3. Magnetic Field, $\vec{B}$ due to a Toroid:

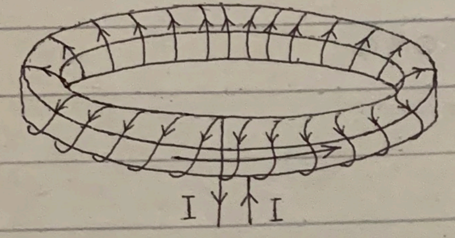
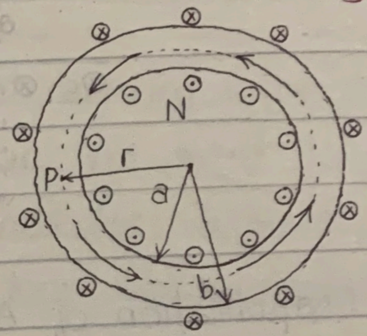


Fig. A Toroid.

Fig. A cut-away view of the ring and the winding.



Let, 'N' be the total number of turns of toroid, and 'I' be the current flowing through each turn of the toroid. The inner and outer radii of the ring are 'a' and 'b', respectively. A cut-away view of the ring and the winding is shown in the figure above. The dot ⊙ indicates the current coming out of the turns and the cross ⊗ indicates the current going into the turns of the toroid.

In the region, $r<a$;

$$\boxed{B=0} \quad ; r<a$$

since, $I_{enc}=0$

In the region, $a<r<b$;

Applying Ampere's Circuit law,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\text{As, } \oint \vec{B}\cdot d\vec{l} = \oint B\,dl\cos 0^\circ = B\oint dl = B\,2\pi r$$

and $I_{enc} = NI$

$$\therefore \quad B\,2\pi r = \mu_0 NI$$

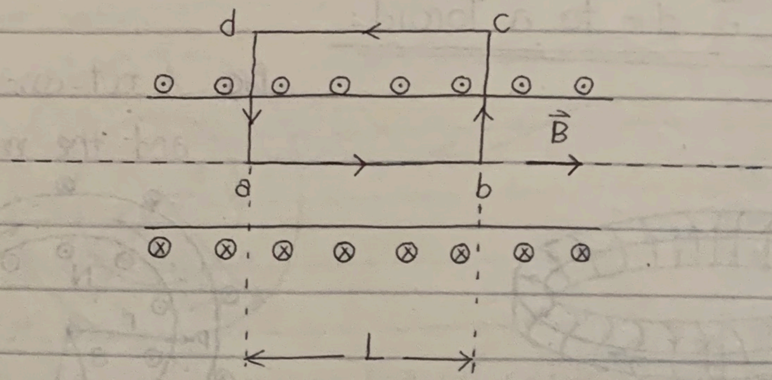
$$\Rightarrow \boxed{B = \frac{\mu_0 NI}{2\pi r}} \quad ; a<r<b$$

In the region, $r>b$;

$$I_{enc} = NI + (-NI) = 0$$

$$\therefore \boxed{B=0} \quad ; r>b$$

4- Magnetic Field, $\vec{B}$ due to a Solenoid :



As a direct application of Ampere's Circuital law, it is clear that there is no magnetic field outside of the solenoid (actually very weak B-field). To determine the $\vec{B}$-field inside, we construct a rectangular contour 'C' of length 'L', that is partially inside and partially outside the solenoid.

The total magnetic field along the path is

$$\oint_C \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \underbrace{\int_b^c \vec{B}\cdot d\vec{l}}_{=0} + \int_c^d \vec{B}\cdot d\vec{l} + \underbrace{\int_d^a \vec{B}\cdot d\vec{l}}_{=0}$$

As, $\vec{B}$ is perpendicular to the sides bc and da.
The third integral on the right hand side also vanishes as $\vec{B}$ is approximately zero, outside the solenoid, so we are left with

$$\oint_C \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} = \int_a^b B\,dl\cos 0^\circ = B\int_a^b dl$$

$$= BL$$

Let, 'n' be the number of turns per unit length of the solenoid and 'I' is the current flowing through each turn of the solenoid. Thus the current enclosed by path abcda is 'nLI'.

Applying Ampere's Circuital Law,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc.}$$

$$B\cancel{L} = \mu_0 n \cancel{L} I$$

$$\Rightarrow \boxed{B = \mu_0 n I} \equiv \text{Magnetic Field in the core of the solenoid.}$$