Ampere's Circuital Law:

This law states that "the line integral of $\vec{B}$ around any closed path is equal to 'μ_0' times the total current flowing through the surface bounded by the path."

Mathematically,

$$\oint_C \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_0 I \equiv \text{integral form of Ampere's law}$$

Since, the current can be expressed in terms of volume current density $\vec{J}(\vec{r})$, as

$$I = \int_S \vec{J}(\vec{r}) \cdot d\vec{a}$$

$$\therefore \oint_C \vec{B}(\vec{r}) \cdot d\vec{l}' = \mu_0 \int_S \vec{J}(\vec{r}) \cdot d\vec{a}'$$

where the path 'C' for the line integral is the contour bounding the surface 'S', and 'I' is the total current through 'S'.

Applying Stoke's theorem to the L.H.S of the above equation, we get

$$\int_S (\vec{\nabla} \times \vec{B}(\vec{r})) \cdot d\vec{a}' = \int_S \mu_0 \vec{J}(\vec{r}) \cdot d\vec{a}'$$

As, 'S' can be any arbitrary open surface bounded by a closed contour 'C', so we can write

$$\boxed{\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})}$$

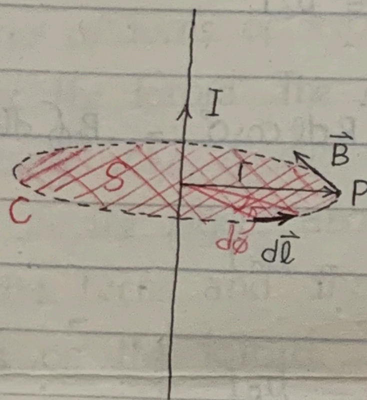
Differential form of Ampere's law for static magnetic fields

Applications of Ampere's Circuital Law:

1. Magnetic Field, $\vec{B}$ due to an infinitely long, straight current carrying wire:

Consider an infinitely long, straight current carrying wire through which current 'I' is flowing in the upward direction. We want to find the magnetic field, $\vec{B}$ at point 'P', whose perpendicular distance from wire is 'r'. For this, we draw a circular Amperian loop whose radius is 'r'.

open surface, S bounded by contour, C



$\vec{B}(\vec{r}) = B(\vec{r})\,\hat{\phi}$

$d\vec{l} = dl\,\hat{\phi} \Rightarrow \vec{B}(\vec{r})\cdot d\vec{l} = B(\vec{r})\,dl\,\underbrace{\hat{\phi}\cdot\hat{\phi}}_{=1}$

According to Ampere's Circuital Law,

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$$

As, $\oint_C \vec{B}(\vec{r})\cdot d\vec{l} = \oint_C B\,dl\cos 0^\circ = B\oint_C dl = B\,2\pi r$

$\oint_C dl = \int_0^{2\pi} r\,d\phi$

has same value at every point lying on the circular Amperian loop, i.e., constant and thus can be taken out of the integral.

$$\therefore \quad B\,2\pi r = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}} \qquad \Rightarrow \vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}$$

The direction of $\vec{B}$ is along the tangent at any point of the circular path.

2. Magnetic Field, $\vec{B}$ due to a coaxial cable:

Consider a coaxial cable consisting of a small center cable conductor of radius 'a' and a coaxial cylindrical outer cable conductor of radius 'b', as shown in the Fig. Assume that the two conductors carry equal total currents of magnitude 'I' in opposite directions.

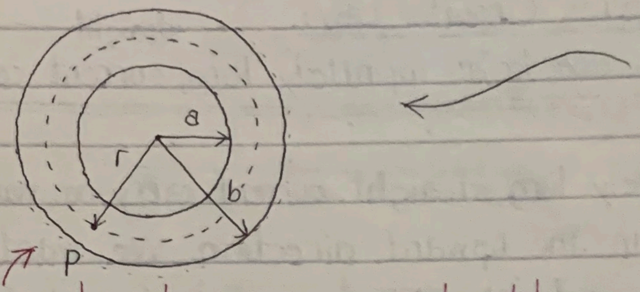


Fig. A Cross-sectional view of coaxial cable.

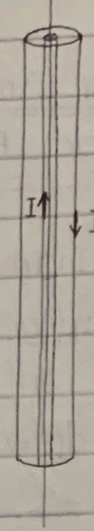


Fig. Coaxial cable

For $a<r<b$;

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$$

As, $\oint_C \vec{B}\cdot d\vec{l} = \oint_C B\,dl\cos 0^\circ = B\oint_C dl = B\,2\pi r$

$$\therefore \quad B\,2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Its direction is given by the right hand rule.

For $r>b$;

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$$

As, the total current enclosed by the Amperian loop

$$= I + (-I) = 0$$

$$\therefore \quad \oint_C \vec{B}\cdot d\vec{l} = 0$$

$$\Rightarrow \quad \boxed{B = 0}$$

i.e., there is no magnetic field outside the coaxial cable.

<:: End of Lecture # 44 :>