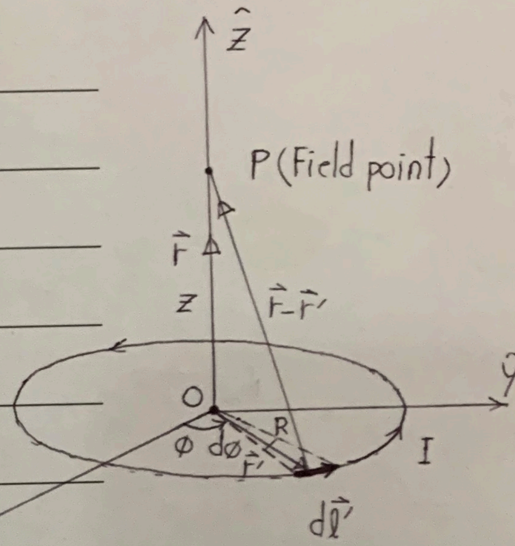
Notes and Reflections

Magnetic Field due to circular current Loop:



$$\vec{r} = z\hat{z}$$

$$\vec{r}' = R\hat{r}$$

$$\vec{r}-\vec{r}' = z\hat{z} - R\hat{r}$$

$$|\vec{r}-\vec{r}'| = (z^2+R^2)^{1/2}$$

$$d\vec{l}' = R\,d\phi\,\hat{\phi}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

= magnetic field due to current element $I d\vec{l}'$, at point 'P'.

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \oint_c \frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

= magnetic field due to entire circular current loop, at point 'P'

'c' is the entire length of the loop.

$$d\vec{l}' \times (\vec{r}-\vec{r}') = R\,d\phi\,\hat{\phi} \times (z\hat{z} - R\hat{r})$$

$$= Rz\,d\phi\,(\hat{\phi} \times \hat{z}) - R^2 d\phi\,(\hat{\phi} \times \hat{r})$$

$$= Rz\,d\phi\,\hat{r} + R^2 d\phi\,\hat{z}$$

$$\therefore \vec{B}(\vec{r}) = \underbrace{\frac{\mu_0 I}{4\pi} \oint_c \frac{Rz\,d\phi}{(z^2+R^2)^{3/2}}\,\hat{r}}_{=0} + \frac{\mu_0 I}{4\pi} \oint_c \frac{R^2 d\phi}{(z^2+R^2)^{3/2}}\,\hat{z}$$

$\hat{r}$: not a constant unit vector.

$\hat{z}$: constant unit vector

$$\hat{r} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y} \quad \text{(in cylindrical coordinates)}$$

Note: $\hat{r}$ doesn't mean a unit vector that represents the direction of $\vec{r}$ here; because in this particular case, a unit vector representing the direction of $\vec{r}$ is $\hat{z}$. $\hat{r}$ is the usual radial unit vector in cylindrical coordinates.

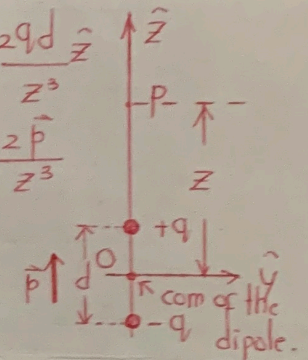
$$\therefore \frac{\mu_0 I}{4\pi}\oint_C \frac{Rz\,d\phi}{(z^2+R^2)^{3/2}}\,\hat{r} = \frac{\mu_0 I}{4\pi}\oint_C \frac{Rz\,d\phi}{(z^2+R^2)^{3/2}}(\cos\phi\,\hat{x}+\sin\phi\,\hat{y})$$

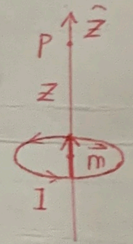
$$= \frac{\mu_0 I R z}{4\pi(z^2+R^2)^{3/2}}\hat{x}\underbrace{\int_0^{2\pi}\cos\phi\,d\phi}_{=0} + \frac{\mu_0 I R z}{4\pi(z^2+R^2)^{3/2}}\hat{y}\underbrace{\int_0^{2\pi}\sin\phi\,d\phi}_{=0}$$

$$= 0$$

$$\therefore \vec{B}(\vec{r}) = \frac{\mu_0 I R^2}{\cancel{4\pi}\,2(z^2+R^2)^{3/2}}\hat{z}\underbrace{\int_0^{2\pi}d\phi}_{=\cancel{2\pi}}$$

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{2qd}{z^3}\hat{z} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{z^3}$$





$$\vec{B}(z) = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{z^3}$$

$$\underset{\vec{r}=z\hat{z}}{\vec{B}(\vec{r})} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}\hat{z} \equiv \vec{B}(z)$$

In the limit $z \to 0$ (Field point lies on the center of the circular current loop)

$$\vec{B} = \lim_{z\to 0}\frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}\hat{z} = \frac{\mu_0 I}{2R}\hat{z} \quad \text{(At the center of the loop)}$$

Let, $z >> R$;

$$(z^2+R^2)^{-3/2} = z^{-3}\Big(1+\underbrace{\frac{R^2}{z^2}}_{<<<1}\Big)^{-3/2}$$

(so, we can apply Binomial expansion)

$$= z^{-3}\Big(1-\underbrace{\frac{3R^2}{2z^2}}+\cdots\Big) \cong z^{-3}$$

neglecting the terms involving $\left(\frac{R}{z}\right)^2, \left(\frac{R}{z}\right)^4$ and so on

$$\therefore \vec{B}(z) = \frac{\mu_0 I R^2}{2z^3}\hat{z} = \frac{\mu_0}{2\pi}\frac{I(\pi R^2)\hat{z}}{z^3} = \frac{\mu_0}{4\pi}\,2\,\frac{I(\pi R^2)\hat{z}}{z^3}$$

$$I(\pi R^2)\hat{z} = I\vec{A} = \vec{m} \text{ (Magnetic dipole moment).}$$

$$\underbrace{\vec{B}(z) = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{z^3}}\quad \text{(on the axis)}$$

Look like electric dipole : $\vec{p} = q\vec{d}$

Also recall ; $\vec{\tau}_B = \vec{m}\times\vec{B}$; $\vec{\tau}_E = \vec{p}\times\vec{E}$ $\quad U_B = -\vec{m}\cdot\vec{B}$; $U_E = -\vec{p}\cdot\vec{E}$

$\Rightarrow$ current loops are "magnetic dipoles".