CHAPTER 10

TESTS OF HYPOTHESES

10.1. Introduction

The estimate based on sample values do not equal to the true value in the population due to inherent variation in the population. The samples drawn will have different estimates compared to the true value. It has to be verified that whether the difference between the sample estimate and the population value is due to sampling fluctuation or real difference. If the difference is due to sampling fluctuation only it can be safely said that the sample belongs to the population under question and if the difference is real we have every reason to believe that sample may not belong to the population under question.

Type I error: Rejecting the hypothesis when it ought to be accepted.

Type II error: Accepting the hypothesis when it ought to be rejected.

10.1.1. Statistical Significance: The probability of the difference between sample estimate and the true value taken with standard error compared with observed difference is very small then we say that there is significant difference between the sample estimate and the population value. That is, the probability is less for the difference between sample estimate and the true value (or population value) not due to sampling fluctuation but due to real difference.

If the above probability is very large then we say that there is no significant difference between the sample estimate and the true value. The difference so obtained is due to sampling fluctuation only.

10.1.2. Levels of Significance: The maximum probability at which we would be willing to risk a type I error is known as the level of significance. In general 5 per cent and 1 per cent are taken as 'levels of significance' thereby indicating that on an average we may go wrong 5 out of 100 cases and 1 out of 100 cases respectively. To say that 5 per cent level of significance, there is 95 per cent confidence in the result with a margin of error 5 per cent. The 5 per cent level of significance is shown in Fig. 10.1.

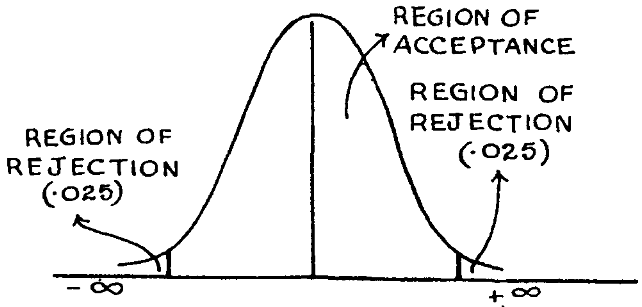

Fig. 10.1. Normal curve.

10.1.3. Degrees of Freedom: It is defined as the difference between the total number of items and the total number of constraints.

If n is the total number of items and k, the total number of constraints then the degrees of freedom (d.f.) is given by d.f. = n—k

Suppose we want to select 10 values with a restriction that the total of the ten values should be equal to 100. Thus, we can select only 9 items at our choice but the tenth item must be chosen in such a way that the total of them should be equal to 100. Therefore, degrees of freedom of selecting 10 items is only 9 with one constraint.

10.1.4. Null Hypothesis: Null hypothesis is the statement about parameters which is likely to be rejected after testing. We start with the hypothesis that the two items are equal.

10.1.5. Standard Normal Deviate Tests: If X follows normal distribution with mean, μ and S.D., σ then $\bar{X}$ also follows normal distribution with mean μ and S.D., $\frac{\sigma}{\sqrt{n}}$. This can be denoted by

$$\bar{X} \rightarrow N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \text{ i.e., } \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \longrightarrow N(0, 1)$$

the expression on the left hand side is called as standard normal variate (or standard normal deviate) which follows normal distribution with mean zero and standard deviation unity. The test of hypothesis based on this deviate is called standard normal deviate test. The confidence limits for the population mean can be obtained as

$$\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

i.e., the population mean, lies between $\bar{X}-1.96 \frac{\sigma}{\sqrt{n}}$ and $\bar{X}+1.96 \frac{\sigma}{\sqrt{n}}$. where 1.96 is the tabulated value of normal distribution at 5 per cent level of significance.

10.2. One Sample Test: Case (i)

Assumptions: 1. Population is normal.

2. The sample is drawn at random.

Conditions: 1. Population S.D., σ is known.

2. Size of the sample may be small or large.

Null hypothesis: $\mu=\mu_0$

$$Z=\frac{|\bar{X}-\mu_0|}{\sigma/\sqrt{n}}$$

we know that 'Z' values follow normal distribution with zero mean and unit S.D. The values of Z on either side of normal curve corresponding to the areas 0.025 and 0.025 are

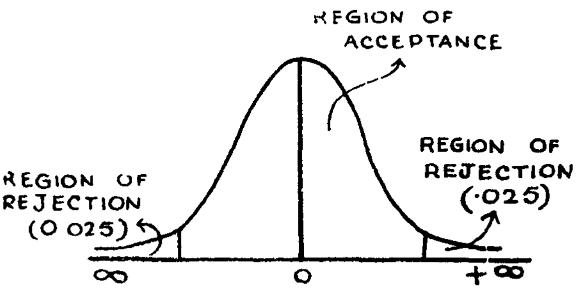

Fig. 10.2. Standard normal curve,