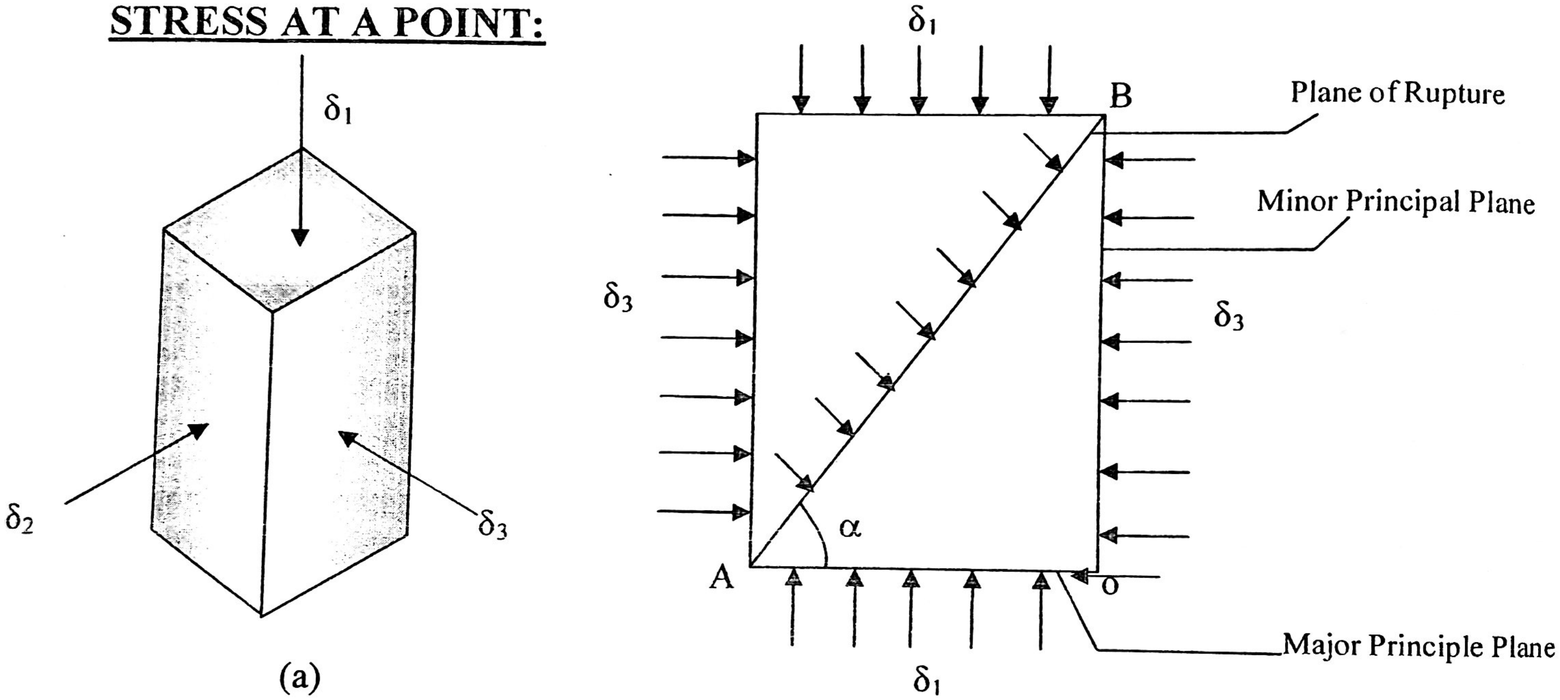
PRINCIPAL STRESS:

The normal stress acting on a principal plane is referred to as a principal stress.

At every point in a soil mass, the applied stress system that exists can be resolved into three principal stresses that are mutually orthogonal as shown in fig. 4a.

The principal planes corresponding to these principal stresses are called the major, intermediate and minor principal planes and are so named from a consideration of the relative magnitude of the stresses.

STRESS AT A POINT:



(a) Fig. 4 (b)

According to the fundamental law of mechanics, to any point there associates three mutually perpendicular planes.

Let σ_1, σ_2 and σ_3 are the three principal stresses acting on an element of soil as shown in Fig.4a.

In a 2-D case, the intermediate stress σ_2 is neglected i.e. $\sigma_2 = 0$. Thus we have a stress system in which major and minor principal stresses are acting, (i.e. σ_1 and σ_3) as shown in fig.4b. In the above figure OA is the major principal plane, OB is the minor principal plane and AB is the plane of rupture, which makes an angle of α with the major principal plane OA. On the plane of rupture normal stress σ_n is acting.

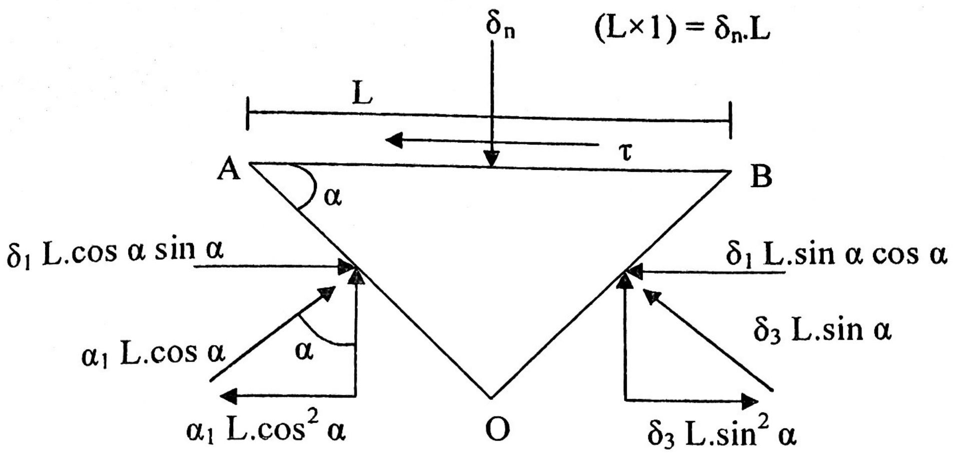


Fig. 5

Let us consider the free-body diagram of element OAB as shown in Fig.5. Let us assume that the thickness of the element of soil is one (1). Then the cross sectional areas of the planes are:

X-area of AB	=	$L \times 1$	=	L being length of plane
X-area of OA	=	$L \cos\alpha \times 1$	=	$L \cos\alpha$
X-area of OB	=	$L \sin\alpha \times 1$	=	$L \sin\alpha$

The forces due to stresses σ_1, σ_3 and σ_n are

Force due to σ_1	=	$\sigma_1 L \cos\alpha$
Force due to σ_3	=	$\sigma_3 L \sin\alpha$
Force due to σ_n	=	$\sigma_n L$

Resolving these forces and the applying equilibrium conditions we get,

$$\Sigma Fx = 0$$

$$\boldsymbol{\sigma_1 L \cos\alpha \sin\alpha - \sigma_3 L \cos\alpha \sin\alpha = \tau L}$$

OR

$$\tau = (\sigma_1 - \sigma_3) \cos\alpha \sin\alpha$$

Now

$$\text{Sin}2\alpha = 2\text{Sin}\alpha\text{Cos}\alpha \quad \text{or}$$

$$\frac{\text{Sin}2\alpha}{2} = \text{Sin}\alpha\text{Cos}\alpha \quad \text{or}$$

$$\boldsymbol{\tau = \frac{(\sigma_1 - \sigma_3)}{2} \text{Sin}2\alpha} \qquad \textbf{Eq-6}$$

$$\Sigma Fy = 0$$

$$\sigma_1 L \cos^2\alpha + \sigma_3 L \sin^2\alpha = \sigma_n L$$

OR

$$\sigma_n = \sigma_1 \cos^2\alpha + \sigma_3 \sin^2\alpha$$

Now

$$\text{Sin}^2\alpha + \text{Cos}2\alpha = 1 \quad \text{or} \quad \text{Sin}^2\alpha = 1 - \text{Cos}^2\alpha$$

Therefore

$$\sigma_n = \sigma_1 \cos^2\alpha + \sigma_3 (1 - \text{Cos}^2\alpha)$$

$$= \sigma_1 \cos^2\alpha + \sigma_3 - \sigma_3 \text{Cos}^2\alpha.$$

$$= \sigma_3 + (\sigma_1 - \sigma_3) \cos^2\alpha.$$

Hence

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)\,Cos^2\alpha$$

Also

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3).\left(\frac{1 + Cos2\alpha}{2}\right)$$

$$= \sigma_3 + \left(\frac{\sigma_1 - \sigma_3}{2}\right) + (\sigma_1 - \sigma_3).\frac{Cos2\alpha}{2}$$

$$\sigma_n = \sigma_3 + \left(\frac{\sigma_1 - \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right).Cos2\alpha$$

$$= \left(\frac{2\sigma_3 + \sigma_1 - \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right).Cos2\alpha$$

$$= \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right).Cos2\alpha \qquad \text{Eq-7}$$

THE MOHR'S CIRCLE DIAGRAM

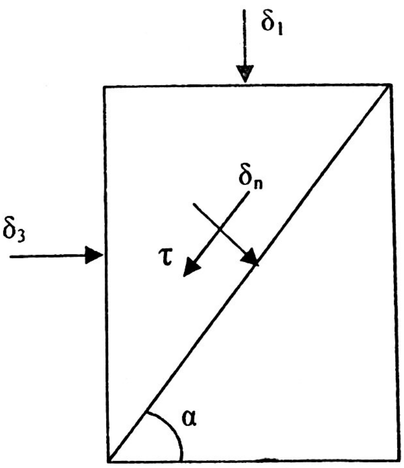


fig. 6

Fig. 6 shows a major principal plane, acted upon by a major principal stress, σ_1 and a minor principal plane, acted upon by a minor principal stress, σ_3.

By considering the equilibrium of an element within the stresses mass Fig. 6 it can be shown that on any plane, inclined at angle α to the direction of the major principal plane, there is a shear stress, τ and a normal stress, σ_n. The magnitudes of these stresses are

$$\tau = \frac{(\sigma_1 - \sigma_3)}{2} \sin 2\alpha$$

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3) \cos^2\alpha = \frac{(\sigma_1 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)}{2} \cos 2\alpha$$

These equations may be represented graphically by an extremely useful device known as: MOHR'S CIRCLE FOR STRESS.

$$\sigma_n - \frac{(\sigma_1 + \sigma_3)}{2} = \frac{(\sigma_1 - \sigma_3)}{2} \cos 2\alpha$$

$$\tau = \frac{(\sigma_1 - \sigma_3)}{2} \sin 2\alpha$$

Squaring both sides of each equation we get

$$\left[\sigma_n - \frac{(\sigma_1 + \sigma_3)}{2}\right]^2 = \frac{(\sigma_1 - \sigma_3)^2}{2} \cos^2 2\alpha$$

$$\tau^2 = \frac{(\sigma_1 - \sigma_3)^2}{2} \sin^2 2\alpha$$

Hence adding two equations:

$$\left[\sigma_n - \frac{(\sigma_1 + \sigma_3)}{2}\right]^2 + \tau^2 = \frac{(\sigma_1 - \sigma_3)^2}{2} \qquad (\sin^2 2\alpha + \cos^2 2\alpha = 1)$$

This is equation of the circle of the form

$$(\sigma_n - a)^2 + \tau^2 = R^2$$

Where the radius is

$$R = \sqrt{[(\sigma_1 - \sigma_3)/2]^2} = (\sigma_1 - \sigma_3)/2$$

And $a = (\sigma_1 + \sigma_3)/2$. The center of the circle lies at a point $[(\sigma_1 + \sigma_3)/2, 0]$. The center of the circle always lies on the σ_n axis.

CONSTRUCTION OF MOHR'S CIRCLE:

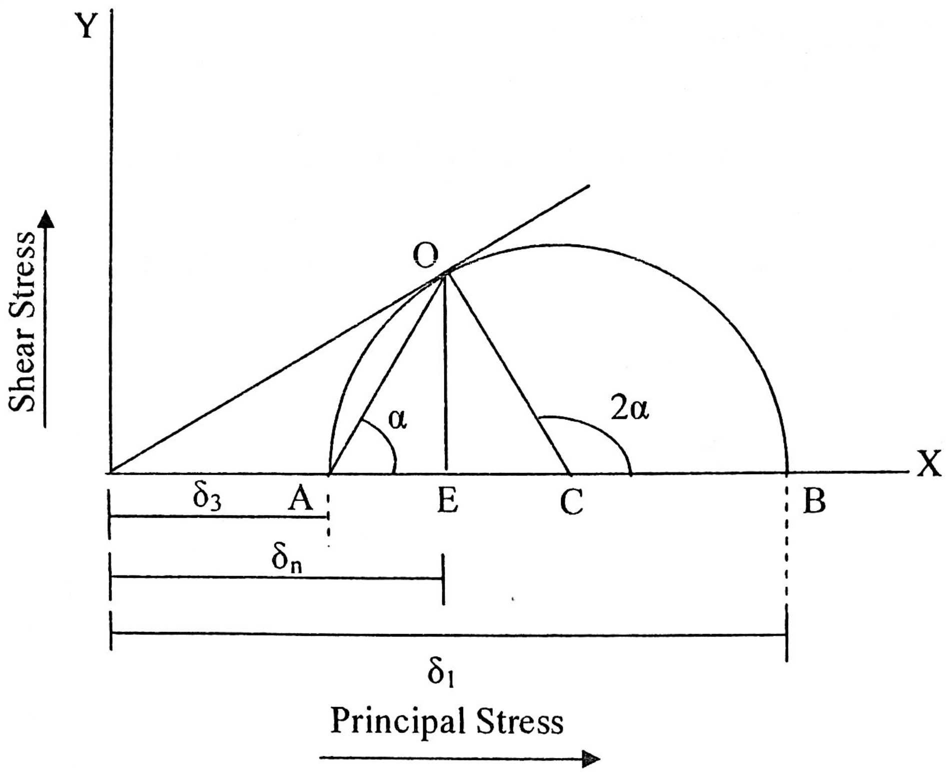


Fig- 7

In order to draw a Mohr's Circle diagram a specific convention must be followed:

1- All normal stresses being plotted along the axis OX, while shear stresses are plotted along the axis OY. For most cases the axis OX is horizontal and OY is vertical.

2- The direction of the major principal stress is parallel to axis OY i.e. the direction of the major principal plane is parallel to axis OX.

3- The tensile stresses are considered positive (Plotted to the right of the origin); the compressive stresses are considered negative (Plotted to the left of the origin). However in soil mechanics it is customary to indicate compressive stresses as positive.

4- The shear stresses are plotted as vertical coordinates. Positive shear stresses are plotted above the origin; negative shear stresses are plotted below the origin.

5- Positive angles on the circle are obtained when measured in the counter clock wise sense; negative angles on the circle are obtained in the clockwise sense. An angle of 2α on the circle corresponds to an angle of α on the element.

To draw the diagram, first lay down the axes OX and OY, then set off OA and OB along the OX axis to represent the magnitudes of the minor and major principal stresses respectively, and finally construct the circle with diameter AB. This circle is the locus of stress conditions for all planes passing through the point A. i.e. a plane passing through A and inclined to the major principal plane at angle α cuts the circle at D. The coordinates of point D are the normal and shear on the plan (Fig. 7).

PROOF:

Normal stress = σ_n = OE = OA + AE

= σ_3 + AD Cosα

= σ_3 + AB Cos$^2\alpha$

= σ_3 + $(\sigma_1 - \sigma_3)$ Cos$^2\alpha$

Shear Stress = τ = DE = DC Sin $(180-2\alpha)$

= DC Sin2α

= $[(\sigma_1 - \sigma_3)/2]$ Sin2α

In Fig. 7 OE and DE represent the normal and shear stress components of the complex stress acting on the plane AD. From the triangle of forces ODE it can be seen that this complex stress is represented in the diagram by the line OD, whilst the angle DOB represents the angle of obliquity (β) of the resultant stress on plane AD.

Shear Strength Parameters (c & φ):

The Shear Strength of a soil is a measure of its resistance to deformation by continuous displacement of its individual soil particles. Soil shear strength is an important consideration in foundation bearing capacity analysis, highway and airfield design and construction, slope stability of earth embankments, and retaining wall construction.

The shear strength of a soil is derived from three basic components:

1. Resistance to displacement because of interlocking of the individual soil particles.
2. Resistance to particle translation because of friction between individual soil particles at their common points of contact.
3. Cohesion between the surfaces of the soil particles. Which of these components, or combinations of components, are actually effective in resisting shear deformation depends on whether the soil is cohesive or cohesion less, and on the soil drainage and consolidation conditions before and during the shearing process.

The first hypothesis on soil shear strength was presented by Coulomb. Coulomb hypothesized that the shear strength of a soil was dependent on the two components, cohesion and friction:

$$S = C + \sigma_n \tan\varphi$$

Where S = Shear Strength (force / Area)

C =Stress on the critical plane(force/Area) or soil shear strength Soil Cohesion (force / Area)

φ = angle of internal friction, degrees

σn = normal can be determined in the field; however, it is often accomplished in the laboratory using one of three common laboratory testing methods:

1. Triaxial shear test.
2. Un-confined compression test.
3. Direct shear test.