

accepted. That is, there is no significant difference between the average daily milk yield of dairy farm and the previous record.

10.3. Two Sample Test: Case (i)

Assumptions: 1. Populations are normal
2. Samples are drawn independently and at random.

Conditions: 1. σ is known.
2. Sizes of samples may be small or large.

Null hypothesis: $\mu_1 = \mu_2$ where μ_1, μ_2 are the population means for the 1st and 2nd populations respectively.

$$Z = \frac{|\bar{X}_1 - \bar{X}_2|}{\sqrt{\sigma^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

where \bar{X}_1, \bar{X}_2 are the means of 1st and 2nd samples with sizes n_1, n_2 respectively.

CONCLUSION: If Z (calculated) $\geq Z$ (tabulated), the null hypothesis is rejected. There is significant difference between two sample means. In other words, the two samples have come from two different populations having two different means. Otherwise, the null hypothesis is accepted.

10.3. Case (ii): In this case the common population S.D. is not known.

Assumptions: 1. Populations are normal.
2. Samples are drawn independently and at random.

Conditions: 1. σ is not known.
2. Sizes of samples are large.

Null hypothesis: $\mu_1 = \mu_2$

$$Z = \frac{|\bar{X}_1 - \bar{X}_2|}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

where $S_1^2 = \frac{1}{n_1} \sum (X_1 - \bar{X}_1)^2$, $S_2^2 = \frac{1}{n_2} \sum (X_2 - \bar{X}_2)^2$

and \bar{X}_1, \bar{X}_2 are the means of 1st and 2nd samples with sizes n_1, n_2 respectively.

CONCLUSION: If Z (calculated) $\geq Z$ (tabulated) at chosen level of significance, the null hypothesis is rejected. Otherwise, it is accepted.

EXAMPLE: A random sample of 90 poultry farms of one variety gave an average production of 240 eggs per bird/year with a S.D. of 18 eggs. Another random sample of 60 poultry farms of another variety gave an average production of 195 eggs per bird/year with a S.D. of 15 eggs. Distinguish between two varieties of birds with respect to their egg production.

Null hypothesis: $\mu_1 = \mu_2$

$$Z = \frac{|240 - 195|}{\sqrt{\frac{(18)^2}{90} + \frac{(15)^2}{60}}} = 16.61$$

CONCLUSION: Z (calculated) $> Z$ (tabulated), 1.96 at 5 per cent level of significance. Here there is significant difference between two varieties of birds with respect to egg production.

10.4. Student's t-distribution

In small samples drawn from a normal population, the ratio of the difference between the sample and population means to its estimated standard error follows a distribution known as t-distribution, where

$$t = \frac{|\bar{X} - \mu|}{\frac{s}{\sqrt{n}}} \quad \text{where } s^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$$

10.4.1. Properties of t-distribution: This distribution is symmetrical about the origin, and unimodal and extends from $-\infty$ to $+\infty$ in both directions. It is known as student's t-distribution, the name 'Student' being the Pen name of W.S. Gosset.

For large n , this distribution tends to standard normal distribution having zero mean and unit variance.

The moments μ_r of the distribution exist only for $r < (n-1)$. All odd order moments are zero by symmetry. The even moments are given by