22. General Equilibrium Theory

A. INTERDEPENDENCE IN THE ECONOMY

In the preceding chapters we have adopted a *partial equilibrium approach*, concentrating on decisions in a particular segment of the economy in isolation of what was happening in other segments, under the *ceteris paribus* assumption.

We examined the utility-maximising behaviour of a household under the assumption that its income was given, although income depends on the amount of labour and other factors of production that the consumer owns and on their prices (wage, rental of capital, etc.). The *ceteris paribus* assumption was useful in that it enabled us to study the individual's demand for different commodities in isolation from influences arising from other parts of the economy.

We studied the production decision of a firm on the assumption that factor prices, the state of technology and the prices of commodities were given. The *ceteris paribus* assumption allowed us to study the cost-minimisation behaviour of a firm in isolation from such factors as the demands for the products, which in turn are influenced by the level of employment, income and tastes of consumers.

Product markets, where buyers and sellers interacted with each other and among themselves to determine prices and levels of outputs of various commodities, were studied under the *ceteris paribus* assumption; relationships with other markets were ignored.

Factor markets, where firms and households as owners of productive resources interacted with each other and among themselves to determine prices and quantities of various factors employed, were also analysed on the basis of the *ceteris paribus* assumption. The interrelationship between the various factor markets and commodity markets were left out of the analysis.

In summary, the basic characteristic of a partial equilibrium approach[1] is the determination of the price and quantity in each market by demand and supply curves drawn on the *ceteris paribus* clause. Each market in the Marshallian methodology is regarded independently of the others.

However, a fundamental feature of any economic system is the interdependence among its constituent parts. The markets of all commodities and all productive factors are interrelated, and the prices in all markets are simultaneously determined. For example, consumers' demands for various goods and services depend on their tastes and incomes. →In turn, consumers' incomes depend on the amounts of resources they own and factor prices. →Factor prices depend on the demand and

[1] The partial equilibrium approach is also known as 'Marshallian approach' after Alfred Marshall, who used it as his basic method of analysis in his *Principles of Economics* (Macmillan, 1920).

supply of the various inputs. →The demand for factors by firms depends not only on the state of technology but also on the demands for the final goods they produce. →The demands for these goods depend on consumers' incomes, which, as we saw, depend on the demand for the factors of production. This circular interdependence of the activity within an economic system can be illustrated with a simple economy composed of two sectors, a consumer sector, which includes households and a business sector, which includes firms.[1] It is assumed that: (*a*) all production takes place in the business sector; (*b*) all factors of production are owned by the households;[2] (*c*) all factors are fully employed; (*d*) all incomes are spent.

The economic activity in the system takes the form of two flows between the consumer sector and the business sector: a real flow and a monetary flow (figure 22.1).

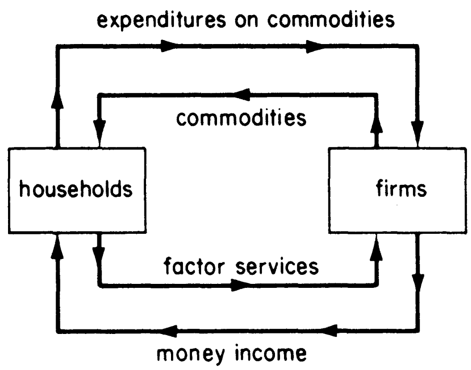

Figure 22.1 Circular flows in a two-sector economy

The *real flow* is the exchange of goods for the services of factors of production: firms produce and offer final goods to the household sector, and consumers offer to firms the services of factors which they own.

The *monetary flow* is the real flow expressed in monetary terms. The consumers receive income payments from the firms for offering their factor services. These incomes are spent by consumers for the acquisition of the finished goods produced by the business sector. The expenditures of firms become the money incomes of the households. Similarly, the expenditures of households become the receipts of firms, which they once again pay the households for the factor services which they supply.

The real flow and the monetary flow, which represent the transactions and the interdependence of the two sectors, move in opposite directions. They are linked by the prices of goods and factor services. The economic system is in equilibrium when a set of prices is attained at which the magnitude of the income flow from firms to households is equal to the magnitude of the money expenditure flow from households to firms.[3]

The interdependence of markets is concealed by the partial equilibrium approach.

[1] The government sector and the foreign sector are excluded from this simple model.

[2] This excludes the production of intermediate goods, i.e. goods produced by some firms and used by other firms as inputs.

[3] In fact, those two streams of payments represent the two traditional ways of measurement of an economy's total income. The payment of incomes by firms to households represents 'the income approach'. The payment of expenditures by households to firms represents the 'product approach'.

Markets consist of buyers and sellers. Thus an economic system consists of millions of economic decision-making units who are motivated by self-interest. Each one pursues his own goal and strives for his own equilibrium independently of the others. In traditional economic theory the goal of a decision-making agent, consumer or producer, is maximisation of something. The consumer maximises satisfaction subject to a budget constraint. The firm maximises profit, subject to the technological constraint of the production function. A worker determines his supply of labour by maximising satisfaction derived from work–leisure opportunities, subject to a given wage rate.

The problem is to determine whether the independent, self-interest motivated behaviour of economic decision-makers is consistent with each individual agent's attaining equilibrium. All economic units, whether consumers, producers, or suppliers of factors, are interdependent. General equilibrium theory deals with the problem of whether the independent action by each decision-maker leads to a position in which equilibrium is reached by all. A general equilibrium is defined as a state in which all markets and all decision-making units are in simultaneous equilibrium. A general equilibrium exists if each market is cleared at a positive price, with each consumer maximising satisfaction and each firm maximising profit. The scope of general equilibrium analysis is the examination of how this state can, if ever, be reached, that is, how prices are determined simultaneously in all markets, so that there is neither excess demand nor excess supply, while at the same time the individual economic units attain their own goals.

The interdependence between individuals and markets requires that equilibrium for all product and factor markets as well as for all participants in each market must be determined simultaneously in order to secure a consistent set of prices. General equilibrium emerges from the solution of a simultaneous equation model, of millions of equations in millions of unknowns. The unknowns are the *prices* of all factors and all commodities and the *quantities* purchased and sold (of factors and commodities) by *each* consumer and each producer. The equations of the system are derived from the maximising behaviour of consumers and producers, and are of two types: *behavioural equations* describing the demand and supply functions in all markets by all individuals, and *clearing-the-market equations.*

In principle a simultaneous-equation system has a solution if the number of *independent equations* is equal to the number of unknowns in the system. This approach has been followed by the founder of general equilibrium analysis Léon Walras, whose system we will outline in the next section.

B. THE WALRASIAN SYSTEM

The most ambitious general equilibrium model was developed by the French economist Léon Walras (1834–1910). In his *Elements of Pure Economics*[1] Walras argued that *all* prices and quantities in *all* markets are determined simultaneously through their interaction with one another. Walras used a system of simultaneous equations to describe the interaction of individual sellers and buyers in all markets, and he maintained that all the relevant magnitudes (prices and quantities of all commodities and all factor services) can be determined simultaneously by the solution of this system.

In the Walrasian model the behaviour of each individual decision-maker is presented by a set of equations. For example, each consumer has a double role: he buys

[1] Léon Walras, *Élements d'Économie politique pure* (Lausanne, 1874). First translated in English by William Jaffé (Allen & Unwin, 1954).

commodities and sells services of factors to firms. Thus for each consumer we have a set of equations consisting of two subsets: one describing his demands of the different commodities, and the other his supplies of factor inputs. Similarly, the behaviour of each firm is presented by a set of equations with two subsets: one for the quantities of commodities that it produces, and the other for the demand for factor inputs for each commodity produced. The important characteristic of these equations is their simultaneity or interdependence. The solution of this system of millions of simultaneous equations defines the 'unknowns' of the model, namely the prices and quantities of all commodities and all factor inputs.

In a general equilibrium system of the Walrasian type there are as many markets as there are commodities and factors of production.

For each market there are three types of functions: demand functions, supply functions and a 'clearing-the-market' equation, which stipulates that the quantities demanded be equal to the quantities supplied.

In a commodity market the number of demand functions is equal to the number of consumers, and the number of the supply functions is equal to the number of firms which produce the commodity.

In each factor market the number of demand functions is equal to the number of firms multiplied by the number of commodities they produce. The number of supply functions is equal to the number of consumers who own (*ex hypothesi*) the factors of production.

A necessary (but not sufficient) condition for the existence of a general equilibrium is that there must be in the system as many independent equations as the number of unknowns. Thus the first task (in establishing the existence of a general equilibrium) is to describe the economy by means of a system of equations, defining how many equations are required to complete (and solve) the system.

For example, assume that an economy consists of two consumers, A and B, who own two factors of production, K and L. These factors are used by two firms to produce two commodities, X and Y. (This is the simple $2 \times 2 \times 2$ general equilibrium model which we will examine in detail in the following section.) It is assumed that each firm produces one commodity, and each consumer buys some quantity of both. It is also assumed that both consumers own some quantity of both factors (but the distribution of ownership of factors is exogenously determined). In this simple model we have the following 'unknowns':

quantities demanded of X and Y by consumers	$2 \times 2 = 4$
quantities supplied of K and L by consumers	$2 \times 2 = 4$
quantities demanded of K and L by firms	$2 \times 2 = 4$
quantities of Y and X supplied by firms	2
prices of commodities Y and X	2
prices of factors K and L	2
Total number of 'unknowns'	18

To find these unknowns we have the following number of equations:

demand functions of consumers	$2 \times 2 = 4$
supply functions of factors	$2 \times 2 = 4$
demand functions for factors	$2 \times 2 = 4$
supply functions of commodities	2
clearing-the-market of commodities	2
clearing-the-market of factors	2
Total number of equations	18

Since the number of equations is equal to the number of unknowns, one should think that a general equilibrium solution exists. Unfortunately, the equality of numbers of equations and unknowns is neither a sufficient nor a necessary condition for the existence of a solution. As we will see in section E and in the Appendix, in the Walrasian system one of the equations is not independent of the others: there is a 'redundant equation' in the system which deprives the system of a solution, since the number of unknowns is larger than the number of independent equations. In this model the absolute level of prices cannot be determined. General equilibrium theorists have adopted the device of choosing arbitrarily the price of one commodity as a *numéraire* (or unit of account) and express all other prices in terms of the price of the *numéraire*. With this device prices are determined only as ratios: each price is given relative to the price of the *numéraire*. (See section E below.) If we assign unity to the price of the *numéraire*, we attain equality of the number of simultaneous equations and unknown variables (the number of unknowns is reduced to 17 in our example). However, the absolute prices are still not determined: they are simply expressed in terms of the *numéraire*. This indeterminacy can be eliminated by the introduction explicitly in the model of a money market, in which money is not only the *numéraire*, but also the medium of exchange and store of wealth. (See Appendix, section III.)

Even if there is equality of independent equations and unknowns, there is no guarantee that a general equilibrium solution exists. The proof of the existence of a general equilibrium solution is difficult. Léon Walras was never able to prove the existence of a general equilibrium. In 1954 Arrow and Debreu[1] provided a proof of the existence of a general equilibrium in perfectly competitive markets, in which there are no indivisibilities and no increasing returns to scale. Furthermore, in 1971 Arrow and Hahn[2] proved the existence of a general equilibrium for an economy with limited increasing returns and monopolistic competition, without indivisibilities. Both proofs are limited to specific market structures and are based on restrictive assumptions, regarding in particular the necessity of 'well-behaved' continuous production and demand functions. Thus the available 'existence proofs' do not hold for the typical real world cases of discontinuities and indivisibilities in production processes. Our current state of knowledge does not enable us to be sure of the existence of a general equilibrium in the real world, which is dominated by oligopolistic firms and production processes which are characterised by indivisibilities. However, the proof of the existence of general equilibrium for a perfectly competitive economy (with no indivisibilities and no increasing returns to scale) is very important, because a perfectly competitive system has certain ideal properties: it results in an *efficient* allocation of resources, as we will see in section E and in Chapter 23.

Apart from the existence problem, two other problems are associated with an equilibrium: the problem of its stability and the problem of its uniqueness. We will illustrate the nature of these problems in the following section. After this digression we will outline the *dynamic process* of reaching general equilibrium in perfectly competitive markets, using basic geometrical analysis. In section E we will present a diagrammatic treatment of the simple two-factor, two-commodity, two-consumer general equilibrium model, concentrating on the *static properties* of such equilibrium in perfectly competitive markets. In the Appendix to this chapter we will extend the general equilibrium model to include any number of factors, commodities and consumers, and we will discuss the problem of introducing money in the Walrasian system.

[1] K. J. Arrow and G. Debreu, 'Existence of an Equilibrium for a Competitive Economy', *Econometrica* (1954), vol. 22, pp. 265–90.

[2] K. J. Arrow and F. H. Hahn, *General Competitive Analysis* (Oliver & Boyd, 1971).

C. EXISTENCE, UNIQUENESS AND STABILITY OF AN EQUILIBRIUM

Three problems arise in connection with a general equilibrium:

1. Does a general equilibrium solution exist? (Existence problem.)
2. If an equilibrium solution exists, is it unique? (Uniqueness problem.)
3. If an equilibrium solution exists, is it stable? (Stability problem.)

These problems can best be illustrated with the partial-equilibrium example of a demand–supply model. Assume that a commodity is sold in a perfectly competitive market, so that from the utility-maximising behaviour of individual consumers there is a market demand function, and from the profit-maximising behaviour of firms there is a market supply function. An equilibrium exists when at a certain positive price the quantity demanded is equal to the quantity supplied. The price at which $Q_D = Q_S$ is the equilibrium price. At such a price there is neither excess demand nor excess supply. (The latter is often called *negative excess demand.*) Thus an equilibrium price can be defined as the price at which the excess demand is zero: the market is cleared and there is no excess demand.

The equilibrium is stable if the demand function cuts the supply function from above. In this case an excess demand drives price up, while an excess supply (excess negative demand) drives the price down (figure 22.2).

The equilibrium is unstable if the demand function cuts the supply function from below. In this case an excess demand drives the price down, and an excess supply drives the price up (figure 22.3).

In figure 22.4 we depict the case of multiple equilibria. It is obvious that at P_1^e there is a stable equilibrium, while at P_2^e the equilibrium is unstable. Finally in figure 22.5 an equilibrium (at a positive price) does not exist.

It should be clear from the above discussion that (*a*) the existence of equilibrium is related to the problem of whether the consumers' and producers' behaviour ensures

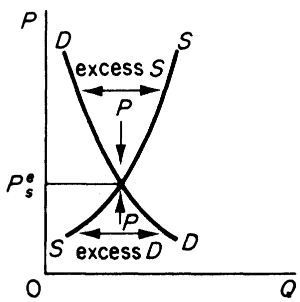

Figure 22.2 Unique, stable equilibrium

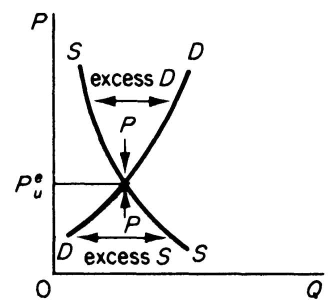

Figure 22.3 Unique, unstable equilibrium

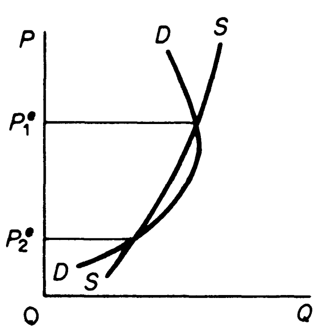

Figure 22.4 Multiple equilibria

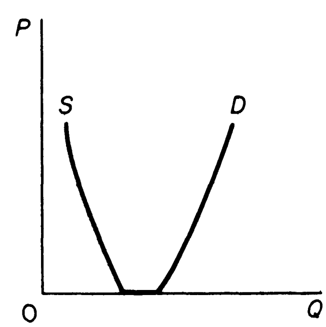

Figure 22.5 No equilibrium exists

that the demand and supply curves intersect (at a positive price); (*b*) the stability of equilibrium depends on the relationship between *the slopes* of the demand and supply curves; (*c*) the uniqueness of equilibrium is related to *the slope of the excess demand function*, that is, the curve which shows the difference between Q_D and Q_S at any one price.

In fact the three basic questions related to the existence, stability and uniqueness of an equilibrium can be expressed in terms of the excess demand function:

$$E_{(P_i)} = Q_{D(P_i)} - Q_{S(P_i)}$$

To see this we redraw below figures 22.2–22.5 in terms of the excess demand function. For each of these cases we have derived the relevant excess demand function by subtracting Q_S from Q_D at all prices.

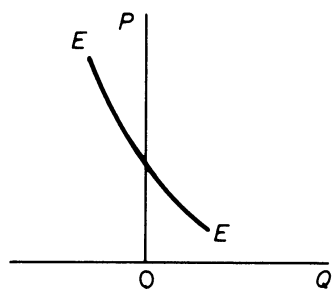

Figure 22.6 Stable equilibrium: slope of $E_{(P)} < 0$

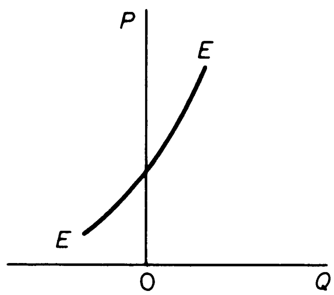

Figure 22.7 Unstable equilibrium: slope of $E_{(P)} > 0$

From the redrawn diagrams (in conjunction with the corresponding ones 22.2–22.5) we can draw the following conclusions.

1. The excess demand function, $E_{(P)}$, intersects the vertical (price)-axis when there is an equilibrium, that is, when the excess demand is zero. If $Q_D = Q_S$, then $E_{(P)} = 0$.
2. There are as many equilibria as the number of times that the excess demand curve $E_{(P)}$ intersects the vertical price-axis (figure 22.8).
3. The equilibrium is stable if the slope of the excess demand curve is negative at the point of its intersection with the price-axis (figure 22.6).
4. The equilibrium is unstable if the slope of the excess demand curve is positive at the point of its intersection with the price-axis (figure 22.7).
5. If the excess demand function does not intersect the vertical axis at any one price, an equilibrium does not exist (figure 22.9).

The above analysis of the existence, stability and uniqueness in terms of excess demand functions can be extended to general equilibrium analysis.[1]

[1] See E. Roy Weintraub, *General Equilibrium Theory*. Macmillan Studies in Economics (Macmillan, 1974).

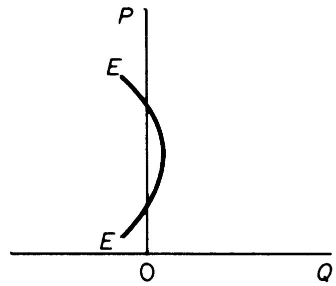

Figure 22.8 Multiple equilibria

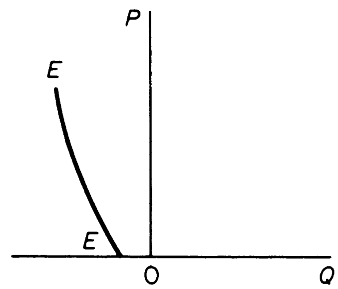

Figure 22.9 No equilibrium exists

D. A GRAPHICAL ILLUSTRATION OF THE PATH TO GENERAL EQUILIBRIUM

In this section we attempt to show how a simple economy with perfectly competitive production and factor markets will have an inescapable tendency toward a general equilibrium solution.

Assumptions

1. Two substitute commodities are produced, X and Y, by two perfectly competitive industries (markets).
2. There are two factors, capital K and labour L, whose markets are perfectly competitive. The quantities of these factors are given (fixed supply).
3. The production functions are continuous, with diminishing marginal rate of factor substitution and decreasing returns to scale.
4. The industry producing X is less capital intensive than the industry producing Y. The K/L ratio is smaller in industry X.
5. Consumers maximise utility and firms maximise profit.
6. The usual assumptions (of large numbers, homogeneous products and factors, and free entry and exit) of perfect competition hold.
7. The system is initially in equilibrium: in all markets demand is equal to supply at a positive equilibrium price.

Assume that an exogenous change in consumers' tastes shifts the demand for X outwards to the right, from D_0 to D_1, causing the price, in the short run, to rise from P_0 to P_1^x and the quantity sold to increase by $X_0 X_1$ units (figure 22.10).

Since X and Y are substitutes (*ex hypothesi*)[1] one should expect the increase in the demand for X to be accompanied by a decrease in the demand for Y.[2] The demand for Y (figure 22.12) shifts to the left, its price drops and the quantity of Y sold decreases by $Y_1 Y_0$ units. (Note that with the assumption of given K and L, the economy cannot produce simultaneously increased quantities of both X and Y. Hence we must assume that an increase in the demand for X is accompanied by a decrease in the demand for Y: there cannot be an increase in the demand for X without a corresponding concurrent decrease in the demand for Y, unless we allow for inflation.)

The increase in P_x creates excess profits for the producers of X (figure 22.11) and losses for the producers of Y (figure 22.13). Firms are thus induced to divert resources from the production of Y to the production of X. This is shown by the movement

[1] The two commodities cannot be complementary in the $2 \times 2 \times 2$ model.

[2] Since the factors of production are given, a simultaneous increase of both X and Y cannot be dealt without complicating the analysis with inflationary phenomena.

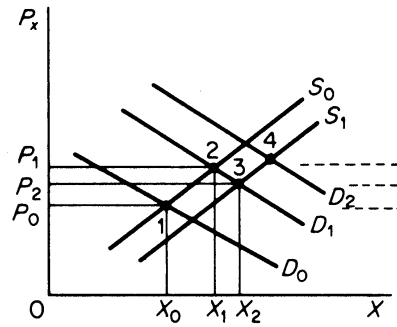

Figure 22.10 Industry X

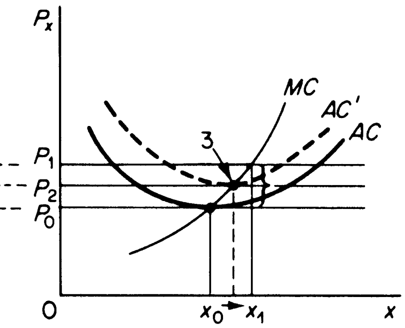

Figure 22.11 A firm in industry X

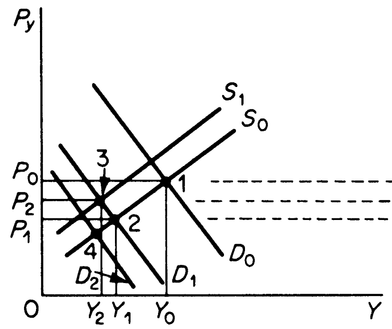

Figure 22.12 Industry Y

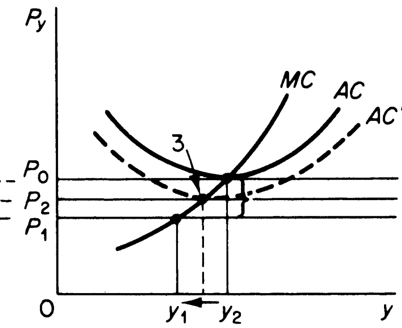

Figure 22.13 A firm in industry Y

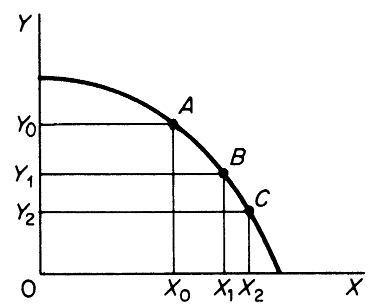

Figure 22.14 Production possibility curve

from point A to point B on the production possibility curve (figure 22.14). This shift reflects the effect of the change in consumers' tastes on the decisions of firms. The increase in P_x induces the producers of X to increase their quantity in order to maximise their profit, given the increase in marginal revenue. (The reaction of a typical firm in industry X is shown in figure 22.11.) Since every firm in the X industry faces the same price the output of each firm increases (each firm produces on the rising part of its AC). The sum of the increases in outputs of *existing* firms is equal to the increase $X_0 X_1$ in figure 22.10.

In industry Y the opposite occurs. The fall in P_y induces firms to decrease their quantity. The reaction of a typical firm in industry Y is shown in figure 22.13. The sum of the decreases in output of the individual firms is equal to the decline $Y_0 Y_1$ in figure 22.12.

In the long run excess profits attract entry in industry X and induce exit of firms from industry Y. Entry and exit affect the demand for factors of production. The markets for labour and capital used in the production of X are shown in figures 22.15 and 22.17.

The expansion of production by existing firms and the entry of new firms in industry X increases the demand for labour and capital. The D_{L_x} and D_{K_x} curves shift outwards and w_x and r_x rise. Employment of these factors rises in industry X (by $L_0 L_1$ and $K_0 K_1$ respectively in figures 22.15 and 22.17). The demand for L and K by a single firm in industry X is shown in figures 22.16 and 22.18. The individual firm can buy any quantity of L and K at the prevailing market price. At the increased price w_1^x the firm will demand l_1 of labour (figure 22.16), and at the increased price r_1^x the firm will demand k_1 of capital (figure 22.18).

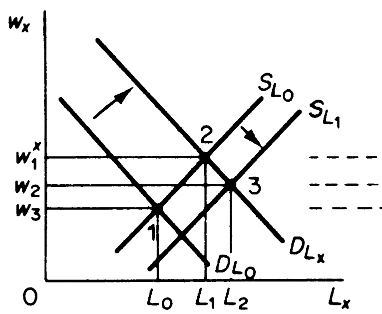

Figure 22.15
Labour market for industry X

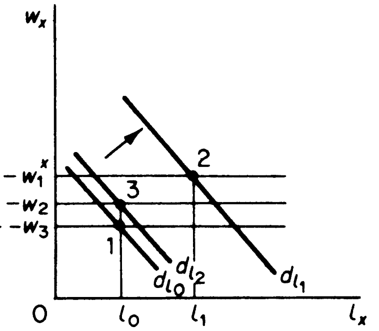

Figure 22.16 Demand for labour by a firm in industry X

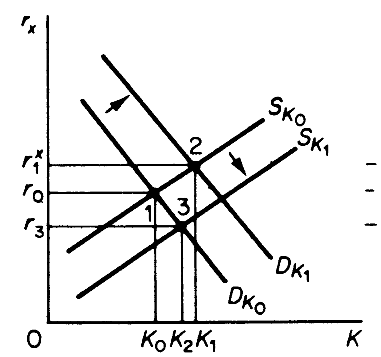

Figure 22.17 Market for capital in industry X

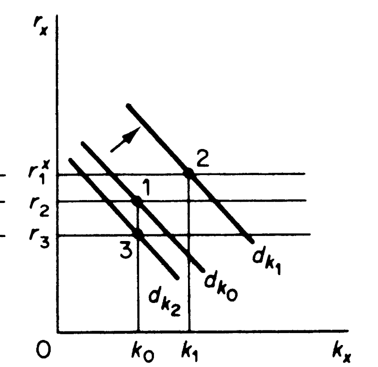

Figure 22.18 Demand for capital by a firm in industry X

The situation in the labour and capital markets for the industry Y will be the reverse. The surviving firms in this industry will reduce their demand for both factors, while the exit of firms in the long run will further reduce the demand for these inputs. The situation for the factor markets in industry Y is shown in figures 22.19 and 22.21. The case of a single firm in industry Y is depicted in figures 22.20 and 22.22.

The above change in the demands for factors in the two industries leads to disequilibrium, because the prices of L and K have risen in the factor markets for industry X, while w and r have fallen in the factor markets of industry Y. However, in perfect factor markets the disequilibrium will be self-correcting, since in the long run there is perfect mobility of factors between the different markets. Thus the owners of L and K will withdraw their services from the Y industry and will seek to have them employed by firms in the X industry where w and r are higher.

The above reactions (mobility) of factor suppliers will result in an upward shift of the supply curve of factors in industry Y and a downward shift of the supply curves of factors in industry X. The shifts are shown in figures 22.15, 22.17, 22.19 and 22.21.

With the assumption that X is more capital intensive than commodity Y the prices of factors will not return to their original levels. w and r will be equalised in the two industries, but the wage level will be higher in the new equilibrium while the price of capital r will be lower in the final equilibrium (this is shown by the point 3 in figures 22.15, 22.17, 22.19, 22.21). The assumption of different factor intensities $((K/L)_x < (K/L)_y)$ has the following repercussions. The demand of the X industry for labour is stronger than the demand for capital. The release of labour by industry Y is smaller than the rate required by X, while the release in capital is larger than the increased need for this factor by industry X. Thus overall the demand for labour will be higher than initially and w will rise. The opposite will occur in the market for capital, where the new equilibrium r will be lower than in the initial situation.

As a result of the new factor prices the individual firms in each industry will adjust their demands for labour and capital. In figures 22.16, 22.18, 22.20 and 22.22 the individual firms are in equilibrium at the points denoted by the digit (3).

The entry on firms in industry X shifts the supply curve downwards to S_1 (figure 22.10). The final equilibrium price of X is lower than in the short run, but higher than in the original equilibrium. The X industry is an increasing cost industry.

The exit of firms from industry Y shifts the supply of this industry upwards (figure 22.12). The final equilibrium price P_2 is higher than the short-run price P_1, but lower than the initial equilibrium level P_0. Industry Y is also an increasing cost industry.

Given the new product prices the individual firms will adjust (independently) their outputs. Firms in industry X will be producing an output lower than in the short run, but higher than in the initial equilibrium (figure 22.11). Firms in industry Y will be producing an output higher than in the short run, but lower than in the initial situation (figure 22.13). The changes in w and r cause the LAC of firms in X to shift

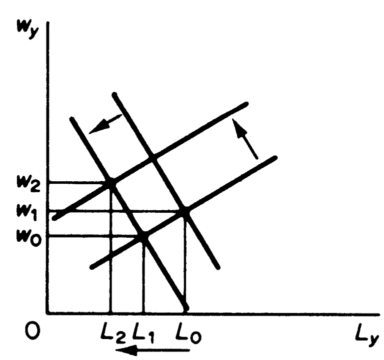

Figure 22.19
Labour market for industry Y

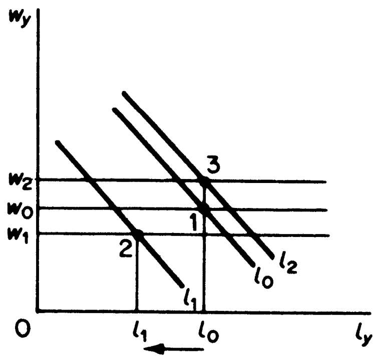

Figure 22.20 Demand for labour by a firm in industry Y

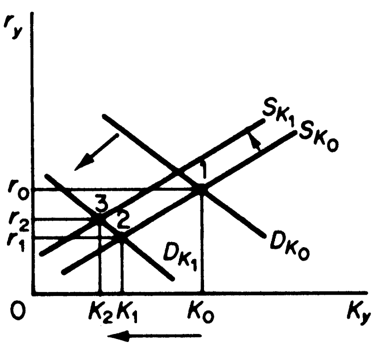

Figure 22.21 Market for capital for industry Y

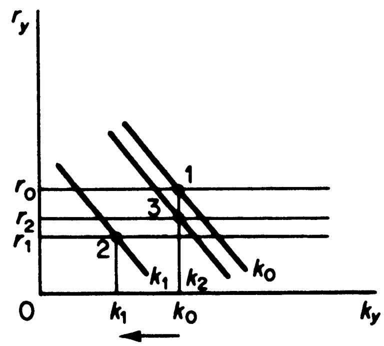

Figure 22.22 Demand for capital by a firm in industry Y

upwards (X is more labour intensive, and w has risen), and the LAC of firms in Y to shift downward (Y is more capital intensive, and r has fallen). Thus in the long run the firms earn just normal profits.

It should be noted that the above-described path towards a general equilibrium would be modified if, apart from the *exogenous* changes, we take into account possible *induced* changes, generated by the adjustments in the system. For example, in our simple model we saw that in the new equilibrium wages were higher and the price of capital lower than initially. If we accept the fact that wage earners have a higher propensity to consume than other factor owners, there will be a further (induced) increase in the demand for X, shifting the demand curve to D_2 in figure 22.10. From this point we can start tracing the additional changes in all other markets, as previously.

The above discussion is not a formal proof of the existence or stability of a general equilibrium. However, the assumption of perfect competition was adequate to enable us to see that the shifts in the supply and demand curves are not random, but precisely interrelated.[1] The curves will continue to shift so long as disequilibria exist. When an equilibrium is attained simultaneously in all markets and among all economic decision-makers within the markets, we say that a general equilibrium exists throughout the entire economy, and the movement of curves will cease until another exogenous force disturbs the system.

In summary: We started from an initial equilibrium, which was disturbed by an *exogenous* change in tastes. We traced the interactions of markets and individual decision-makers, in perfect product and factor markets. The system under our assumptions generated reactions which led to a new equilibrium. It should be stressed that this effect is certain only with perfectly competitive markets and continuous production functions with decreasing returns to scale. If the system is not perfectly competitive we cannot be sure that a general equilibrium will be attained. The changes in initial conditions will be reflected in the various markets, but the economic signals are likely to be weaker. For example, in imperfect competition firms produce where $P > MC$. A shift in the demand for X will tend to increase production in this industry, but given $P > MC$, factors will tend to be underemployed in the X industry, and a rise in wages may create disequilibrium in the resource markets, leading to further disequilibria elsewhere in the system.

E. A GRAPHICAL TREATMENT OF THE TWO-FACTOR, TWO-COMMODITY, TWO-CONSUMER ($2 \times 2 \times 2$) GENERAL EQUILIBRIUM MODEL

In this section we use graphical analysis to show the general equilibrium of a simple economy in which there are two factors of production, two commodities (each produced by a firm) and two consumers. This is known as the $2 \times 2 \times 2$ general equilibrium model.[2]

Throughout this section we will restrict our analysis to the perfectly competitive market system, since with free competition it has been proved that a general equilibrium solution exists (given some additional assumptions about the form of the production and demand functions).[3] Furthermore we will be concerned with the *static*

[1] The problems arising from discontinuities and increasing returns to scale in perfectly competitive markets will be discussed in Chapter 23.

[2] For a rigorous treatment of the $2 \times 2 \times 2$ Model see H. G. Johnson, *Two-Sector Model of General Equilibrium* (Aldine Press, New York, 1971).

[3] See section C above and the Appendix to this chapter.

properties of general equilibrium and not with the dynamic process of reaching the state of such an equilibrium, the latter having been sketched in the preceding section.

1. The assumptions of the 2 × 2 × 2 model

1. There are two factors of production, labour (L) and capital (K), whose quantities are given exogenously. These factors are homogeneous and perfectly divisible.
2. Only two commodities are produced, X and Y. Technology is given. The production functions of the two commodities are represented by two isoquant maps, with the usual properties. The isoquants are smooth and convex to the origin, implying diminishing marginal rate of factor (technical) substitution along any isoquant. Each production function exhibits constant returns to scale. Finally, it is assumed that the two production functions are independent: there are no external economies or diseconomies for the production activity of one product arising from the production of the other.[1]
3. There are two consumers in the economy, A and B, whose preferences are represented by ordinal indifference curves, which are convex to the origin, exhibiting diminishing marginal rate of substitution between the two commodities. It is assumed that consumer choices are independent: the consumption patterns of A do not affect B's utility, and vice versa. Bandwagon, snob, Veblenesque and other 'external' effects are ruled out.[2] Finally, it is assumed that the consumers are sovereign, in the sense that their choice is not influenced by advertising or other activities of the firms.
4. The goal of each consumer is the maximisation of his own satisfaction (utility), subject to his income constraint.
5. The goal of each firm is profit maximisation, subject to the technological constraint of the production function.
6. The factors of production are owned by the consumers.
7. There is full employment of the factors of production, and all incomes received by their owners (A and B) are spent.
8. There is perfect competition in the commodity and factor markets. Consumers and firms pursue their goals faced by the same set of prices (P_x, P_y, w, r).

In this model a general equilibrium is reached when the four markets (two commodity markets and two factor markets) are cleared at a set of equilibrium prices (P_x, P_y, w, r) and each participant economic agent (two firms and two consumers) is simultaneously in equilibrium. The general equilibrium solution thus requires the determination of the values of the following variables:

The total quantities of the two commodities X and Y, which will be produced by firms and bought by the consumers.

The allocation of the given K and L to the production of each commodity (K_x, K_y, L_x, L_y).

The quantities of X and Y which will be bought by the two consumers (X_A, X_B, Y_A, Y_B).

The prices of commodities (P_x and P_y) and of the factors of production (wage w, and rental of capital r).

The distribution of factor ownership between the two consumers (K_A, K_B, L_A, L_B). The quantities of factors multiplied by their prices define the income distribution between A and B, and hence their budget constraint.

[1] The effects of externalities in production are discussed in Chapter 23.

[2] The effects of externalities in consumption are discussed in Chapter 23.

2. Static properties of a general equilibrium state (configuration)

Three static properties are observed in a general equilibrium solution, reached with a free competitive market mechanism:

(*a*) Efficient allocation of resources among firms (equilibrium of production).

(*b*) Efficient distribution of the commodities produced between the two consumers (equilibrium of consumption).

(*c*) Efficient combination of products (simultaneous equilibrium of production and consumption).

These properties are called *marginal conditions of Pareto optimality* or *Pareto efficiency*. A situation is defined as Pareto optimal (or efficient) if it is impossible to make anyone better-off without making someone worse-off. The concept of Pareto optimality will be discussed in detail in Chapter 23. In the following paragraphs we discuss briefly the three optimality properties that are observed in a general equilibrium state.

(a) Equilibrium of production (efficiency in factor substitution)

Equilibrium of production requires the determination of the efficient distribution of the available productive factors *among the existing firms* (efficiency in factor substitution).

From Chapter 3 we know that the firm is in equilibrium if it chooses the factor combination (for producing the most lucrative level of output) which minimises its cost. Thus the equilibrium of the firm requires that

$$\begin{bmatrix}\text{slope of}\\ \text{isoquant}\end{bmatrix} = \begin{bmatrix}\text{slope of}\\ \text{isocost}\end{bmatrix}$$

or

$$MRTS_{L,K} = \frac{w}{r}$$

where w and r are the factor prices prevailing in the market and $MRTS$ is the marginal rate of technical substitution between the factors.

The joint equilibrium of production of the two firms in our simple model can be derived by the use of the Edgeworth box of production.[1] On the axes of this construct we measure the given quantities of the factors of production, $\bar{K}$ and $\bar{L}$ (figure 22.23). The isoquants of commodity X are plotted with origin the south-west corner and the isoquants of Y are plotted with origin the north-east corner. The locus of points of tangency of the X and Y isoquants is called the *Edgeworth contract curve of production*.[2] This curve is of particular importance because it includes the efficient allocations of K and L between the firms.

Each point of the Edgeworth box shows a specific allocation of K and L in the production of commodities X and Y. Such an allocation defines six variables: the

[1] A detailed description of the construction of the Edgeworth box of production is given in Chapter 3, p. 100.

[2] In constructing the Edgeworth box we have assumed that X is less capital intensive than Y. If the K/L ratio were the same for the two products the contract curve would be a straight line.

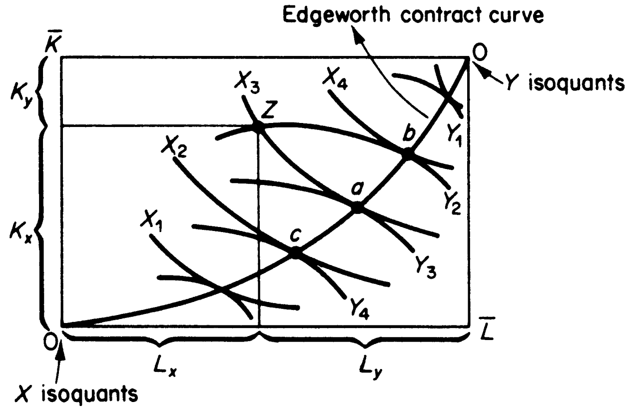

Figure 22.23 Edgeworth box of production

amounts of Y and X produced and the amounts of capital and labour allocated to the production of Y and X. For example point Z shows that:

X_3 is the quantity produced of commodity X
Y_2 is the quantity produced of commodity Y
K_x is the amount of capital allocated to the production of X_3
K_y is the amount of capital allocated to the production of Y_2
L_x is the amount of labour allocated to the production of X_3
L_y is the amount of labour allocated to the production of Y_2

However, not all points of the Edgeworth box represent efficient allocations of the available resources. Given that K and L are limited in supply, their use should produce the greatest possible output. An allocation of inputs is efficient if the produced combination of X and Y is such that it is impossible to increase the production of one commodity without decreasing the quantity of the other.[1] From figure 22.23 we see that efficient production takes place *on* the Edgeworth contract curve. It is impossible to move to a point off this curve without reducing the quantity of at least one commodity. Point Z is a point of inefficient production, since a reallocation of K and L between the two commodities (or firms) such as to reach any point from a to b leads to a greater production of one or both commodities.

Since the Edgeworth contract curve of production is the locus of tangencies of the X and Y isoquants, at each one of its points the slopes of the isoquants are equal:

$$\begin{bmatrix}\text{slope of} \\ X \text{ isoquant}\end{bmatrix} = \begin{bmatrix}\text{slope of} \\ Y \text{ isoquant}\end{bmatrix}$$

or

$$MRTS^{x}_{L,K} = MRTS^{y}_{L,K}$$

In our simple general equilibrium model the firms, being profit maximisers in competitive markets, will be in equilibrium only if they produce somewhere *on* the Edgeworth contract curve. This follows from the fact that the factor prices facing the

[1] The above definition of efficiency is also known as *Pareto efficiency* or *Pareto optimality* after the name of the Italian economist Vilfredo Pareto (*Cours d'Économie politique* (Lausanne, 1897)). This concept is further discussed in Chapter 23.

producers are the same, and their profit maximisation requires that each firm equates its $MRTS_{L,K}$ with the ratio of factor prices w/r:

$$MRTS^x_{L,K} = MRTS^y_{L,K} = \frac{w}{r} \qquad (1)$$

In summary. The general equilibrium of production occurs at a point where the $MRTS_{L,K}$ is the same for all the firms, that is, at a point which satisfies the Pareto-optimality criterion of efficiency in factor substitution: the general equilibrium of production is a Pareto-efficient allocation of resources. The production equilibrium is not unique, since it may occur at any point along the Edgeworth contract curve: there is an infinite number of possible Pareto-optimal production equilibria. However, with perfect competition, one of these equilibria will be realised, the one at which the ('equalised' between the firms) $MRTS_{L,K}$ is equal to the ratio of the market factor prices w/r. That is, with perfect competition general equilibrium of production occurs where condition (1) is satisfied.

If the factor prices are given, from the Edgeworth box of production we can determine the amounts of X and Y which maximise the profits of firms. However, in a general equilibrium, these quantities must be equal to those which consumers want to buy in order to maximise their utility. Consumers decide their purchases on the basis of the prices of commodities, P_x and P_y. Thus, in order to bring together the production side of the system with the demand side, we must define the equilibrium of the firms in the product space, using as a tool the *production possibility curve* of the economy.[1] This is derived from the Edgeworth contract curve of production, by mapping its points on a graph on whose axes we measure the quantities of the final commodities X and Y. From each point of the Edgeworth contract curve of production we can read off the maximum obtainable quantity of one commodity, given the quantity of the other. For example, point a in figure 22.23 shows that, given the quantity of X is X_3, the maximum quantity of Y that can be produced (with the given factors $\bar{K}$ and $\bar{L}$) is Y_3. The X_3, Y_3 combination is presented by point a' in figure 22.24. Similarly, point b of the Edgeworth contract curve of production shows that, given X_4, the maximum amount of Y that the economy can produce is Y_2. Point b' in figure 22.24 is the mapping of b from the factor space to the production space.

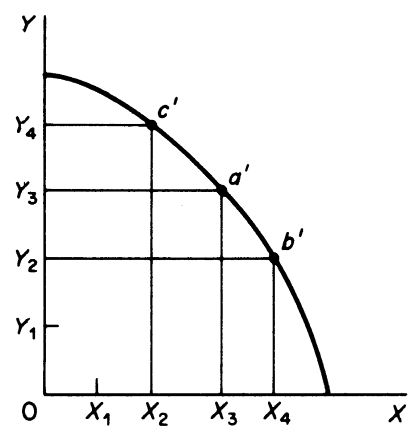

Figure 22.24 Production possibility curve

[1] The production possibility curve has been introduced in Chapter 3 in relation to the equilibrium of a multi-product firm.

In summary, the production possibility curve of an economy is the locus of all Pareto-efficient outputs, given the resource endownment ($\bar{K}$ and $\bar{L}$) and the state of technology. This curve shows the maximum quantity of a good obtainable, given the quantity of the other good. At any point on the curve all factors are optimally (efficiently) employed. Any point inside the curve is technically inefficient, implying unemployed resources. Any point above the curve is unattainable, unless additional resources or a new technology or both are found.

The production possibility curve is also called the *product transformation curve* because it shows how a commodity is 'transformed' into another, by transferring some factors from the production of one commodity to the other.

The negative of the slope of the production possibility curve is called the marginal rate of (product) transformation, $MRPT_{x,y}$ and it shows the amount of Y that must be sacrificed in order to obtain an additional unit of X. The economic meaning of the transformation curve is the rate at which a commodity can be transformed into another. By definition

$$MRPT_{x,y} = -\frac{dY}{dX}$$

Since dY/dX is negative, the $MRPT$ is a positive number. It can be shown that the $MRPT_{x,y}$ is equal to the ratio of the marginal costs of the two products

$$MRPT_{x,y} = -\frac{dY}{dX} = \frac{MC_x}{MC_y}$$

Proof

(1) By definition

$$MC_x = \frac{d(TC_x)}{dX}$$

and

$$MC_y = \frac{d(TC_y)}{dY} \tag{1a}$$

(2) Dividing these expressions we obtain

$$\frac{MC_x}{MC_y} = \frac{d(TC_x)}{d(TC_y)} \cdot \frac{dY}{dX} \tag{2a}$$

(3) But

$$d(TC_x) = w \cdot (dL_x) + r \cdot (dK_x)$$

and

$$d(TC_y) = w \cdot (dL_y) + r \cdot (dK_y)$$

so that

$$\frac{d(TC_x)}{d(TC_y)} = \frac{w \cdot (dL_x) + r \cdot (dK_x)}{w \cdot (dL_y) + r \cdot (dK_y)} \tag{3a}$$

(4) In order to remain *on* the *PPC* the factors released from the decrease in commodity Y must be equal to the factors absorbed by the increase in the production of X, that is

$$dL_x = -dL_y$$

and

$$dK_x = -dK_y \tag{4a}$$

Substituting (4) in (3) we find

$$\frac{d(TC_x)}{d(TC_y)} = \frac{w\cdot(-dL_y) + r\cdot(-dK_y)}{w\cdot(dL_y) + r\cdot(dK_y)} = -1 \tag{5a}$$

(5) Finally, substituting (5) in (2), we obtain

$$\frac{MC_x}{MC_y} = (-1)\frac{dY}{dX} = -\frac{dY}{dX} = \text{(slope of } PPC) = MRPT_{x,y} \qquad \text{Q.E.D.}$$

In perfect competition the profit-maximising producer equates the price of the commodity produced to the long-run marginal cost of production:

$$MC_x = P_x \quad \text{and} \quad MC_y = P_y$$

Therefore the slope of the production possibility curve is also equal to the ratio of the prices at which X and Y will be supplied by perfectly competitive industries:

$$MRPT_{x,y} = \frac{MC_x}{MC_y} = \frac{P_x}{P_y} \tag{2}$$

Given the commodity prices, general equilibrium of production is reached at the point on the production transformation curve that has a slope equal to the ratio of these prices. Such a general equilibrium of production is shown in figure 22.25. Assume that the market prices of commodities define the slope of the line AB. The ratio $0A/0B$ measures the ratio of the marginal cost of and hence the supply price of X to that of Y.

The general equilibrium product-mix from the point of view of firms is given by point T. The two firms are in equilibrium producing the levels of output Y_e and X_e.

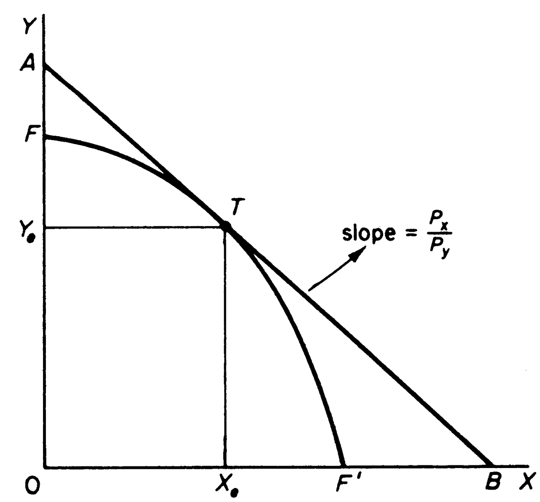

Figure 22.25 General equilibrium of production with perfect competition

(b) Equilibrium of consumption (efficiency in distribution of commodities)

We must now show how each consumer, faced with the market prices P_x and P_y, reaches equilibrium, that is, maximises his satisfaction. From the theory of consumer behaviour (Chapter 2) we know that the consumer maximises his utility by equating the marginal rate of substitution of the two commodities (slope of his indifference curves) to the price ratio of the commodities. Thus the condition for consumer equilibrium is

$$MRS_{x,y} = \frac{P_x}{P_y}$$

Since both consumers in perfectly competitive markets are faced with the same prices the condition for joint or general equilibrium of both consumers is

$$MRS^A_{x,y} = MRS^B_{x,y} = \frac{P_x}{P_y} \tag{3}$$

This general equilibrium of consumption for the product mix Y^e, X^e is shown in figure 22.26. We construct an Edgeworth box for consumption with the precise dimensions Y^e and X^e by dropping from point T (on the product transformation curve) lines parallel to the commodity axes. We next plot the indifference curves of consumer A with origin the south-west corner, and the indifference curves of B with origin the north-east corner.

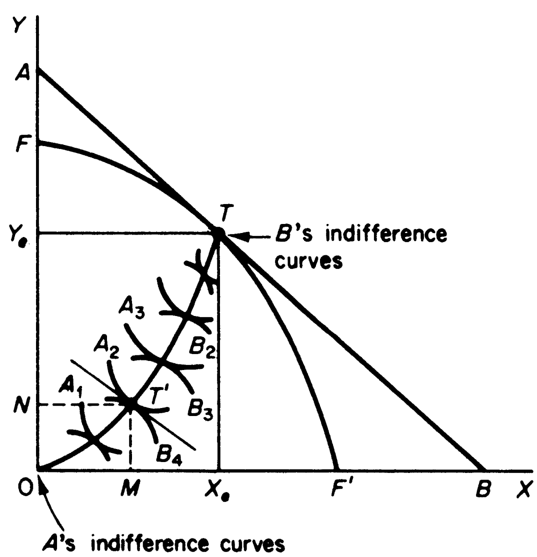

Figure 22.26

Any point in the Edgeworth consumption box shows six variables: the total quantities Y^e and X^e, and a particular distribution of these quantities between the two consumers. However, not all distributions are efficient in the Pareto sense. A Pareto-efficient distribution of commodities is one such that it is impossible to increase the utility of one consumer without reducing the utility of the other.[1] From figure 22.26 it is seen that only points of tangency of the indifference curves of the two consumers represent Pareto-efficient distributions. The locus of these points is called the *Edgeworth contract curve of consumption*. It should be clear that at each point of this curve the following equilibrium condition is satisfied

$$MRS^A_{x,y} = MRS^B_{x,y}$$

[1] See also Chapter 23.

Thus for a given product-mix (such as T, which we are considering) there is an infinite number of possible Pareto-optimal equilibria of distribution: the equilibrium of consumption is not unique, since it can occur at any point of the contract curve of consumption. However, with perfect competition, only one of these points is consistent with the general equilibrium of the system. This is the point of the contract curve where the ('equalised') $MRS_{x,y}$ of the consumers is equal to the price ratio of the commodities, that is, where condition (3) is fulfilled.

In figure 22.26 the equilibrium of the consumers is defined by point T'. Consumer A reaches the utility level implied by the indifference curve A_2, buying $0M$ of X and $0N$ of Y. Consumer B reaches the utility level implied by the indifference curve B_4, buying the remaining quantities MX_e of X and NY_e of Y.

(c) Simultaneous equilibrium of production and consumption (efficiency in product-mix)

From the discussion of the preceding two sections it follows that the general equilibrium of the system as a whole requires the fulfilment of a third condition, namely that the marginal rate of product transformation (slope of the PPC) be equal to the marginal rate of substitution of the two commodities between the consumers

$$MRPT_{x,y} = MRS^A_{x,y} = MRS^B_{x,y}$$

In perfect competition this condition is satisfied, since, from expression (2)

$$MRPT_{x,y} = \frac{P_x}{P_y}$$

and from expression (3)

$$MRS^A_{x,y} = MRS^B_{x,y} = \frac{P_x}{P_y}$$

so that

$$MRPT_{x,y} = MRS^A_{x,y} = MRS^B_{x,y} \tag{4}$$

This is the third condition of Pareto efficiency. It refers to *the efficiency of product substitution* (or optimal composition of output). Since the $MRPT$ shows the rate at which a good can be transformed into another in production, and the MRS shows the rate at which the consumers are willing to exchange one good for another, the system is not in equilibrium unless the two ratios are equal. Only then the production sectors' plans are consistent with the household sectors' plans, and the two are in equilibrium. A simple numerical example may illustrate the argument. Suppose that the $MRPT_{x,y}$ is $2Y/X$, while the $MRS_{x,y} = Y/X$. The economy can produce two units of Y by sacrificing one unit of X, while the consumers are willing to exchange one unit of X for one unit of Y. In figure 22.27 the inequality of the two ratios is shown by points c and d.

Apparently firms produce a smaller quantity of Y and a larger quantity of X relative to the preferences of the consumers. Given the assumption of consumer sovereignty, firms must reduce X and increase the production of Y for the attainment of general equilibrium.

The economic meaning of the third efficiency criterion is that the combination of outputs must be optimal from both the consumers' and the producers' point of view.

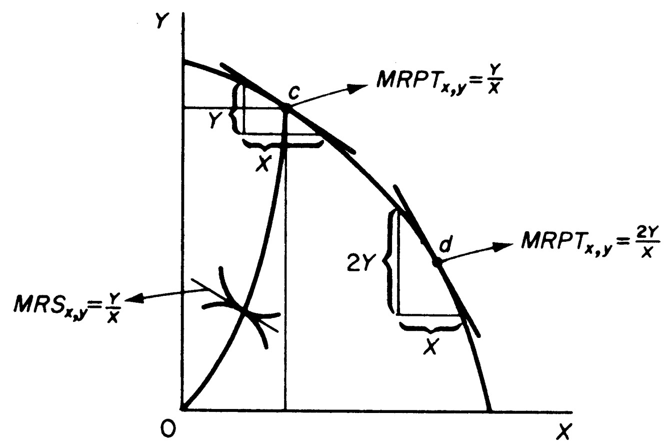

Figure 22.27 $MRPT_{x,y} = 2Y/X$, $MRS_{x,y} = Y/X$. Disequilibrium of production and consumption

In summary, with perfect competition (and no discontinuities and with constant returns to scale) the simple two-factor, two-commodity, two-consumer system has a *general equilibrium solution*, in which three Pareto-efficiency conditions are satisfied:

1. The *MRS* between the two goods is equal for both consumers. This *efficiency in distribution* implies optimal allocation of the goods among consumers.
2. The *MRTS* between the two factors is equal for all firms. This *efficiency in factor substitution* implies optimal allocation of the factors among the two firms.
3. The *MRS* and the *MRPT* are equal for the two goods. This *efficiency in product-mix* implies optimal composition of output in the economy and thus optimal allocation of resources.[1]

Whether such a general equilibrium solution (on the *PPC*) is desirable for the society as a whole is another question, which is the core of Welfare Economics. In the next chapter we will be concerned with the significance for the economic welfare (of the society as a whole) of the existence of a general equilibrium solution reached with perfect competition.

3. General equilibrium and the allocation of resources

In figure 22.26 the general equilibrium solution is shown by points *T* (on the production possibility curve) and *T'* (on the Edgeworth contract curve). These points define six of the 'unknowns' of the system, namely the quantities to be produced of the two commodities (X_e and Y_e), and their distribution among the two consumers (X_e^A, X_e^B, Y_e^A, Y_e^B). In this section we examine the determination of the allocation of resources between *X* and *Y*. The determination of the remaining unknowns (prices of factors and commodities, and the distribution of income between the two consumers) is examined in two separate sections below.

Point *T* on the production transformation curve (figure 22.26) defines the equilibrium product mix Y_e and X_e. Recalling that the *PPC* is the locus of points of the Edgeworth contract curve of production mapped on the product space, point *T* corresponds to a given point on this contract curve, say *T''* in figure 22.28. Thus *T''* defines the allocation of the given resource endowments in the production of the

[1] Because in perfect competition all three marginal conditions for Pareto-optimal resource allocation are satisfied, perfect competition is considered as an 'ideal' market structure, in the sense that the scarce resources are used in the most efficient way.

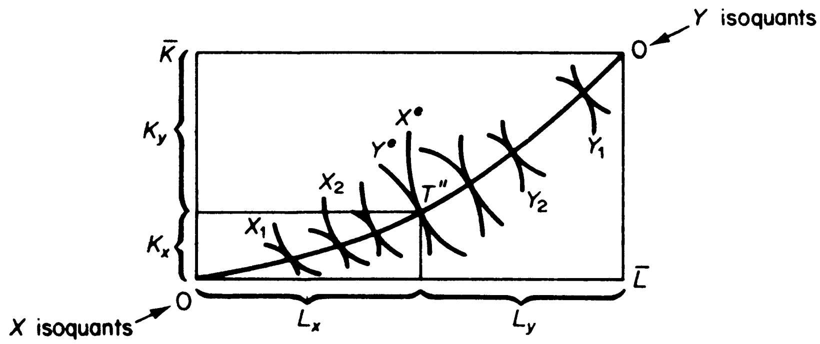

Figure 22.28 Allocation of resources to the production of X_e and Y_e

general equilibrium commodity mix. The production of X_e absorbs L_x of labour and K_x of capital, while Y_e employs the remaining quantities of factors of production, L_y and K_y. Thus four more 'unknowns' have been defined from the general equilibrium solution.

4. Prices of commodities and factors

The next step in our analysis is to show the determination of prices in the general equilibrium model, under perfect competition.

In the simple 2 × 2 × 2 model there are four prices to be determined, two commodity prices, P_x and P_y, and two factor prices, the wage rate w, and the rental of capital r. We thus need four independent equations.[1] However, given the assumptions of the simple model, we can derive only three independent relations.

1. Profit maximisation by the individual firm implies least-cost production of the profit-maximising output. This requires that the producer adjusts his factor mix until the *MRTS* of labour for capital equals the w/r ratio:

$$MRTS^x_{L,K} = \frac{w}{r} = MRTS^y_{L,K} \tag{5}$$

In other words the individual producer maximises his profit at points of tangency between the isoquants and isocost lines whose slope equals the factor price ratio.

2. In perfect factor and output markets the individual profit-maximising producer will employ each factor up to the point where its marginal physical product times the price of the output it produces just equals the price of the factor

$$w = (MPP_{L,x}) \cdot (P_x) = (MPP_{L,y}) \cdot (P_y) \tag{6}$$

$$r = (MPP_{K,x}) \cdot (P_x) = (MPP_{K,y}) \cdot (P_y) \tag{7}$$

3. The individual consumer maximises his utility by purchasing the output mix which puts him on the highest indifference curve, given his income constraint. In other words maximisation of utility if attained when the budget line, whose slope is equal to the ratio of commodity prices P_x/P_y, is tangent to the highest utility curve, whose slope is the marginal rate of substitution of the two commodities

$$MRS^A_{y,x} = \frac{P_y}{P_x} = MRS^B_{y,x} \tag{8}$$

[1] Note that the graphical analysis of the general equilibrium solution required the *ratios* of the input prices. The *absolute values* of prices were of no importance in our simple model, where the equilibrium of firms and consumers is attained by equating ratios of prices to various transformation (of commodities) and substitution (of factors) ratios.

Although we have four relations between the four prices, one of them is not independent. Because, dividing (6) and (7), we obtain

$$\frac{w}{r} = \frac{MPP_{L,x}}{MPP_{K,x}} = \frac{MPP_{L,y}}{MPP_{K,y}} = MRTS_{L,K}$$

which is the same as expression (5). Thus we have three independent equations in four unknowns. Apparently the absolute values of w, r, P_x and P_y are not uniquely determined (although the general equilibrium solution is unique).

Prices in the Walrasian system are determined only up to a ratio or a scale factor. We can express any three prices in terms of the fourth, which we choose arbitrarily as a *numéraire* or unit of account. For example assume that we choose P_x as the *numéraire*.

The remaining three prices can be determined in terms of P_x as follows:

From (5) we obtain

$$w = r(MRTS_{L,K}) \tag{9}$$

From the first part of expression (7) we have

$$r = (MPP_{K,x})P_x \tag{10}$$

Substituting (10) in (9) we obtain

$$w = (MRTS_{L,K})(MPP_{K,x})P_x \tag{11}$$

Finally from (8) we obtain

$$P_y = (MRS_{y,x})(P_x) \tag{12}$$

Equations (10), (11) and (12) give the *relative* prices w, r and P_y, that is, the prices relative to the *numéraire* P_x:

$$\frac{P_y}{P_x} = (MRS_{y,x}) \tag{13}$$

$$\frac{w}{P_x} = (MRTS_{L,K})(MPP_{K,x}) \tag{14}$$

$$\frac{r}{P_x} = (MPP_{K,x}) \tag{15}$$

The terms in brackets are *known* values, that is, values determined by the general equilibrium solution and the maximising behaviour of economic decision-makers with a given state of technology and given tastes.

Note that any good can serve as *numéraire*, and the change of *numéraire* leaves the relative prices unaffected. We can also assign any numerical value to the price of the *numéraire*. For convenience P_x is assigned the value of 1. But if, for example, we choose to set $P_x = £b$, then the price of y in £ will be:

$$\hat{P}_y = b \cdot \frac{P_y}{P_x} \text{ (pounds)}$$

This, however, does *not* mean that the absolute level of the prices of the system is determined. It simply illustrates the fact that we can assign to the price of the *numéraire* any value we choose.

The reason that the prices are determined only up to a ratio is that money has not been introduced in the system as a commodity used for transactions or as a store of wealth. In a system with perfect certainty, where, for example, nobody would think of holding money, only *relative* prices matter. The three equations (13)–(15) establish the price ratios implied by the unique general equilibrium solution, and the absolute values of prices are of no importance. However, the general equilibrium model can be completed by adding one (or more) monetary equation. Then the absolute values of the four prices can be determined. Unless a market for money is explicitly introduced, the price side of the model depends on an endogenous *numéraire*. Some attempts to introduce money in the system are discussed in the Appendix to this chapter.

5. Factor ownership and income distribution

For the simultaneous equilibrium of production and consumption, consumers must earn the 'appropriate' incomes in order to be able to buy the quantities of the two commodities (X_A, X_B, Y_A, Y_B) implied by point T' in figure 22.26.

Consumers' income depends on the distribution of factor ownership (quantities of factors which they own) and on factor prices. We saw in the preceding paragraph that the prices of factors are determined only up to a ratio. This, however, is adequate for the required income distribution, if the ownership of factors by A and B is determined. For this purpose we require four independent relations, given that we have four unknowns (K_A, K_B, L_A, L_B).

From the assumption of constant returns to scale we can make use of Euler's 'product exhaustion theorem' (see Chapter 21). This postulates that, with constant returns to scale, the total factor income is equal to the total value of the product of the economy (in perfect factor markets, where inputs are paid their marginal product):[1]

$$[(P_x)(X) + (P_y)(Y)] = [wL + rK] \tag{16}$$

or

$$[\text{value of total output}] = [\text{total factor income}]$$

[1] To illustrate Euler's exhaustion theorem we use the specific form of the Cobb–Douglas production function, which we developed in Chapter 3. For one product we have

$$X = b_0 \cdot L^{b_1} \cdot K^{b_2}$$

In Chapter 3 we saw that the sum $b_1 + b_2$ provides a measure of the returns to scale.

If $b_1 + b_2 = 1$ there are constant returns to scale. We also showed that b_1 and b_2 are the elasticities of output with respect to the factor inputs

$$b_1 = \frac{\partial X}{\partial L} \cdot \frac{L}{X} \quad \text{and} \quad b_2 = \frac{\partial X}{\partial K} \cdot \frac{K}{X}$$

where

$$\frac{\partial X}{\partial L} = MPP_L \quad \text{and} \quad \frac{\partial X}{\partial K} = MPP_K$$

In perfect factor markets

$$w = \frac{\partial X}{\partial L} \cdot P \quad \text{and} \quad r = \frac{\partial X}{\partial K} \cdot P$$

Thus we may write

The income of each consumer is, by definition, spent, so that

$$[wL_A + rK_A] = [(P_x)(X_A) + (P_y)(Y_A)] \tag{17}$$

$$[wL_B + rK_B] = [(P_x)(X_B) + (P_y)(Y_B)] \tag{18}$$

Finally by the assumption of full employment of factors of production we have

$$L_A + L_B = L \tag{19}$$

$$K_A + K_B = K \tag{20}$$

The above five equations give only three independent relations, (16), (19) and (20), because the addition of (17) and (18) is implied by Euler's theorem:

$$w(L_A + L_B) + r(K_A + K_B) = P_x(X_A + X_B) + P_y(Y_A + Y_B)$$

or

$$wL + rK = (P_x)(X) + (P_y)(Y)$$

Thus we have three independent equations in four unknowns (K_A, K_B, L_A, L_B), whose values cannot be uniquely determined. The general equilibrium solution does not give absolute values for the distribution of ownership of the factors and money incomes between consumers *A* and *B*. This indeterminacy can be resolved only partly if one fixes arbitrarily the value of one of the four factor endowments, and then allocate the remaining three so as to make the individual incomes of *A* and *B* such as to lead them willingly to the consumption pattern implied by point *T'* in figure 22.26. It should be clear that different distribution of resources among the two consumers can result in different product combinations, that is, different general equilibrium solutions. (See also Chapter 23.)

The conclusion of this paragraph may be summarised as follows. The general equilibrium solution defines the total value of the product in the economy. With constant returns to scale this value is equal to the total income of the consumers. However, the individual incomes of *A* and *B* are not uniquely determined endogenously. One has to make a consistent assumption about the factor ownership

---

$$b_1 = \frac{wL}{PX} = \text{share of } L \qquad \text{and} \qquad b_2 = \frac{rK}{PX} = \text{share of } K$$

Summing, we obtain

$$(b_1 + b_2) = \frac{wL}{PX} + \frac{rK}{PX}$$

With constant returns to scale we have

$$1 = \frac{wL}{PX} + \frac{rK}{PX}$$

so that

$$PX = wL + rK$$

or

$$\begin{bmatrix}\text{value}\\ \text{of output}\end{bmatrix} = \begin{bmatrix}\text{income}\\ \text{of } L\end{bmatrix} + \begin{bmatrix}\text{income}\\ \text{of } K\end{bmatrix}$$

Thus with constant returns to scale and perfect factor markets, the incomes of inputs exhaust the value of the output.

distribution among the two consumers, so that their incomes are compatible with the purchasing pattern of X_e and Y_e implied by the general equilibrium solution (T and T' in figure 22.26).

It should be stressed that the above result of factor and income distribution follows from the assumption of fixed amounts of L and K owned by the consumers and supplied to the firms irrespective of prices. The factor supplies did not depend (in this simple model) on the prices of factors and the prices of commodities. The model could be solved simultaneously for input allocations among X and Y, total output mix and commodity-distribution between the two consumers, and only subsequently could we superimpose on this solution the ownership of factors and money-income distribution problem. In the Appendix to this chapter we will abandon the fixed factor supply assumption. We will make the supply of factors by the individual consumers a function of prices of factors and prices of commodities. We will retain the assumption that each consumer has given amounts of factors, but will supply only such quantities as will maximise his utility. This will resolve the indeterminacy of income distribution, but not the indeterminacy of the absolute levels of prices. The latter can be resolved only with the introduction of money in the Walrasian system, which deals only with 'real' magnitudes, not monetary ones.

F. CONCLUDING REMARKS

There are several reasons why the study of general equilibrium theory is important.

First. General equilibrium theory, despite its obvious shortcomings, is the most complete existing model of economic behaviour. General equilibrium theory, by viewing the economy as a vast system of mutually interdependent markets, makes the student aware of the tremendous complexity of the real world. At its present stage, general equilibrium theory is largely non-operational and unrealistic.[1] However, the general equilibrium model can be improved so as to become more flexible, more realistic and, hence, more useful for analysing the real world.

Second. Under certain assumptions the general equilibrium system has a solution: it yields a set of price ratios which lead to an optimal allocation of resources.

Third. This solution and its optimality properties can be used as a norm to judge the significance and implications of deviations of the various markets from this 'ideal' state of equilibrium.

Fourth. General equilibrium theory can be helpful in the resolution of macroeconomic controversies. If two macromodels are both consistent with statistical data (in that neither is refuted by empirical tests), then one might argue that the model which has closer links to individual optimising behaviour may be considered more nearly correct, since it has a better grounding in the wider body of traditional economic knowledge.[2]

At the present time the fourth of the above issues is more important, given the reopened debate between 'Keynesians' and 'Classics'. The participants in this controversy take recourse to general equilibrium theory in an attempt to give more credibility to their positions. (See Appendix, section III.)

[1] For an attempt to develop operational general equilibrium models see S. Adelman and S. Robinson, *Income Distribution Policy in Developing Countries: A Case Study of Korea* (Stanford University Press, 1976). Also A. C. Harberger, 'The Incidence of the Corporation Income Tax', *Journal of Political Economy*, vol. 70 (1962). Also J. Shoven and J. Whalley, 'A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S.', *Journal of Public Economics* (1972).

[2] See E. Roy Weintraub, *General Equilibrium Theory*, Macmillan Studies in Economics (Macmillan, 1974).

Appendix

In the first section of this Appendix we extend the simple general equilibrium model to the case of any number of factors, commodities and consumers.

In the second section we discuss briefly the problems of existence, stability and uniqueness of a general equilibrium solution.

Finally in section III we present a summary of the attempts that have been made to introduce money in the Walrasian system.

SECTION I

Extension of the Simple General Equilibrium Model to any Number of Households, Commodities, and Factors of Production: The H × M × N General Equilibrium Model

Assume that the economy consists of H households. The subscript h ($h = 1, 2, \ldots, H$) is used to denote a particular household. There are M commodities produced, each by one firm.[1] The subscript m ($m = 1, 2, \ldots, M$) is used to denote a particular commodity (and the corresponding producing firm). Finally suppose that there are N factors of production owned by the households. The subscript n ($n = 1, 2, \ldots, N$) is used to denote a particular factor of production.

The letter q_m denotes the quantities of commodities; the symbol v_n denotes the quantities of factors of production. The prices of commodities are denoted by the symbol P_m and the prices of factors by w_n.

The supply of factors by household is not fixed, but is a function of all the prices in the system. All the other assumptions of the 2 × 2 × 2 model hold for the general model. In particular there is perfect competition in all markets, so that consumers and firms are faced by the same set of prices.

A. Number of Equations

1. The household sector

Consumers demand commodities produced by the business sector, and supply services of factors to firms. The demands for goods and supplies of factors are the result of the utility-maximising behaviour of households.

The demand for any commodity depends on the prices of all commodities, the household's income, and its tastes, which, by assumption, are represented by smooth indifference curves convex to the origin. The income of the household depends on the supply of factor services, which, in turn, depend on the prices of factors and the prices of commodities.

[1] The model can be extended to include many firms, each producing some quantity of all commodities. This would complicate the presentation without adding any significant insight to the general model.

The utility of a household is a function of the quantities of commodities and of factors. Maximisation of the utility function (subject to the budget constraint) gives rise to demand equations for commodities and supply equations of factor services. The determinants of the quantities of goods and factors are the prices in all the markets and the household's tastes. Tastes are implicitly included in the particular form of the utility function.

The demand function for the mth commodity by the hth consumer is of the general form

$$q_{mh} = f_{mh}(P_1, P_2, \ldots, P_M; w_1, w_2, \ldots, w_N). \qquad \begin{matrix} m = 1, 2, \ldots, M \\ h = 1, 2, \ldots, H \end{matrix}$$

(Tastes are implicit in the f_{mh} symbol of the right-hand side.)

There are MH demand functions in the general system.

The supply function for the nth factor by the hth consumer is of the general form

$$v_{nh} = f_{nh}(P_1, P_2, \ldots, P_M; w_1, w_2, \ldots, w_N). \qquad \begin{matrix} n = 1, 2, \ldots, N \\ h = 1, 2, \ldots, H \end{matrix}$$

(Tastes are again implicit in the f_{nh} symbol.)

There are NH supply functions for productive services.

2. The business sector

Firms demand factors of production and supply quantities of finished commodities to the household sector. Their goal is profit maximisation, subject to technological constraints.

The demand for the nth productive factor to be used in the manufacture of the mth commodity depends on the quantity of this particular commodity, the prices of all factors of production and the production function

$$v_{nm} = f_{nm}(q_m; w_1, w_2, \ldots, w_N). \qquad \begin{matrix} m = 1, 2, \ldots, M \\ n = 1, 2, \ldots, N \end{matrix}$$

(The particular production function of the firm is implicit in the f_{nm} symbol. It is assumed that the production isoquants are smooth and convex to the origin, and there are constant returns to scale.)

There are NM demand equations for productive services.

The supply of any commodity in perfect competition is defined, in the long run, by the equality of the revenue to the long average cost

$$P_m q_m = \sum_{n=1}^{n} v_{nm} w_n. \qquad \begin{matrix} m = 1, 2, \ldots, M \\ n = 1, 2, \ldots, N \end{matrix}$$

There are M supply equations for the M commodities produced in the economy.

3. Equilibrium conditions (or 'clearing of the market' equations)

In order to have equilibrium in all markets the quantity supplied must be equal to the quantity demanded, or, equivalently, the excess demand must be zero in each market.

Equilibrium in each commodity market requires that the total demand of households for that product to equal the total supply by firms of that product. Hence we have M 'clearing of the commodity market' equations of the form

$$\sum_{h=1}^{H} q_{mh} = q_m \qquad m = 1, 2, \ldots, M$$

Similarly, for each factor market to be in equilibrium the total demand by firms for that factor must be equal to the total supply by households of that factor. Thus we have N 'clearing of the factor market' equations of the form

$$\sum_{m=1}^{M} v_{nm} = \sum_{h=1}^{H} v_{nh} \qquad n = 1, 2, \ldots, N$$

In total the H × M × N general model consists of the following equations

	Number of equations
I. Behavioural equations	
1. Demand functions for commodities	MH
2. Supply functions for commodities	M
3. Demand functions for factors	NM
4. Supply functions for factors	NH
II. 'Clearing of the market' equations	
5. Clearing of the commodity markets	M
6. Clearing of the factor markets	N

B. NUMBER OF UNKNOWNS

The unknowns that must be determined by the solution of the general model are:

(*a*) The quantities of goods demanded by the households, q_{mh}. There are MH such quantities.

(*b*) The quantities of goods supplied by the firms, q_m. There are M such variables.

(*c*) The quantities of productive factors demanded by the firms, v_{nm}. There are NM such quantities.

(*d*) The quantities of productive factors supplied by the households, v_{nh}. There are NH such quantities.

(*e*) The prices of commodities that clear their markets, P_m. There are M such prices.

(*f*) The prices of factors that clear their markets, w_n. There are N such prices.

In total the number of unknowns is

$$MH + M + NM + NH + M + N$$

C. EXISTENCE OF A SOLUTION

A first step in determining whether a solution exists is to compare the number of equations with the number of variables whose values must be determined. A necessary condition for the existence of a solution is that the number of unknowns must be equal to the number of *independent* equations. In the above generalised model the number of equations is equal to the number of the unknowns

$$MH + M + NM + NH + M + N$$

However, the independent equations are one short of the number of unknowns. There is one redundant (not independent) equation. Thus the entire system is underdetermined, and we cannot find unique values for all commodity and factor prices and for all commodity and factor quantities.

Proof of a redundant (not independent) equation

To show that one of the equations in the system is redundant we use the following relations.

1. Total consumers' expenditures

If we multiply the q_{mh} demands for finished commodities by their prices, p_m, and sum over all individual households and commodities, we obtain

$$\sum_{h=1}^{H} \sum_{m=1}^{M} q_{mh} p = \begin{bmatrix} \text{total consumers'} \\ \text{expenditures} \end{bmatrix} \tag{1}$$

2. Total consumers' income

If we multiply the v_{nh} supplies of productive services by their prices, w_n, and sum over all households and factors, we obtain

$$\sum_{h=1}^{H} \sum_{n=1}^{N} v_{nh} w_n = \begin{bmatrix} \text{total consumers'} \\ \text{income} \end{bmatrix} \tag{2}$$

By assumption all income is spent, so that the total expenditures of households are equal to the total income of households

$$\sum_{h=1}^{H} \sum_{m=1}^{M} q_{mh} p_m = \sum_{h=1}^{H} \sum_{n=1}^{N} v_{nh} w_n \tag{3}$$

$$\begin{bmatrix} \text{total expenditures} \\ \text{of households} \end{bmatrix} = \begin{bmatrix} \text{total income} \\ \text{of households} \end{bmatrix}$$

This is the aggregate budget restriction. The expression may be interpreted to mean that in a general equilibrium system where money appears only as a unit of account, the market value of aggregate supply is identically equal to the market value of aggregate demand, or expenditures are identically equal to receipts. This is known as *Walras's Law*. The significance of the *identity*[1] is that the relation holds true for *all values* of the prices, not just the equilibrium values. With the assumptions of this model it is *impossible* to have a discrepancy between the values of aggregate demand and aggregate supply. A corollary of Walras's Law is that if the first $N - 1$ markets in a system are in equilibrium, then the Nth is necessarily in equilibrium as well (see below).

3. The total revenues and costs of firms

We split the revenues and costs in two parts, the one referring to the $M - 1$ commodities

$$\sum_{m=1}^{M-1} p_m q_m = \sum_{n=1}^{N} \sum_{m=1}^{M-1} v_{nm} w_n \tag{4}$$

$$\begin{bmatrix} \text{total revenue} \\ \text{from the } M-1 \\ \text{commodities} \end{bmatrix} = \begin{bmatrix} \text{total cost} \\ \text{of the } M-1 \\ \text{commodities} \end{bmatrix}$$

[1] The distinction between an identity and an equality may be illustrated with an algebraic example:
The expression $(a + b)^2 = a^2 + 2ab + b^2$ is an identity, because it holds true for all values of a and b.
The expression $a - 4b = 6$ is an equality, because it holds true only for certain values of a and b.

and the other referring to the Mth commodity

$$P_M q_M = \sum_{n=1}^{N} v_{nM} w_n \tag{5}$$

$$\begin{bmatrix}\text{revenue}\\ \text{from } M\end{bmatrix} = [\text{cost of } M]$$

4. By definition the total incomes received by households from the supply of productive services (expression 2) must be equal to the total payments (costs) by firms for buying these services

$$\left(\sum_{h=1}^{H}\sum_{n=1}^{N} v_{nh} w_n\right) = \left(\sum_{n=1}^{N}\sum_{m=1}^{M-1} v_{nm} w_n\right) + \left(\sum_{n=1}^{N} v_{nm} w_n\right) \tag{6}$$

$$\begin{bmatrix}\text{total}\\ \text{consumers' income}\end{bmatrix} = \begin{bmatrix}\text{total cost of}\\ M-1 \text{ commodities}\end{bmatrix} + \begin{bmatrix}\text{cost of the}\\ M\text{th commodity}\end{bmatrix}$$

Expression (6) may be rewritten as follows

$$\left(\sum_{h=1}^{H}\sum_{n=1}^{N} v_{nh} w_n\right) - \left(\sum_{n=1}^{N}\sum_{m=1}^{M-1} v_{nm} w_n\right) = \left(\sum_{n=1}^{N} v_{nM} w_n\right) \tag{7}$$

5. Similarly, the total expenditures of households (expression 1) must be equal to the total receipts (revenue) of firms from the sale of all M finished commodities

$$\left(\sum_{h=1}^{H}\sum_{m=1}^{M} q_{mh} p_m\right) = \left(\sum_{m=1}^{M-1} p_m q_m\right) + (p_M q_M) \tag{8}$$

$$\begin{bmatrix}\text{total consumers'}\\ \text{expenditures}\end{bmatrix} = \begin{bmatrix}\text{consumers'}\\ \text{expenditures}\\ \text{on the } M-1\\ \text{commodities}\end{bmatrix} + \begin{bmatrix}\text{consumers'}\\ \text{expenditure}\\ \text{on the } M\text{th}\\ \text{commodity}\end{bmatrix}$$

This expression may be rearranged to give

$$\left(\sum_{h=1}^{H}\sum_{m=1}^{M} q_{mh} p_m\right) - \left(\sum_{n=1}^{M-1} p_m q_m\right) = (p_M q_M) \tag{9}$$

We observe that the left-hand sides of expressions (7) and (9) are equal (because of the relations (3) and (4)), so that the right-hand sides must also be equal, i.e.

$$p_M q_M = \sum_{n=1}^{N} v_{nM} w_n \tag{10}$$

This relation is identical to expression (5), which thus is a redundant equation. In other words, since

$$\begin{bmatrix}\text{total consumers'}\\ \text{income}\end{bmatrix} = \begin{bmatrix}\text{total consumers'}\\ \text{expenditures}\end{bmatrix}$$

and

$$\begin{bmatrix}\text{total revenues}\\ \text{of firms}\\ \text{from the } M-1\\ \text{commodities}\end{bmatrix} = \begin{bmatrix}\text{total costs}\\ \text{of firms}\\ \text{for the } M-1\\ \text{commodities}\end{bmatrix}$$

then *necessarily* relation (5) must be satisfied. Q.E.D.

The indeterminacy can be resolved by choosing arbitrarily any commodity or factor as the *numéraire* and express the prices of all the other commodities and factors in terms of its price (which for convenience is set equal to 1, just as the price of one £ is 1). In this way there is one less unknown to be determined by the solution of the model.

The system with the *numéraire* is called a *real* model, because prices are stated in terms of a commodity instead of being expressed in terms of money. It is obvious that the above general equilibrium system of equations determines uniquely the *price ratios*, but not the absolute level of prices. Despite the indeterminacy of absolute prices all the quantities of commodities and factors are uniquely determined. The absolute level of prices can be determined if the real general equilibrium model is augmented to include a market for money. (See section III below.)

D. Uniqueness of the solution

The detection of the redundant equation and the use of the *numéraire* do not guarantee that the general equilibrium solution is unique or economically meaningful. To secure an economically meaningful solution we must impose non-negativity constraints on the values of the unknowns (i.e. prices and quantities must be non-negative). If these restrictions are not imposed we cannot be sure that a solution exists at all. However, if to the non-negativity constraints we impose the additional requirements of convexity (of the production isoquants and the households' indifference curves), as well as the condition of constant returns to scale, we can be sure that with perfect competition the general equilibrium model has a unique and economically meaningful solution.[1]

SECTION II

Some Comments on the Existence, Stability and Uniqueness of General Equilibrium[2]

1. Existence of general equilibrium

There are two views about the desirability of proving the existence of solutions to general equilibrium systems. The first view is adopted, among others, by Dorfman, Samuelson and Solow,[3] who argue that general equilibrium analysis becomes practically useless if it is not known whether an economic system can ever attain or tend towards a general equilibrium. This view apparently stresses the operationality of a model.

The second view is that a model can have value even if it is non-operational because one cannot prove the existence of a solution to it. The general equilibrium model, in this view, is useful, even if it does not have a solution, because it shows the complexities of the interdependence between markets and between individual decision-makers. According to this view, counting equations and variables, while it

[1] See K. J. Arrow and G. Debreu, 'Existence of an Equilibrium for a Competitive Economy', *Econometrica* (1954), pp. 265–90.

[2] This section is based on D. Simpson's monograph *General Equilibrium Analysis* (Blackwell, 1975).

[3] R. Dorfman, P. Samuelson, and R. Solow, *Linear Programming and Economic Analysis* (McGraw-Hill, 1958).

can prove nothing positively about the existence of equilibrium, does show whether the system is consistent with the existence of equilibrium, that is, whether in principle an equilibrium may exist.[1]

We have seen that recent research has provided a proof of the existence of a general equilibrium in an economy with perfectly competitive markets, where there are no discontinuities and non-increasing returns to scale. Thus perfect competition guarantees the existence of a general equilibrium.

2. Stability of general equilibrium

Very little is known about the stability of a general equilibrium solution. Walras maintained that the equilibrium is stable. Supposing that the system was initially out of equilibrium, Walras argued that an equilibrium would eventually be attained by an iterative process. Suppose that an external agency moves through each market in turn, adjusting each price to its equilibrium value. When the process is complete the last market will be in equilibrium, and because of interdependence among markets, all the others will be out of equilibrium; but Walras argued that the disequilibrium would not be as large as initially, because he assumed that supply and demand responded more to 'own-price' changes than to 'other-price' changes. With this process repeated the system would eventually reach equilibrium by trial and error, or groping (*tâtonnement*) by the market mechanism. While this adjustment may, in theory, take place timelessly, in practice it would be difficult to maintain that it would not require real time. Walras did not attempt to discuss the time required for the attainment of a new equilibrium. In general the *tâtonnement* process is an unsatisfactory device, since it essentially evades the adjustment issue.

Some theorems about the stability of general equilibrium systems have been proved for a limited number of special cases of quite restricted generality.[2] They are of two types. First, assuming some specific form of disequilibrium (usually an excess demand), the behaviour of the system is investigated and theorems are proved about the conditions under which an equilibrium in whose neighbourhood the system is defined is a stable one. The second type of stability theorems concerns such questions as what kind of general equilibrium systems are likely to have equilibria which satisfy these conditions. Investigating such questions, Arrow and Hurwicz found that in none of their selected cases was a Walrasian system shown to be unstable. However, other examples have shown the Walrasian system to be unstable. These findings led Kuenne (1967) to suggest that a possible way to avoid instability is to abandon the assumption that market equilibrium is attained in a recontracting *tâtonnement* process, and to substitute for it an equilibrating process which permits non-equilibrium transactions to occur. Non-*tâtonnement* equilibrium may prove a better process for reaching equilibrium. (It may be significant that the literature on non-equilibrium behaviour has recently been expanding.)

Two results of the investigations into stability properties of Walrasian systems may be worth mentioning.

(*a*) Under the usual disequilibrium behaviour assumptions and the usual proper-

[1] D. Simpson, *General Equilibrium Analysis* (Blackwell, 1975), p. 50.

[2] Strictly speaking, these theorems have been proved about the stability of multiple exchange systems which may be regarded as reduced forms of the Walrasian system. See K. Arrow and L. Hurwicz, 'On the Stability of the Competitive Equilibrium', *Econometrica* (1958). Also, L. Meltzer, 'The Stability of Multiple Markets: the Hicks Conditions', *Econometrica* (1945). Also M. Morishima, 'On the three Hicksian Laws of Comparative Statics', *Review of Economic Studies* (1960).

ties of a Walrasian general equilibrium system an equilibrium is stable if all commodities are strict gross substitutes. Gross substitutes are commodities whose excess demands have the following slopes:

$$\frac{\partial E_i}{\partial P_j} < 0 \qquad \text{for all } i = j$$

and

$$\frac{\partial E_i}{\partial P_j} > 0 \qquad \text{for all } i \neq j$$

(*b*) If a system satisfies the weak axiom of revealed preference in *the aggregate*, then the system is stable (i.e. will return to the initial equilibrium following a disturbance of any magnitude). The weak axiom of revealed preference can be stated as follows:

If p_i^0 and q_i^0 are prices and quantities satisfying demand functions, and p_i^1 and q_i^1 are another such set, then $\sum p_i^0 q_i^0 \geq \sum p_i^0 q_i^1$ must imply that $\sum p_i^1 q_i^1 < \sum p_i^1 q_i^0$. For an individual consumer this axiom implies that his preferences do not change as prices change. But applied 'in the aggregate', i.e. to market demand functions, it is a very strong assumption. It implies that the economy as a whole has a single set of 'preferences', which remain unchanged as prices change. Normally one would expect changes in prices to change the distribution of income, and hence to change 'preferences'.

In summary we may conclude that in a Walrasian system with the usual disequilibrium behaviour assumption, if the second-order equilibrium conditions are fulfilled by all the relevant functions (continuity and convexity of indifference curves and isoquants) the equilibrium attained will *probably* be stable.

3. Uniqueness of general equilibrium

Uniqueness is a property of an equilibrium solution which is of less interest than either existence or stability. It is true that *local* uniqueness may be a desirable property if one is carrying out a comparative static analysis. Otherwise it is not all that evident that it is a particularly desirable property.

However, it can be proved that if an equilibrium exists for a system characterised *either* by gross substitutability *or* by the weak axiom for aggregate excess demands then that equilibrium is unique.

Uniqueness proofs have the disadvantage of restrictive (strong) assumptions. For example, the 'well-behaved' property of the production function excludes a wide range of plausible production behaviour.

SECTION III

Money and General Equilibrium

General equilibrium theory has traditionally been developed as a part of microeconomics. The general equilibrium model developed in this chapter is the most disaggregative structure of microeconomic analysis.

Yet general equilibrium theory is the link between microeconomics and macroeconomics. Macroeconomics is general equilibrium theory with some of the many markets grouped together for convenience. Thus we may say that:

A general equilibrium system is simply a totally disaggregated macroeconomic model.

or A macromodel is an aggregated general equilibrium model.

or A general equilibrium system is a complete microeconomic model and at the same time a detailed approach to macroeconomics.

However, to make the general equilibrium apparatus suitable for the analysis of macrophenomena such as inflation or involuntary unemployment the introduction of money is essential.

In this section we will examine briefly some monetary general equilibrium models. The goal is to provide a view of what a *unified theory* might be like, that is, a theory capable of dealing with the aggregative structure of macroeconomic theory as well as with the disaggregative structure of microeconomic analysis.

In recent years monetary general equilibrium models have attracted a lot of attention in economic literature. A great controversy, still unresolved, has developed between two 'schools', centred around the importance of the 'Keynesian revolution'. Probably the most representative view of the 'traditionalist school' is expressed by Patinkin, who argues that Keynes's theory of equilibrium with involuntary unemployment is a special case of the general equilibrium model, augmented to include money. At the other extreme we have the 'ultra-Keynesian school', with main proponents Clower, Hahn, Leijonhufvud, who have refuted Patinkin's arguments. We will attempt to present a summary of the main issues of the controversy, which is related to the monetisation of the real general equilibrium system.

To facilitate the analysis we will use a simplified model, in which there is no production. The households have *given quantities of commodities*, which they exchange until a general equilibrium of all the individuals is reached. This is known as a *pure exchange system*. We will also express the model in terms of 'excess demand' functions.

Assume that there are N commodities held by consumers in the economy. We can write N market demand equations, each of the form

$$q_{Dn} = f_{Dn}(P_1, P_2, \ldots, P_N) \qquad n = 1, 2, \ldots, N$$

and N market supply equations, each of the form

$$q_{sn} = f_{sn}(P_1, P_2, \ldots, P_N) \qquad n = 1, 2, \ldots, N$$

The excess demand for any one market, E_n, is defined as the difference between the quantity demanded and the quantity supplied at *any* set of prices

$$E_n = f_{Dn}(P_1, P_2, \ldots, P_N) - f_{sn}(P_1, P_2, \ldots, P_N)$$

It is clear that in the pure exchange system the excess demand is a function of the set of prices of the given commodities.

The equilibrium condition for each market

$$q_{Dn} = q_{sn}$$

can be equivalently written in terms of the excess demand function since the equality of the quantity demanded to the quantity supplied implies that the excess demand is zero. Thus the condition for a general equilibrium is that in all markets the excess demand must be zero

$$E_n = f_n(P_1, P_2, \ldots, P_N) = 0 \qquad n = 1, 2, \ldots, N$$

The pure exchange system contains one dependent (redundant) equation, so that its solution can determine only $N - 1$ *relative prices* or *price ratios*

$$\frac{P_1}{P_n}, \frac{P_2}{P_n}, \ldots, \frac{P_{n-1}}{P_n}$$

where P_n is the price of the Nth commodity, chosen arbitrarily as the *numeraire*. The 'real' system of market equations (or, equivalently, excess demand equations) determines 'real' or 'relative prices'. The absolute level of prices remains indeterminate. The pure exchange system is homogeneous of degree zero in absolute prices. That is, if the values of all the prices are increased equiproportionately the quantities of the commodities exchanged are left unchanged.

To resolve the indeterminacy and obtain the absolute levels of prices some theorists suggested the introduction in the 'real' exchange system of the well-known equation of exchange

$$MV = PT$$

or, given

$$PT = \sum p_n q_n$$

$$MV = \sum_{n=1}^{N} p_n q_n$$

where M = given stock of money,
V = velocity of circulation of money.

The equation of exchange is an additional independent equation to the 'real' system of N commodity markets. It can be seen that a doubling of the money supply (given V and the quantities of the N commodities, q_n) would double all absolute prices. However, the relative prices would remain unchanged, since they are determined by the market demands and supplies (or excess demands) of real commodities.

With this approach a dichotomy was introduced in the general equilibrium model:

(*a*) real factors determine relative prices
(*b*) monetary factors determine the absolute levels of prices.

The above augmented general equilibrium system is theoretically unsatisfactory, because the N market equations are behavioural, while the equation of exchange is an identity. To rectify this the Cambridge version of the equation of exchange was used

$$M = k\left(\sum_{n=1}^{N} p_n q_n\right)$$

where

$$k = \frac{1}{V} = \text{'the Cambridge multiplier'}$$

In this version the velocity of circulation of money is not a constant, but a behavioural variable.

We may express the Cambridge equation of exchange in terms of excess demand for money, since the left-hand side is the supply of money and the right-hand side is the demand for money. If we denote money as the $(N + 1)$th commodity we may write the excess demand for money as

$$E_{n+1} = \frac{1}{V} \sum_{n=1}^{N} p_n q_n - M$$

Given V, the excess demand for money is homogeneous of degree one in the absolute (money) prices and the quantity of money, i.e. if all prices *and* the quantity of money increase by the same proportion the excess demand for money will increase by an identical proportion.

In summary, the classical pure exchange general equilibrium system with money introduced into it consists of $N + 1$ equations:

N equations for excess demands in the goods markets, which are homogeneous of degree zero in absolute (money) prices, and one equation for the excess demand for money, which is homogeneous of degree one in money prices and the quantity of money.

The difference in the degree of homogeneity between the commodity markets and the money market is a contradiction, implying inconsistent behaviour of consumers. Consumers' demand for money is a decision to hold cash balances now in order to make purchases in the future. Thus an excess demand for money is identically equal to an excess supply of commodities. Yet the excess demand for money is homogeneous of degree one in money prices *and* the quantity of money, while the excess supply of goods is homogeneous of degree one in money prices alone. This contradiction renders the monetised classical general equilibrium system unsatisfactory.

Patinkin's system (or the 'neoclassical synthesis')

Patinkin (among others) attempted to rectify the above contradiction. His approach is often called 'the neoclassical synthesis', because it consists of the combination of modern monetary theory and the 'classical' (Walrasian) general equilibrium theory.

Patinkin expressed the n excess demands for commodities as functions of the n prices and the quantity of money, all divided by the general price level, defined as the weighted average of all the commodity prices

$$P = \sum_{n=1}^{N} z_n p_n$$

where z_i are given weights, with $\sum z_i = 1$.

Patinkin's model consists of $N + 2$ equations, N for the excess demands for commodities, one for the excess demand for money, and the above definition of the general price level

$$E_1 = f_1\left(\frac{P_1}{P}, \frac{P_2}{P}, \frac{P_3}{P}, \ldots, \frac{M}{P}\right)$$

$$E_2 = f_2\left(\frac{P_1}{P}, \frac{P_2}{P}, \frac{P_3}{P}, \ldots, \frac{M}{P}\right)$$

$$\vdots \qquad \vdots$$

$$E_n = f_n\left(\frac{P_1}{P}, \frac{P_2}{P}, \frac{P_3}{P}, \ldots, \frac{M}{P}\right)$$

$$E_{n+1} = f_{n+1}\left(\frac{P_1}{P}, \frac{P_2}{P}, \frac{P_3}{P}, \ldots, \frac{M}{P}\right) - \frac{M}{P}$$

$$P = \sum_{n=1}^{N} z_n p_n$$

In this model all the excess demand equations are homogeneous of degree zero in money prices and the stock of money: if all prices and the money stock change by the same percentage, relative prices and the real money stock remain unchanged, and so do the demands for commodities. In the Patinkin system an increase in the quantity of money leads to an increase in the price level, but the relative prices remain unchanged. It can be shown that if the money stock changes to λM, then the new equilibrium prices will be λP_i and λP. This results from the assumptions of Patinkin's model, which does *not* make use of the equation of exchange. Let us see how an increase in the money supply works in Patinkin's system. Assume that the money balances increase. Relative prices remain unchanged. But the increased money balances mean that excess demands arise in all goods markets. Positive excess demands lead to rises in prices everywhere. Price increases will cause real balances (M/P) to fall, and this process will continue for as long as real balances are larger than in the initial equilibrium. Prices will continue to rise until they have risen in proportion to the quantity of money, and so brought real balances down to their initial value.

Although there are $N + 2$ equations in $N + 1$ unknowns,

$$\left(\frac{P_1}{P}, \frac{P_2}{P}, \ldots, \frac{M}{P}\right)$$

one of the equations in Patinkin's system is redundant: since demand for money is equivalent to supply of goods, Walras's Law implies that one market equation is dependent on the others. Thus Patinkin's system is determinate. Even if one decides to consider the excess-demand-for-money equation as the redundant one (and hence eliminate it from the system) the excess demand for money still appears in the form of an excess supply (negative excess demand) of commodities.

The important contribution of Patinkin is that he presented a system in which all the excess demand equations have the same degree of homogeneity: they are homogeneous of degree zero in money prices *and* the quantity of money. Furthermore, without the use of the equation of exchange, Patinkin preserved the pure classical proposition that an increase in the money stock leads to an equiproportional increase in all prices, so that the relative prices remain unchanged. Despite money having been fully integrated into the neoclassical model, in equilibrium it is still neutral.

Patinkin used a variant of the above model to attack Keynes's *General Theory*. Keynes maintained that it is possible to have equilibrium with unemployment. Setting this proposition in the framework of a general equilibrium model of Patinkin's type, Keynes's theory implies that the labour market is in disequilibrium, while all the other markets for goods and money are in equilibrium. In Patinkin's models this situation cannot arise if prices of goods and factors are flexible, since, from Walras's Law, if the $N - 1$ markets are in equilibrium, then the Nth market must also be in equilibrium. A Keynesian equilibrium with involuntary unemployment can arise, according to Patinkin, only if wages are rigid. Thus the Keynesian equilibrium is a special case of the general 'neoclassical', equilibrium model: in Patinkin's view, Keynes's *General Theory* is neither 'general' nor a 'theory'.

The Keynesian counter-revolution

Although the introduction of real money balances represents a step forward in the development of the neoclassical theory of general equilibrium, this theory has serious shortcomings.

The main difficulty with Patinkin's neoclassical synthesis arises from the treatment of expectations. In the neoclassical monetary general equilibrium model the demand for money to hold arises from the wish of consumers to allocate their consumption over time, so as to maximise their utility. The inter-temporal choices are based on *expectations* of the market participants, but the neoclassical model assumes that these expectations are held with certainty: consumers are assumed to be perfectly informed about the exchange opportunities, present and future.

Keynes stressed the function of money as a store of wealth: consumers demand money not only for carrying out their transactions, but also to satisfy their precautionary and speculative needs. These motives for holding money arise from uncertainty and from risk-aversion, and, according to Keynes, these are the really important functions of money: 'the importance of money essentially flows from it being a link between the present and the future'; 'partly on reasonable and partly on instinctive grounds, our desire to hold money as a store of wealth is a barometer of the degree of our distrust of our own calculations and conventions concerning the future ...' (J. M. Keynes, 'The General Theory of Employment', *Quarterly Journal of Economics*, 1937). This fundamental uncertainty about the future cannot be dealt within the neoclassical models, which have an essentially static framework in which consumers have full information. Thus the neoclassical apparatus distorts the Keynesian revolution.

Clower[1] and Leijonhufvud[2] have stressed the fundamental uncertainty about the future, which enters the choice between holding money balances and commodities, and which can lead to the Keynesian type of equilibrium with unemployment. Within the neoclassical framework such equilibrium cannot arise because (by the assumption of certainty and complete information) price changes convey always the correct signals to the market participants so that resources are allocated in an optimal way and are fully employed.

Clower reintroduced the distinction between effective demands for goods, demands based on *realised* income, and notional demands, which are based on incomes *anticipated* (*expected*) by consumers. Only with full employment, Clower argued, would these coincide. In the real world expectations and realisations rarely coincide. If realised income falls short of expected income there will be an excess supply of goods, which leads to unemployment. As expectations are adjusted (downwards) the excess supply of goods would be absorbed and the commodity markets could be in equilibrium while the labour market would be in disequilibrium (unemployment).

Leijonhufvud argued that under uncertainty the choices of consumers between money to hold and commodity purchases are likely to give rise to excess demands and price changes which convey the wrong signals to producers. Such misleading information is inherent in uncertainty, and is at the heart of the failure of the resource allocation mechanism, that is, the creation of persistent unemployment, while commodity markets are in equilibrium. This is the situation with which Keynes was concerned. Given the uncertainty of the real world this sort of Keynesian equilibrium with unemployment can be considered as the general rather than the special case. In this view the neoclassical model is a special case of Keynes's general theory: the neoclassical monetary general equilibrium model deals with the special case of expectations held with certainty.

The neoclassical system never really analyses a monetary economy in which money is a store of wealth. Furthermore in the neoclassical system money as a medium of exchange is 'neutral': changes in monetary variables do not have any

[1] R. W. Clower, 'The Keynesian Counter-Revolution: A Theoretical Appraisal', in *The Theory of Interest Rates*, ed. F. H. Hahn and F. P. R. Brechling (Macmillan, 1965).

[2] A. Leijonhufvud, *On Keynesian Economics and the Economics of Keynes* (Oxford University Press, 1968).

effect upon the 'real' sector of the economy. This system can be criticised for continuing to treat a monetised economy as if it were in fact a barter economy from the point of view of exchange. Until Keynes no one had suspected that the existence of money as a medium of exchange might be a source of long-run disequilibrium in a market economy.

In conclusion we may say that the controversy is by no means settled. So far no general equilibrium model has satisfactorily been monetised. And it is very doubtful whether money can operationally be incorporated in a static general equilibrium system for the following reasons.

1. Money involves inter-temporal choices and requires a dynamic framework, whereas most general equilibrium models are static, single period.

2. Money is inescapably linked with uncertainty, whereas most general equilibrium models are built on the assumption of full information and expectations about the future which are held with confidence by the market participants.

3. Money has both short-run and long-run implications. General equilibrium systems are concerned with the nature of long-run equilibrium.

4. The 'neutrality' of money implied by the neoclassical general equilibrium systems is incompatible with the more sophisticated monetary institutions of the real world, such as the capital markets and the government monetary policy.

23. Welfare Economics

Welfare economics is concerned with the evaluation of alternative economic situations (states, configurations) from the point of view of the society's well-being.

To illustrate this definition assume that the total welfare in a country is W, but given the factor endowments (resources) and the state of technology, suppose that this welfare could be larger, for example W^*. The tasks of welfare economics are (*a*) to show that in the present state $W < W^*$, and (*b*) to suggest ways of raising W to W^*.

A. CRITERIA OF SOCIAL WELFARE

To evaluate alternative economic situations we need some criterion of social well-being or welfare. The measurement of social welfare requires some ethical standard and interpersonal comparisons, both of which involve subjective value judgements. Objective comparisons and judgements of the deservingness or worthiness of different individuals are virtually impossible.

Various criteria of social welfare have been suggested by economists at different times. We will discuss briefly some of these criteria.

1. GROWTH OF GNP AS A CRITERION OF WELFARE

Adam Smith implicitly accepted the growth of the *wealth* of a society, that is, the growth of the gross national product, as a welfare criterion. He believed that economic growth resulted in the increase of social welfare because growth increased employment and the goods available for consumption to the community. To Adam Smith, economic growth meant bringing W closer to W^*.

The growth criterion implies acceptance of the *status quo* of income distribution as 'ethical' or 'just'. Furthermore, growth may lead to a reduction in social welfare, depending on who avails mostly from it. However the growth criterion highlights the importance of *efficiency* in social welfare. Given that social welfare depends on the amount of goods and services (as well as on their distribution) efficiency is a necessary prerequisite for the maximisation of the level of welfare. We have discussed briefly the conditions of efficiency in Chapter 22, and we will examine them in detail in a subsequent section of this chapter. We note here that economic efficiency can be defined objectively, and the modern welfare economics is mainly concerned with the examination (comparison) of the (Pareto)-efficiency of different economic situations. However, efficiency, although a necessary condition, is not sufficient to guarantee the maximisation of social welfare. Efficiency does not dispose of the need of having an ethical standard of comparing alternative economic states.

2. BENTHAM'S CRITERION

Jeremy Bentham, an English economist, argued that welfare is improved when 'the greatest good (is secured) for the greatest number'. Implicit in this dictum is the assumption that the total welfare is the sum of the utilities of the individuals of the society. The application of Bentham's ethical system to economics has serious shortcomings. To illustrate the pitfalls in Bentham's criterion let us assume that the society consists of three individuals, A, B, and C, so that

$$W = U_A + U_B + U_C$$

According to Bentham $\Delta W > 0$ if $(\Delta U_A + \Delta U_B + \Delta U_C) > 0$. However, assume that the change which resulted in the changes in the individual utilities is such, that A's and B's utility increases, while C's utility decreases, but $(\Delta U_A + \Delta U_B) > |\Delta U_C|$. In other words, two individuals are better-off while the third is worse-off after the change has taken place, but the sum of the increases in utilities of A and B is greater than the decrease in the utility of C.

Bentham's criterion, obviously, implies that A and B have a greater 'worthiness' than C. That is, implicit in Bentham's criterion is an interpersonal comparison of the deservingness of the members of the society. There is another difficulty with the application of Bentham's criterion. This criterion cannot be applied to compare situations where 'the greatest good' and the 'greatest numbers' do not exist simultaneously. For example assume that in a situation $U_A = 200$, $U_B = 50$, $U_C = 30$, so that the total utility in the society is 280. In another situation assume that a change occurred and $U_A = 100$, $U_B = 80$, and $U_C = 80$, so that the total utility is 260. The first situation has 'the greatest good' (280 > 260), but the second involves a more even distribution (of a smaller 'total good') among the 'greatest number'.

3. A 'CARDINALIST' CRITERION

Several economists proposed the use of the 'law of diminishing marginal utility' as a criterion of welfare. Their argument can be illustrated by the following example. Assume that the society consists of three individuals; A has an income of £1000, while B and C have an income of £500 each. Consumer A can buy double quantities of goods as compared to B and C. However, given the law of diminishing marginal utility, A's total utility is less than double the total utility of either B or C, because A's marginal utility of money is less than that of B or C. Thus $W < W^*$. To increase social welfare income should be redistributed among the three individuals. In fact cardinal welfare theorists would maintain that social welfare would be maximised if income was equally distributed to all members of the society.

The cardinalist approach to welfare has a serious flaw: it assumes that all individuals have identical utility functions for money, so that with an equal income distribution all would have the same marginal utility of money. This assumption is too strong. Individuals differ in their attitudes towards money. A rich person may have a utility for money function that lies far above the utility (for money) function of poorer individuals. In this case a redistribution of income (towards more equality) might reduce total welfare. Opponents of the cardinalist approach pointed out also that welfare effects of an equal distribution of income cannot be examined in isolation from the effects on resource allocation (which would follow the redistribution of income) and the incentives for work of the various individuals. An equal income distribution may induce some people to work less, thus leading to a reduction in total GNP. Similarly, equal incomes in all employments may lead to an allocation of resources which produces a smaller total output. In both cases income equality results in (Pareto) inefficiency in the use of resources and a reduction in social welfare.

4. THE PARETO-OPTIMALITY CRITERION

This criterion refers to economic efficiency which can be objectively measured. It is called *Pareto criterion* after the famous Italian economist Vilfredo Pareto (1848–1923). According to this criterion any change that makes at least one individual better-off and no one worse-off is an improvement in social welfare. Conversely, a change that makes no one better-off and at least one worse-off is a decrease in social welfare.

The criterion can be stated in a somewhat different way: a situation in which it is impossible to make anyone better-off without making someone worse-off is said to be *Pareto-optimal* or *Pareto-efficient.*

For the attainment of a Pareto-efficient situation in an economy three marginal conditions must be satisfied: (*a*) Efficiency of distribution of commodities among consumers (efficiency in exchange); (*b*) Efficiency of the allocation of factors among firms (efficiency of production); (*c*) Efficiency in the allocation of factors among commodities (efficiency in the product-mix, or composition of output). Before examining these marginal conditions we discuss briefly the main weaknesses of the Pareto criterion.

The Pareto criterion cannot evaluate a change that makes some individuals better-off and others worse-off. Since most government policies involve changes that benefit some and harm others it is obvious that the strict Pareto criterion is of limited applicability in real-world situations. A simplified version of it is presented in section E below.

Furthermore, a Pareto-optimal situation does not guarantee the maximisation of the social welfare. For example, we said in Chapter 22 that any point on the production possibility curve represents a Pareto-efficient situation. To decide which of these points yields maximum social welfare we need an interpersonal comparison of the individual consumer's utility. In a subsequent section we will show that the Pareto-optimal state is a necessary but not sufficient condition for maximum social welfare.

Let us examine now the three marginal conditions that must be satisfied in order to attain a Pareto-efficient situation in the economy. We will assume that the assumptions of the 2 × 2 × 2 model, developed in Chapter 22, hold.

(a) Efficiency of distribution of commodities among consumers

Applying the Pareto optimality criterion to the case of distribution of commodities Y and X, we can say that a distribution of the given commodities X and Y between the two consumers is efficient if it is impossible by a redistribution of these goods to increase the utility of one individual without reducing the utility of the other. In figure 23.1 we show the Edgeworth box for the given commodities X and Y. We have shown in Chapter 22 that only points *on* the Edgeworth contract curve satisfy the Pareto-optimality condition. Any other distribution off the contract curve is inefficient. For example, point z is inefficient, since a redistribution of the commodities such as to reach any point between a and b increases the utility of both consumers. A movement to a increases the utility of B without reducing the utility of A. Similarly, the distribution implied by b increases the utility of A without reducing the utility of B. Thus all the points from a to b represent improvements in social welfare compared with the distribution at z. By reversing the argument it can be seen that a movement from a point on the contract curve to a point off it results in a decrease in social welfare. Thus the contract curve shows the locus of Pareto-optimal or -efficient

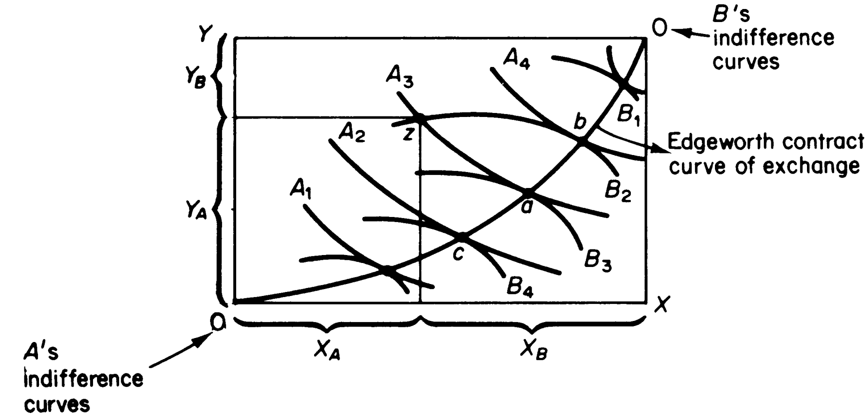

Figure 23.1 Edgeworth box of exchange

distribution of goods between consumers. This curve is formed from the points of tangency of the two consumers' indifference curves, that is, points where the slopes of the indifference curves are equal. In other words, at each point of the contract curve the following condition is satisfied

$$MRS^A_{x,y} = MRS^B_{x,y}$$

Therefore we may state the marginal condition for a Pareto-efficient distribution of given commodities as follows:

The marginal condition for a Pareto-optimal or -efficient distribution of commodities among consumers requires that the MRS *between two goods be equal for all consumers.*

(*b*) Efficiency of allocation of factors among firm-producers

To derive the marginal condition for a Pareto-optimal allocation of factors among producers we use an argument closely analogous to the one used for the derivation of the marginal condition for optimal distribution of commodities among consumers. In the case of allocation of given resources K and L we use the Edgeworth box of production which we explained in detail in Chapter 22. Such a construct is shown in figure 23.2.

Only points on the contract curve of production are Pareto-efficient. Point H is inefficient, since a reallocation of the given K and L between the producers of X and Y such as to reach any point from c to d inclusive results in the increase of at least one commodity without a reduction in the other.

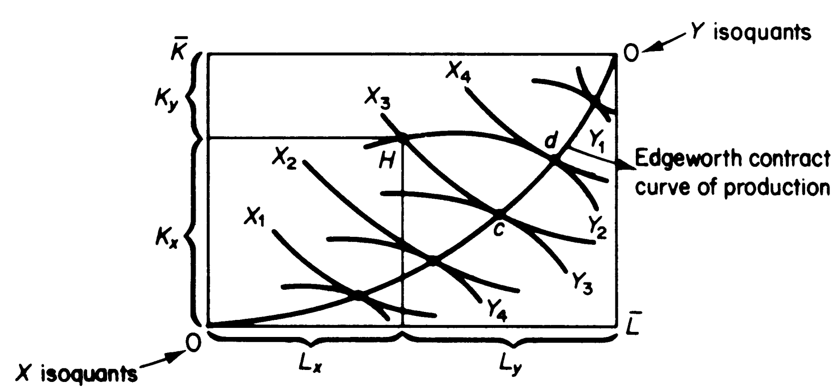

Figure 23.2 Edgeworth box of production

The contract curve is the locus of points of tangency of the isoquants of the two firms which produce X and Y, that is, points where the slopes of the isoquants are equal. Thus at each point of the contract curve the following condition holds

$$MRTS^x_{L,K} = MRTS^y_{L,K}$$

Therefore we may state the marginal condition for a Pareto-optimal allocation of factors among firms as follows:

The marginal condition for a Pareto-optimal allocation of factors (inputs) requires that the MRTS *between labour and capital be equal for all commodities produced by different firms.*

(c) Efficiency in the composition of output (product-mix)

The third possible way of increasing social welfare is a change in the product-mix. To define the third marginal condition of a Pareto-optimal state in an economy we will use the production possibility curve, which we derived in Chapter 22. Recall that the slope of the *PPC* is called the 'marginal rate of (product) transformation' ($MRPT_{x,y}$), and it shows the amount of Y that must be sacrificed in order to obtain an additional unit of X. In other words the $MRPT$ is the rate at which a good can be transformed into another.

The marginal condition for a Pareto-optimal or -efficient composition of output requires that the MRPT *between any two commodities be equal to the* MRS *between the same two goods:*

$$MRPT_{x,y} = MRS^A_{x,y} = MRS^B_{x,y}$$

Since the $MRPT$ *shows the rate at which a good can be transformed into another* (on the production side), and the MRS shows the rate at which consumers are willing to exchange a good for another, the rates must be equal for a Pareto-optimal situation to be attained. Suppose that these rates are unequal. For example assume

$$MRPT_{x,y} = \frac{2Y}{1X} \qquad \text{and} \qquad MRS_{x,y} = \frac{1Y}{1X}$$

that is,

$$MRPT_{x,y} > MRS_{x,y}$$

The above inequality shows that the economy can produce two units of Y by sacrificing one unit of X, while the consumers are willing to exchange one unit of Y for one unit of X. Clearly the economy produces too much of X and too little of Y relatively to the tastes of consumers. Welfare therefore can be increased by increasing the production of Y and decreasing the production of X. (This example was presented in more detail in Chapter 22, page 503.)

In summary. A Pareto-optimal state in the economy can be attained if the following three marginal conditions are fulfilled:

1. The $MRS_{x,y}$ between any two goods be equal for all consumers.
2. The $MRTS_{L,K}$ between any two inputs be equal in the production of all commodities.
3. The $MRPT_{x,y}$ be equal to the $MRS_{x,y}$ for any two goods.

A situation may be Pareto-optimal without maximising social welfare. However, welfare maximisation is attained only at a situation that is Pareto-optimal. In other words, Pareto optimality is a necessary but not sufficient condition for welfare maximisation. All points on the *PPC* are Pareto-optimal. The choice among these alternative Pareto-optimal states requires some measure or criterion of social welfare. In a subsequent section we will use one such criterion, namely Bergson's social welfare function.

5. THE KALDOR–HICKS 'COMPENSATION CRITERION'

Nicholas Kaldor[1] and John Hicks[2] suggested the following approach to establishing a welfare criterion.

Assume that a change in the economy is being considered, which will benefit some ('gainers') and hurt others ('losers'). One can ask the 'gainers' how much money they would be prepared to pay in order to have the change, and the 'losers' how much money they would be prepared to pay in order to prevent the change. If the amount of money of the 'gainers' is greater than the amount of the 'losers', the change constitutes an improvement in social welfare, because the 'gainers' *could* compensate the 'losers' and still have some 'net gain'. Thus, the Kaldor–Hicks 'compensation criterion' states that a change constitutes an improvement in social welfare if those who benefit from it could compensate those who are hurt, and still be left with some 'net gain'.

The Kaldor–Hicks criterion evaluates alternative situations on the basis of monetary valuations of different persons. Thus it implicitly assumes that the marginal utility of money is the same for all the individuals in the society. Given that the income distribution is unequal in the real world, this assumption is absurd. Assume, for example, that the economy consists of two individuals, *A*, who is a millionaire, and *B*, who has an income of £4000. Suppose that the change (being considered by the government) will benefit *A*, who is willing to pay £2000 for this change to happen, while it will hurt *B*, who is prepared to pay £1000 to prevent the change. According to the Kaldor–Hicks criterion the change will increase the social welfare (since the 'net gain' to *A*, after he compensates *B*, is £1000). However, the gain of £2000 gives very little additional utility to millionaire *A*, while the 'loss' of £1000 will decrease a lot the well-being of *B*, who has a much greater marginal utility of money than *A*. Thus the total welfare will be reduced if the change takes place. Only if the marginal utility of money is equal for all the individuals would the Kaldor–Hicks criterion be a 'correct' welfare measure. This criterion ignores the existing income distribution. In fact this criterion makes implicit interpersonal comparisons, since it assumes that the same amounts of money have the same utility for individuals with different incomes.

6. THE BERGSON CRITERION: THE SOCIAL WELFARE FUNCTION

The various welfare criteria so far discussed show that when a change in the economy benefits some individuals and hurts others it is impossible to evaluate it without making some value judgement about the deservingness of the different individuals or groups. Bergson[3] suggested the use of an explicit set of value judgements in the form of a *social welfare function*. A social welfare function is analogous to the individual consumer's utility function. It provides a ranking of alternative states

[1] N. Kaldor, 'Welfare Propositions in Economics and Interpersonal Comparisons of Utility', *Economic Journal* (1939), pp. 549–52.

[2] J. Hicks, 'The Foundations of Welfare Economics', *Economic Journal* (1939), pp. 696–712.

[3] A. Bergson, 'A Reformulation of Certain Aspects of Welfare Economics', *Quarterly Journal of Economics* (1937–8), pp. 310–34.

(situations, configurations) in which different individuals enjoy different utility levels. If the economy consists of two individuals the social welfare function could be presented by a set of social indifference contours (in utility space) like the ones shown in figure 23.3. Each curve is the locus of combinations of utilities of *A* and *B* which yield the same level of social welfare. The further to the right a social indifference contour is, the higher the level of social welfare will be. With such a set of social indifference contours alternative states in the economy can be unambiguously evaluated. For example a change which would move the society from point *b* to point *c* (or *d*) increases the social welfare. A change moving the society from *a* to *b* leaves the level of social welfare unaltered.

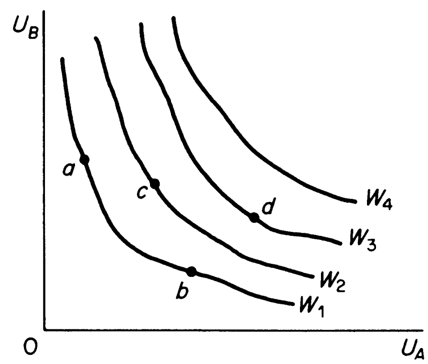

Figure 23.3 Bergson's welfare contours

The problem with the social welfare function is that there is no easy method of constructing it. Its existence is axiomatically assumed in welfare economics (see below). Somebody in the economy must undertake the task of comparing the various individuals or groups and rank them according to what he thinks their worthiness is. A democratically elected government could be assumed to make such value judgements which would be acceptable by the society as a whole. This is implicitly or explicitly assumed when use is made of the apparatus of the social welfare function.

It should be noted that the social welfare function cannot be used to derive social (or community) indifference curves in output space (analogous to the indifference curves of a single individual) without taking into account the distribution of income among the various individuals in the economy. In a subsequent section we will examine the conditions under which community indifference curves in output space can be derived from the social welfare function.[1]

B. MAXIMISATION OF SOCIAL WELFARE

In this section we will examine the conditions of social welfare maximisation in the simple two-factor, two-commodity, two-consumer model. The assumptions of our analysis are listed below.

1. There are two factors, labour *L*, and capital *K*, whose quantities are given (in perfectly inelastic supply). These factors are homogeneous and perfectly divisible.

[1] P. A. Samuelson, 'Social Indifference Curves', *Quarterly Journal of Economics* (1956), pp. 1–22.

2. Two products, X and Y, are produced by two firms. Each firm produces only one commodity. The production functions give rise to smooth isoquants, convex to the origin, with constant returns to scale. Indivisibilities in the production processes are ruled out.

3. There are two consumers whose preferences are represented by indifference curves, which are continuous, convex to the origin and do not intersect.

4. The goal of consumers is utility maximisation and the goal of firms is profit maximisation.

5. The production functions are independent. This rules out joint products and external economies and diseconomies in production.

6. The utilities of consumers are independent. Bandwagon, snob and Veblen effects are ruled out. There are no external economies or diseconomies in consumption.

7. The ownership of factors, that is, the distribution of the given L and K between the two consumers, is exogenously determined.

8. A social welfare function, $W = f(U_A, U_B)$, exists. This permits a unique preference-ordering of all possible states, based on the positions of the two consumers in their own preference maps. This welfare function incorporates an ethical valuation of the relative deservingness or worthiness of the two consumers.

The problem is to determine the welfare-maximising values of the following variables:

(*a*) The welfare-maximising commodity-mix, that is the total quantity of X and Y (production problem).

(*b*) The welfare-maximising distribution of the commodities produced between the two consumers, X_A, X_B, Y_A, Y_B (distribution problem).

(*c*) The welfare-maximising allocation of the given resources in the production of X and Y, L_x, L_y, K_x, K_y (allocation problem).

In summary. In the $2 \times 2 \times 2$ model we have ten unknowns, and we have to find the values of these unknowns which maximise the social welfare.

The social welfare function is one of the tools which we will use in finding the situation which maximises social welfare. For this purpose, however, we need another tool, the *grand utility possibility frontier*. This shows the maximum utility attainable by B, given the utility enjoyed by A from any given product-mix.

1. DERIVATION OF THE GRAND UTILITY POSSIBILITY FRONTIER

We have seen in Chapter 22 that each product-mix (combination of X and Y) can be distributed optimally among the two consumers in an infinite number of ways, represented by the points of the Edgeworth contract curve of exchange corresponding to that particular output combination. Take, for example, the combination $Y_0 X_0$, denoted by point a on the production possibility curve FF' of figure 23.4. Each point on the contract curve $0a$ implies a different distribution of the two commodities between the two consumers, and hence a different combination of utilities. For example, point c denotes the utility combination A_2 (for consumer A) and B_8 (for consumer B). We can plot this utility combination in the utility space, that is a graph on whose axes we measure the utility of the two consumers (in ordinal utility indexes). Point c' in figure 23.5 represents the $A_2 B_8$ utility combination; it shows the maximum utility attainable by B (B_8), given the utility enjoyed by A (A_2) from the distribution of $Y_0 X_0$ as shown by point c in figure 23.4. We say that c in output space 'maps' into point c' in utility space.

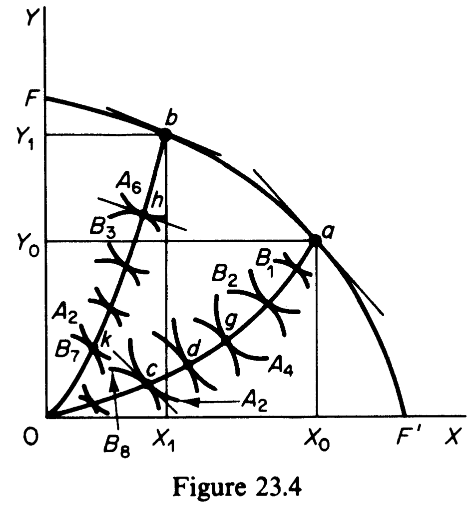

Figure 23.4

We may repeat this process for any other distribution of the given $Y_0 X_0$ product-mix. For example, at point g in figure 23.4 the two consumers enjoy the utilities A_4 and B_3. This utility combination is shown by point g' in the utility space of figure 23.5. Point g in output space maps into g' in utility space. Point g' shows the maximum utility attainable to B (B_3) given the utility of A (A_4) from the distribution of $Y_0 X_0$ denoted by point g in figure 23.4. By mapping all the points of the contract curve $0a$ into corresponding points in the utility space we obtain the utility possibility frontier for the particular commodity-mix $Y_0 X_0$ (curve SS' in figure 23.5). The utility possibility curve SS' is drawn for the specific product-mix $Y_0 X_0$, and it shows maximum utility possibilities when the economy produces this specific combination of commodities. Since there is an infinite number of points on the *PPC* curve, there must be an infinite number of utility possibility curves, each such curve for each product-mix on the production possibility curve. For example, assume that the economy produces the output-mix $Y_1 X_1$ denoted by point b on the *PPC* curve of figure 23.4. The points on the Edgeworth contract curve $0b$ show Pareto-optimal distributions of the product-mix $Y_1 X_1$. Point h shows the utility combination $A_6 B_3$, which is depicted by point h' in the utility space of figure 23.5. Similarly, point k in output space is mapped into point k' in utility space. The remaining points of the $0b$ contract curve are mapped into the points of the utility possibility frontier RR' in figure 23.5.

In summary, each point on the *PPC* gives rise to a utility possibility frontier. The *envelope* of these utility possibility frontiers is *the grand utility possibility frontier* of the economy.

There is an alternative way of deriving the grand utility possibility frontier, which is much simpler. It makes use of the third marginal condition of Pareto optimality, that the slope of the *PPC* be the same as the 'equalised' *MRS* of the two commodities for the two consumers ($MRPT_{x,y} = MRS^A_{x,y} = MRS^B_{x,y}$). For any commodity combination produced in the economy, such as a on the *PPC* in figure 23.4, we pick the point on the corresponding contract curve ($0a$) which has the same slope as the *PPC* at a. At this point of the contract curve (c in figure 23.4) $MRPT = MRS$. From the utilities of consumers A and B associated with c we can obtain c' in the utility space of figure 23.5; c' shows the utilities of A and B when the product-mix $Y_0 X_0$ (denoted by a) is distributed between them in a way satisfying the Pareto-optimality condition $MRPT = MRS$. Point c' shows the maximum or *grand utility* attainable to the society from the output combination $Y_0 X_0$. Thus only point c' of the SS' utility frontier belongs to the 'envelope' (or 'grand') utility possibility frontier.

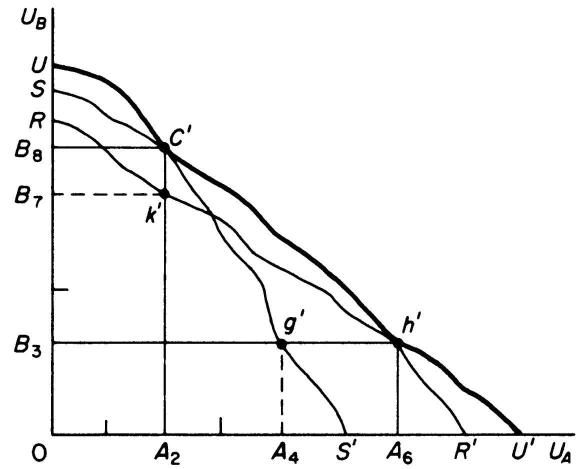

Figure 23.5 The grand utility possibility frontier

For each point on the *PPC* we can repeat the above procedure and obtain a point in utility space. For example, if the product-mix is *b*, point *h* on the corresponding 0*b* contract curve indicates the optimal distribution of this product-mix between the two consumers. The 'grand' utility associated with this distribution is represented by point *h'* in the utility space of figure 23.5. Repetition of this procedure for each point on the *PPC* yields the 'envelope' utility possibility frontier. This is shown by the curve *UU'* in figure 23.5. Thus, the grand utility possibility frontier is the locus of utility combinations (of the two consumers) which satisfy the marginal condition $MRPT = MRS$ for each commodity-mix. Each point of the 'envelope' shows the maximum of *B*'s utility for any given feasible level of *A*'s utility, and vice versa.

It should be clear from the above discussion that all points on the grand utility possibility frontier satisfy all the Pareto marginal conditions of efficiency: efficiency in production, efficiency in distribution, efficiency in product composition.

2. DETERMINATION OF THE WELFARE-MAXIMISING STATE: THE 'POINT OF BLISS'

In figure 23.6 the grand utility possibility frontier is combined with the social welfare function shown by the set of social indifference contours.

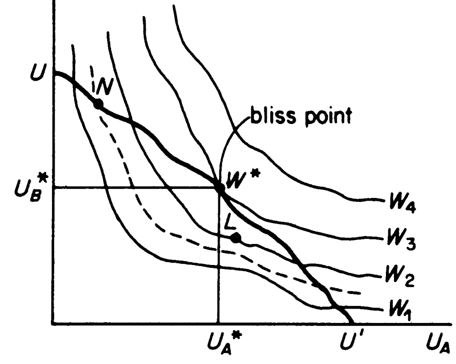

Figure 23.6 Maximisation of social welfare

Social welfare is maximised at the point of tangency of the 'envelope' utility possibility frontier with the highest possible social indifference contour. This point is called 'the point of bliss'. It is denoted by W^* in figure 23.6. The maximum social welfare attainable in our example is the level implied by the indifference contour W_3. The two consumers will enjoy the levels of utility U_A^* and U_B^*.

We can now see why the Pareto-optimality is a necessary but not sufficient condition for welfare maximisation. The welfare maximisation will occur at a point *on* the 'envelope' utility possibility frontier, and we saw that all points on this frontier satisfy all three conditions of Pareto optimality. Thus, the point of welfare maximisation is a Pareto-optimal state. However, a large number of points *below* the grand utility frontier, although not Pareto-optimal, yield a higher level of social welfare than points *on* the utility frontier. For example, point N in figure 23.6 is a Pareto-optimal situation while point L is not. Yet L lies on a higher social indifference contour than point N. However, it can be shown that, given any inefficient point (below the 'envelope' utility frontier), there will exist some point(s) on the grand utility frontier that represents an improvement in social welfare.

C. DETERMINATION OF THE WELFARE-MAXIMISING OUTPUT-MIX, COMMODITY DISTRIBUTION AND RESOURCE ALLOCATION

From the bliss point we can determine the optimising (welfare-maximising) values of the ten unknowns of the $2 \times 2 \times 2$ model:

(*a*) the total quantities of the two products X^* and Y^*;
(*b*) the quantities of X^* and Y^* that will be consumed by the two consumers: X_A^*, X_B^*, Y_A^*, Y_B^*;
(*c*) the quantities of capital and labour which will be used in the production of the optimal product-mix, L_x, L_y, K_x, K_y.

By retracing our steps we will show that the maximum welfare configuration (state) is determinate, in that it yields unique values to the above ten unknowns involved.

The constrained bliss point W^* is associated with a unique commodity-mix on the production possibility curve, because: W^* defines the utility combination $U_A^* U_B^*$ $\rightarrow$ which belongs to a particular utility frontier. $\rightarrow$ But recall that each utility frontier is derived from a single (unique) point on the production possibility curve. Thus, assume that the utility possibility frontier, to which $U_A^* U_B^*$ belong, corresponds to the unique point W' on the production possibility curve FF' in figure 23.7. Point W' defines the welfare-maximising commodity combination Y^*X^*.

We next construct the Edgeworth box of exchange with precise coordinates the welfare-maximising levels of outputs Y^* and X^*. By examining the contract curve $0W'$ we can locate the one point where the utilities of the two consumers are U_A^* and U_B^*. At this point (W'' in figure 23.7) the 'equalised' slope of the indifference curves of consumers A and B is equal to the slope of the PPC at W'. Point W'' defines the distribution of X^* and Y^* between the two consumers (X_A^*, X_B^*, Y_A^* and Y_B^* in figure 23.7).

Finally, point W on the production possibility curve defines point W''' on the contract curve of the Edgeworth box of production. (The PPC was derived, point by point, from the contract curve of the Pareto-efficient input locus.) In figure 23.8 point W''' is the point of tangency of the isoquants Y^* and X^* implied by the bliss point. By drawing perpendiculars through W''' we define the amount of capital K and labour L

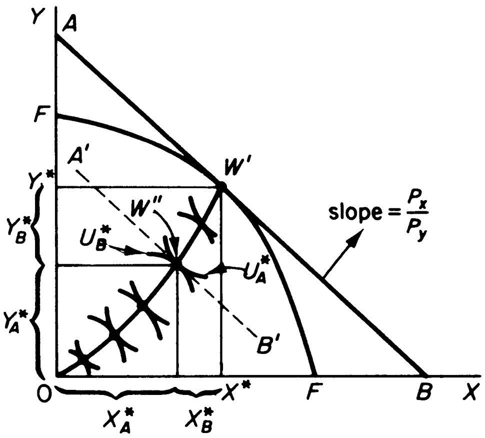

Figure 23.7

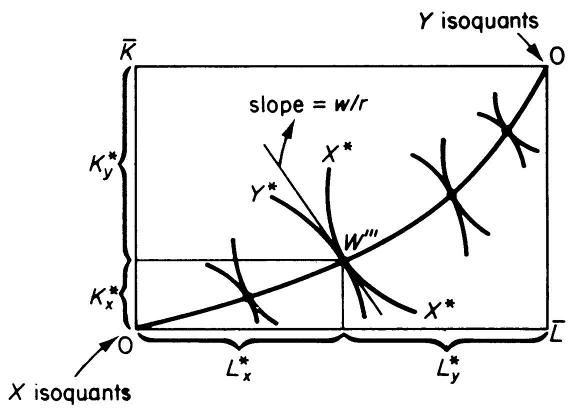

Figure 23.8 Optimal allocation of resources

which will be used in the production of Y^* and X^*. The welfare-maximising resource allocation is shown by the length of the segments marked by the brackets and denoted by the symbols L_x^*, L_y^*, K_x^* and K_y^*.

In summary, the maximum-welfare configuration is determinate. We have solved (found unique values) for the total outputs, the distribution of these outputs between the two consumers, and for the labour and capital to be used in the production of the welfare-maximising outputs.[1]

[1] An interesting consequence of the concave shape of the *PPC* is that the *value* of the product-mix that maximises social welfare, estimated at the 'shadow-prices' embeded in the system is at a maximum. For the definition of the 'prices' implied by the solution of the welfare-maximisation problem, see below, p. 537). An examination of figure 23.9 makes this clear. The line *AB*, whose slope is the ratio of the 'prices' of the commodities, P_x/P_y, can be thought of as an 'isovalue' line. The equation of the line *AB* is derived from the relation

$$V = P_x \cdot X + P_y \cdot Y$$

where V = total value of the output.

Solving for Y we obtain the *AB* line

$$Y = (1/P_y) \cdot V - (P_x/P_y) \cdot X$$

Given the prices, we can form a family of isovalue curves, by assigning different values to V. The higher the line, the greater the value of the total output will be.

D. WELFARE MAXIMISATION AND PERFECT COMPETITION

We have demonstrated that under certain assumptions an economy can reach the point of maximum social welfare. It should be stressed that the bliss point (and the solution of the system for the values of the ten variables that are the unknowns in the welfare-maximisation problem of the $2 \times 2 \times 2$ model) depends only on technological relations: the problem of welfare maximisation is purely 'technocratic'. Recall that the bliss point is attained by equalising the slopes of isoquants, the slopes of the indifference curves, and the slope of the production possibility curve to the (equalised) slope of the indifference curves. Thus, the welfare-maximising solution does not depend on prices. However, in Chapter 22 we have shown that perfect competition can lead to a general equilibrium situation where the three marginal conditions of Pareto optimality are satisfied. The analysis of Chapter 22 can now be extended to show that the general equilibrium solution reached with perfect competition is the same as the situation implied by the bliss point of the welfare maximisation problem.

(*a*) Profit maximisation by the individual firm implies that whatever output the firm may choose as the most profitable must be produced at a minimum cost. Cost minimisation is attained if the firm chooses the input combination at which the marginal rate of technical substitution of the two factors is equal to the input price ratio

$$MRTS_{L,K} = \frac{w}{r}$$

Since in perfect competition all firms are faced by the same set of factor prices, it follows that

$$MRTS^x_{L,K} = MRTS^y_{L,K} = \frac{w}{r}$$

Thus in figure 23.8 the slope of the contract curve at W'''' must be equal to w/r in perfectly competitive input markets, since W'''' is the point of the (equalised) slope of the isoquants Y^* and X^*.

(*b*) Utility maximisation by each individual requires the choice of the product-mix where the marginal rate of substitution of the two commodities is equal to the ratio

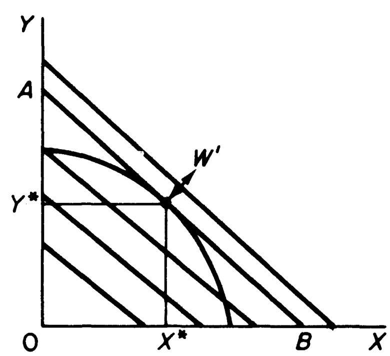

Figure 23.9

Now, the welfare-maximising commodity-mix is defined by the point of tangency of the production possibility curve with the line *AB*, which is the highest possible isovalue line in our example. Hence, at the price ratio implied by the line *AB*, point *W'* defines the welfare-maximising quantities of the two commodities, and at the same time the highest output-value.

of their price

$$MRS_{x,y} = \frac{P_x}{P_y}$$

Since in perfect competition all consumers are faced by the same commodity prices, it follows that

$$MRS^A_{x,y} = MRS^B_{x,y} = \frac{P_x}{P_y}$$

Thus in figure 23.7 the slope of the contract curve at W'' must be equal to the P_x/P_y ratio in perfectly competitive commodity markets, since W'' is the point of the (equalised) slope of the indifference curves of the two consumers.

(*c*) We have shown that at the bliss point the (equalised) slope of the indifference curves (the 'common' $MRS_{x,y}$) is equal to the slope of the production possibility curve at W''. Thus at W'', which defines the product-mix that maximises social welfare, we have

$$MRPT_{x,y} = \frac{P_x}{P_y}$$

(*d*) Finally, a firm in a perfectly competitive market maximises its profit by setting its marginal cost equal to the market price of the commodity. Consequently we have

$$MRPT_{x,y} = \frac{P_x}{P_y} = \frac{MC_x}{MC_y}$$

Thus we have established that a perfectly competitive system guarantees the attainment of maximum social welfare. This is the result of the maximising behaviour of firms and consumers. In a perfectly competitive (free enterprise) system, each individual, in pursuing his own self-interest, is led by an 'invisible hand' to a course of action that increases the general welfare of all.[1]

However, perfect competition is only one way for attaining welfare maximisation. A decentralised socialist system, for example, in which the government has somehow estimated 'shadow' prices and directs its individual economic units to maximise their 'gains', can in principle achieve the same results as a perfectly competitive system.[2]

In summary. Implicit in the purely 'technocratic' problem of welfare maximisation is a set of 'prices'. Decentralised decisions in response to these 'prices' by atomistic profit maximisers and utility maximisers will result in just that levels of inputs, outputs and commodity-distribution that the bliss point requires. The individual maximisers can be acting in a perfectly competitive system, or in a decentralised social system of the Lerner–Lange type, where bureaucrats have established somehow the 'prices' that maximise social welfare and coerce the citizens to act in response to these 'prices'.

The above '*duality theorem*' is the kernel of modern welfare economics. This 'duality theorem' may be stated as follows. Welfare maximisation can be attained by maximising behaviour of individuals, given the technological relations of the production function, ordinal indexes of the utility of consumers, and given a social welfare function. The welfare maximisation is independent of prices. However, implicit in the

[1] See Adam Smith, *An Inquiry into the Nature and Causes of the Wealth of Nations* (London, Methuen, 1950).

[2] See Abba P. Lerner, *The Economics of Control* (Macmillan, New York, 1944). Also R. Dorfman, P. A. Samuelson, and R. Solow, *Linear Programming and Economic Analysis* (McGraw-Hill, 1958).

logic of this purely technocratic formulation is a set of constants. These constants can be the prices of a perfectly competitive economy, or the 'shadow prices' of a socialist economy. Thus, if these 'prices' (constants) are known (as, for example, the prices of a perfectly competitive system), and individual profit maximisers and utility maximisers act in response to these prices, their behaviour will lead to the maximisation of social welfare.[1]

E. CRITIQUE AND EXTENSIONS

We have examined the maximisation problem of modern welfare economics within the framework of the simple $2 \times 2 \times 2$ static model. We turn now to the examination of its assumptions, to see which are simplifying, in the sense that they can be relaxed without essentially affecting the model, and which are restrictive, in the sense that if they do not hold the model collapses.

1. EXTENSION TO MANY FACTORS, PRODUCTS AND CONSUMERS

The model can be extended to any number of resources, commodities and consumers. We have examined the general $H \times M \times N$ model in the Appendix to Chapter 22.

2. CORNER SOLUTIONS

We have implicitly assumed that the two sets of isoquants provide a smooth locus of 'internal' tangencies, that is, the contract curve of production was assumed to lie inside the Edgeworth box. This assumption is not particularly harmful to the model. To cover cases where the tangency of the isoquants occurs outside the Edgeworth box of production (in which case the solution will be *on* the axes of the box, that is, there will be a 'corner solution') we must state the condition of equilibrium of production in the form of an inequality rather than an equality of the $MRTS_{L,K}$. In particular the condition

$$MRTS^x_{L,K} = MRTS^y_{L,K}$$

must be replaced by the condition

$$(MRTS^x_{L,K}) < (MRTS^y_{L,K})$$

or

$$\begin{Bmatrix}\text{slope of the}\\ X\text{ isoquants}\end{Bmatrix} < \begin{Bmatrix}\text{slope of the}\\ Y\text{ isoquants}\end{Bmatrix}$$

To understand this consider the figures 23.10 and 23.11.
Point a in figure 23.10 represents a 'corner solution'. At a the slope of the X isoquant is smaller than the slope of the Y isoquant, so that a shows the maximum output of X (i.e. X^*) given the output of Y (Y^* in figure 23.10).

Point b in figure 23.11 is *not* an equilibrium solution, because it does not show the maximum of X, given Y. Actually, given Y, the maximum quantity of X is at point b'. (Apparently $X' > X$). Note that at b the slope of the X isoquant is larger than the

[1] See F. M. Bator, 'The Simple Analytics of Welfare Maximisation', *American Economic Review* (1957), pp. 22–59.

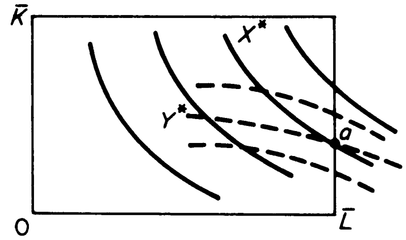

Figure 23.10

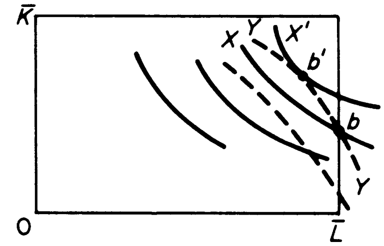

Figure 23.11

slope of the Y isoquant, so that the 'inequality' equilibrium condition is not satisfied.

In summary, for a corner solution to occur the Y isoquant must cut (on the axis of the Edgeworth box) the X isoquant from below.

3. EXISTENCE OF COMMUNITY INDIFFERENCE CURVES IN OUTPUT SPACE

The bliss point, where the social welfare function was maximised, was defined in *utility space*, that is, on the graph on whose axes we measure the utilities of the two consumers (in ordinal indexes), while the utility-maximising position of an individual consumer is determined in *output space* (i.e. on a graph on whose axes we measure the quantities of the two commodities). The social welfare contours are *not* social or community indifference curves (equivalent to a single individual's indifference curves). For a *single individual*, a given XY combination (in output space) belongs to a unique indifference curve and has a unique slope ($MRS^A_{x,y}$). However, if the particular XY combination is distributed between *two consumers*, a community indifference curve passing through XY (in output space) would not have a unique slope, because this slope would be sensitive to the way that the product mix XY is distributed among the two individuals. Recall that a particular product-mix can be distributed optimally among A and B in an infinite number of ways, along the contract curve of the Edgeworth box of exchange corresponding to this product mix. Each distribution (of the given product-mix) has a different (equalised) MRS for the two individuals, because it corresponds to a different utility combination. These utility combinations, corresponding to the different distributions, form the utility possibility frontier for the particular commodity-mix. Accordingly, the community MRS at a given point in commodity space (i.e. the slope of a community indifference curve) will vary with movements along the corresponding utility possibility frontier (i.e. with the distribution of the XY combination between A and B).

In summary, we can say that a point in output space maps to a *curve* in utility space; and a point in utility space maps into a *curve* in output space. Not just one, but many possible XY combinations can yield a specified $U_A U_B$ mix. It is this reciprocal point-line phenomenon that lies at the heart of Samuelson's proof of the non-existence of community indifference curves. The community MRS for a given fixed Y and X combination depends on how X and Y are distributed among A and B, i.e. on which $U_A U_B$ point on the Edgeworth contract curve of exchange is chosen. Hence the slope of a community indifference curve for a given XY mix is not uniquely determined.

However, if one can decide which is the most desirable $U_A U_B$ combination for a given 'basket' of X and Y, then the equalised MRS of the two individuals at this utility combination can be considered as the unique MRS of the community as a whole, so that the community indifference curve at the XY point will have a unique slope. Based on this, Samuelson[1] proved that one can derive community indifference curves by continuous redistribution of 'incomes' until the welfare function (axiomatically assumed to exist) is maximised in utility space. To illustrate this assume an initial distribution (point *a* in figure 23.12) which yields a total welfare of W_0 (point *a'* in figure 23.13). With continuous redistribution the community can reach point *e* on W_2 where the social welfare is maximised for the particular $Y_0 X_0$ output mix. (We saw that *e* is mapped into a *single* point in output space.)

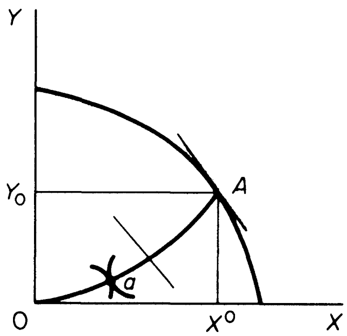

Figure 23.12

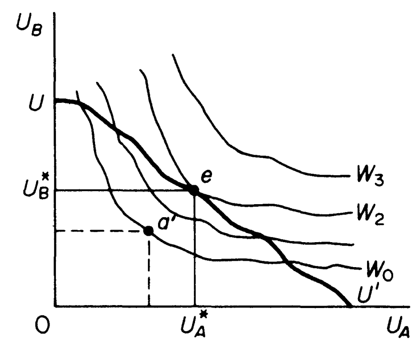

Figure 23.13

If we repeat this process for all output combinations, we can derive a set of *social indifference curves in the output space.* Note the two basic assumptions of Samuelson's proof of 'existence' of social indifference curves: (*a*) A social welfare function exists; (*b*) continuous redistribution of 'incomes' is possible.

In summary, the maximisation of social welfare procedure can be used to 'establish' the existence of community indifference curves in output space. These curves are called 'community indifference curves *corrected for income distribution*'. They are widely used in international trade theory and in other fields of economics. Bator (op. cit.) argues that these community indifference curves reveal the society's ranking of preferences, as reflected in a social welfare function which is defined by a government elected by political consensus. Bator believes that the rankings of the society, based on these community indifference curves is more objective than that of any 'arbitrary ethic standard', adopted subjectively by policy-makers. We think that this argument is 'circular', since the community indifference curves (in output space) are derived from a social welfare function which is assumed to exist axiomatically, and which incorporates the subjective value judgements of those who supposedly have defined it. The point is that, given the subjective nature of the welfare function, the community indifference curves will also reflect subjective valuations.

4. ELASTIC SUPPLY OF FACTORS

The $2 \times 2 \times 2$ model assumed given quantities of factors. This assumption can be relaxed. The supply of factors can be expressed as a function of all the prices in the system. This allows for some elasticity in the supply of factors. The analytical effect is to make the *PPC* a function of the factors of production. This approach has been adopted in the generalised $H \times M \times N$ model, presented in the Appendix to Chapter 22.

[1] See P. A. Samuelson, 'Social Indifference Curves', *Quarterly Journal of Economics* (1956) pp. 1–22.

5. JOINT AND INTERMEDIATE PRODUCTS

It was assumed in the 2 × 2 × 2 model that all products were final products, and joint products were ignored. The model can easily be extended to take into account joint production of some goods, as well as the fact that some products are partly bought by final consumers and partly by other firms which use them in their production as inputs. Analytically this is done by introducing the relevant products as inputs in the production functions of the firms.[1]

6. DECREASING RETURNS TO SCALE

We have assumed constant returns to scale. If this assumption is relaxed and instead it is assumed that there are decreasing returns to scale, some problems do arise.

In the case of decreasing returns the total value of output will exceed the total payments to the factors of production: product is not 'exhausted' by factor payments. This creates ambiguity about how to treat the imbalance between total value of output and total payments to factor owners in a general equilibrium framework.

Traditionally, decreasing returns to scale (increasing costs) are attributed to managerial diseconomies arising from the 'entrepreneurial factor' which is scarce even in the long run. Entrepreneurship, in this approach, defines the size of the firm (which is indeterminate in perfect competition with constant or increasing returns to scale), but is not treated explicitly as an input.

In any case no serious problems arise by assuming decreasing returns to scale. The required convexity of the production possibility curve is preserved, although the mathematical treatment of the problem becomes more tedious with the introduction of non-linearities.

7. EXTERNALITIES: DIVERGENCE BETWEEN PRIVATE AND SOCIAL COSTS AND BENEFITS

In our welfare-maximisation analysis we have assumed that there is no direct interaction among producers (independence of production functions), among households (independence of utilities) and between producers and households.

Furthermore, in asserting the Pareto-optimal properties of perfect competition we have implicitly assumed that all benefits and costs of producers and consumers are reflected in market prices, and that there is no divergence between private and social costs and benefits. However, there are many situations in which the effects of an individual's action is not fully reflected in market prices, and there is a difference between private and social costs and benefits.

When the action of an economic decision-maker creates benefits for others, for which he is not paid, there occurs an *external economy* for the others (and the economy as a whole). When the action of an individual agent creates costs for others for which he does not pay, there occurs an *external diseconomy* for the others (and the society as a whole).

The term *externalities* refers to both external economies and diseconomies. If externalities exist, the model breaks down for two reasons: first, the Pareto-optimality conditions are violated; second, the constants embedded in the system lose their significance as 'prices', because they do not reflect all the costs and benefits

[1] For a simple treatment of this case see C. E. Ferguson, *Microeconomic Theory* (3rd ed., Irwin, 1972) chap. 15.

of an action to the society as a whole. In other words, externalities create a divergence between private and social costs and benefits. Because externalities are not reflected in market prices, these prices provide misleading information (signals) for an optimal allocation of resources.

We will examine separately the externalities in production and the externalities in consumption.

A. Externalities in production

(*a*) *Divergence between private and social costs*

We will illustrate this case with an example. Assume that commodity X is alcohol, which for simplicity we assume is manufactured in a perfectly competitive market. Each firm is in equilibrium when

$$MC_x = P_x$$

where MC_x is the cost of the individual firm, or the private cost. This does not include the cost of pollution of the environment that the firm creates, nor the costs of accidents and deaths caused by drunken consumers. These are external costs to the firm. Suppose that the Health Department obtains an estimate of these costs. The marginal social cost (MSC) is the sum of the private cost (MC_x) and the marginal external cost (MEC), that is

$$MSC_x = MC_x + MEC$$

Obviously we have a divergence of private and social cost of alcohol. Since $MC_x = P_x$, it follows that $P_x < MSC_x$, which implies that the allocation of resources to the production of alcohol is not socially optimal; since the firm does not pay the full cost the production of alcohol in the economy is excessive. If the firm were made to pay the full social costs it would produce a smaller amount of alcohol, defined by the point where

$$P_x = MSC_x$$

This is shown in figure 23.14. The marginal social cost curve lies above the private marginal cost curve, given $MSC_x > MC_x$. The vertical difference between these two curves is the marginal external costs incurred in the production and consumption of alcohol. If the firm does not pay the external costs, its profit-maximising output is X_0. However, if the government required the firm to pay the external costs the firm would reduce its output to X_1.

In summary, when the private cost is smaller than the social cost, adherence to the rule $MC_x = P_x$ leads to overproduction of X. By an analogous argument it can be

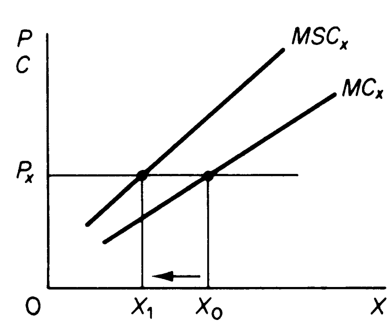

Figure 23.14

shown that if $MC_x > MSC_x$, the level of production of X will be less than the socially optimum. Divergence between private and social costs (and benefits) results to misallocation of resources in a perfectly competitive system.

(*b*) *Divergence between price and social benefit*

Even if the price is equal to the MSC there is no guarantee that social welfare is maximised, because price may be different than the social benefit. For example assume that an environmentalist uses unleaded petrol for running his car, paying $P_g = MC_g$. By using unleaded gasoline the environmentalist keeps the air cleaner, thus creating a benefit to others who breathe in a less polluted atmosphere. Since they do not pay for this benefit we have an externality for the society as a whole. If we add the value of this benefit to the price we obtain the marginal social benefit (MSB). Apparently $P_g < MSB_g$, and since the firm produces where $P_g = MC_g$, it follows that the production of gasoline is less than the socially optimal quantity. If the government added the external benefit on the price of petrol, the consumers would pay the full MSB_g, and each firm would increase its production. This is shown in figure 23.15.

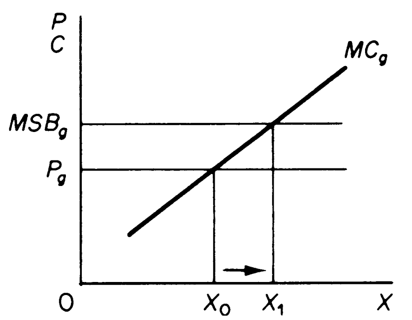

Figure 23.15

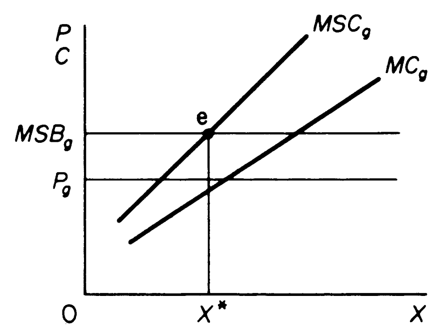

Figure 23.16

The MSB curve is above the P_g curve at all levels of output. If consumers pay the full MSB_g, the firm would increase its output by the amount X_0X_1. If we take into account any *external costs* of lead-free petrol the marginal cost curve would shift to the left (figure 23.16) and equilibrium would be reached at point *e*, where

$$MSC_g = MSB_g$$

From the above discussion we may conclude that when externalities exist, the condition for a socially optimal production is the equality of the MSC and the MSB. In a multi-product economy the condition for optimal resource allocation is

$$\frac{MSB_x}{MSC_x} = \frac{MSB_y}{MSC_y} = \cdots = \frac{MSB_M}{MSC_M} = 1$$

(*c*) *External economies in production*

We said that the presence of externalities in production invalidates the conditions required for social welfare maximisation. The question is how important are externalities in the real world. Some examples may illustrate the extent of the problem.

(i) A new highway reduces the transport cost of individual firms. Since they do not pay for the construction of the highway the MSC is higher than the private marginal cost.

(ii) The expansion of an industry (for example the motorcar industry) creates additional demand for the industries that supply it with raw materials, intermediate products and machinery. This increase in demand in the other industries may allow them to reap economies of large-scale production and lower the price of their product. This is an external economy to all the buyers of the product of these industries.

(iii) A new training programme of corporation A increases the supply of skilled labour for other firms in the industry. Since the latter do not pay the training costs there is an external economy for them.

(iv) If an apple producer grows more apple trees the production of a honey producer in the vicinity of the orchard will increase, because his bees have access to a larger number of apple trees. This is an external economy to the honey producer.

(v) If a honey producer expands his beehives the production of oranges of the owner of a near-by orange grove will increase, since bees help pollinate oranges. This is an external economy for the orange producer.

(vi) If a firm adopts a production method which reduces the pollution of a lake where it disposes its wastes the fish catch will increase. This is an external economy for fishermen in that lake.

(vii) Government-financed research creates external economies for all the firms which benefit from such research.

(*d*) *External diseconomies in production*

(i) Pollution of lakes, rivers and the sea by firms reduces the fish population and hence creates an external diseconomy for the fishing industry. Pollution of this sort creates also health hazards, and therefore a diseconomy for all consumers in adjoining areas.

(ii) Pollution of the air from the smoke of factories or fumes of cars and airplanes has similar diseconomies.

(iii) Establishment of a new industry in an area increases the wage rates to all firms in that area.

(iv) Creation of new shopping centres increases traffic in the neighbourhood, causing a diseconomy to the inhabitants of that area.

The above examples show that externalities in production are very important in the real world. The model developed in this chapter cannot deal with externalities.

B. Externalities in consumption

We have assumed that the utility level (satisfaction) of one consumer is independent of the consumption pattern of the other. This is a far cry from reality. Changes in fashion create strong imitation patterns (bandwagon effects). The desire 'to keep up with the Joneses' creates conspicuous consumption patterns (Veblen effect) which decrease the utility of at least some consumers. Snob behaviour is a widespread phenomenon in western societies. The utility of snobs is greatly affected by the purchases of other people. A smoker decreases the utility of non-smokers in a restaurant or a theatre. The change of cars every year by some consumers decreases the utility of those who cannot afford such change. If one keeps one's car in good condition, the safety of others, and hence their utility, is increased.

If externalities in consumption are present the equalisation of the marginal rate of substitution of commodities among consumers does not lead to Pareto optimality and maximisation of social welfare. Consider figures 23.17 and 23.18, which show the indifference maps of consumers A and B. Assume that initially the two consumers are at points a and b where $MRS^A_{x,y} = MRS^B_{x,y}$, i.e. the condition of Pareto optimality in exchange holds if there are no externalities in consumption. Consumer A has a utility of 100 while consumer B has a utility of 80. (Utility indexes are used.)

Suppose that A is *not* affected by B's consumption, but B's utility is reduced by A's

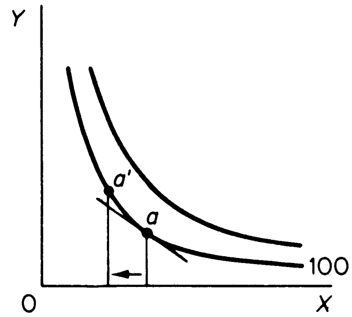

Figure 23.17
A's indifference map

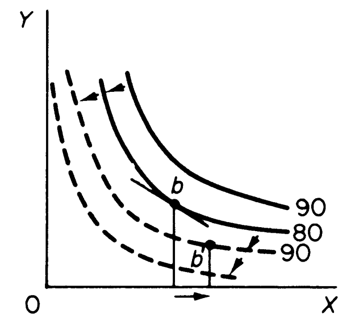

Figure 23.18
B's indifference map

consumption of commodity X (for example tobacco), but not of Y (for example alcohol). Under these conditions a redistribution of the same total output of X and Y between the two consumers can increase social welfare. Assume that redistribution is such as to decrease the consumption of X by consumer A. This *shifts* the utility map of B downwards, showing the increase in the utility of B arising from the reduction in the consumption of X by consumer A. Redistribution is such that consumer A moves to point a' and consumer B moves to b'. The total welfare has increased, despite the fact that at the new equilibrium the $MRS_{x,y}$ is *not* the same for the two consumers. In the new situation consumer A is on the same indifference curve, while B has moved to a higher indifference curve (point b' lies on the shifted indifference curve with utility 90). We conclude that, when externalities in consumption exist, adherence to the equalisation of the MRS of the two consumers does not ensure Pareto optimality.

8. KINKED ISOQUANTS

We have assumed that the isoquants are continuous and smooth, without kinks. Such smooth curvatures are mathematically convenient, because calculus can be applied in the solution of the problem of welfare maximisation. Kinked isoquants cause indeterminacy in marginal rates of substitution, that is, lead to a breakdown of calculus techniques. However, kinked isoquants can be handled with the linear programming techniques. The mathematics become more complicated, but the model retains its essential properties. All the efficiency conditions can be restated so as to take into account the kinked isoquants. Furthermore the existence of implicit 'prices' embedded in the maximum-welfare problem is, if anything, even more striking in linear programming.[1]

9. CONVEX ISOQUANTS

We have assumed that the isoquants are convex to the origin and there are constant returns to scale. These assumptions ensure the concavity of the production possibility curve, which is essential for the solution of the welfare-maximisation problem. In this paragraph we examine the effects of the relaxation of the assumption of convexity of the production isoquants. In the next section we will examine the effects of increasing returns to scale.

[1] See R. Dorfman, P. A. Samuelson, and R. Solow, *Linear Programming and Economic Analysis* (McGraw-Hill, 1958).

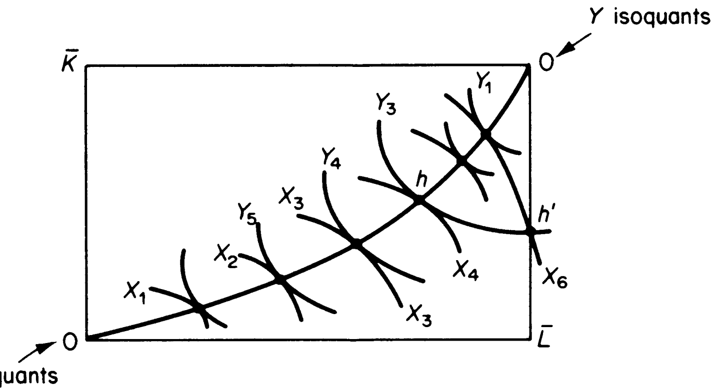

Figure 23.19

If the production isoquants are concave to the origin the Edgeworth contract curve of production becomes the locus of tangencies (of the isoquants of X and Y) where the output of X is *minimum* for a given level of Y, and vice versa. Consider figure 23.19, where both sets of isoquants are concave. Any point on the contract curve FF' shows the minimum quantity of X for a given quantity of Y. For example, point h shows the combination $Y_3 X_4$. However, given Y_3 the maximum amount of X is X_6 (at a 'corner solution'), while X_4 shows the minimum amount of X given Y_3.

We conclude that with concave isoquants the welfare-maximisation condition that the marginal rate of technical substitution of X and Y be equalised ($MRTS^x_{L,K} = MRTS^y_{L,K}$) will result in input combinations that give a minimum of one commodity for specified amount of the other, that is, a configuration which does not maximise social welfare.

10. INCREASING RETURNS TO SCALE

Returns to scale are related to the *position* of the isoquants, *not* to their shape. Assume that the isoquants are convex. Increasing returns to scale are shown by isoquants that are closer and closer (for output levels which are multiples of the original level) along any ray from the origin: to double output the firm needs less than double inputs. (See Chapter 3.)

In our simple model we assumed constant returns to scale. We also saw that decreasing returns to scale do not create serious difficulties except, perhaps, with the treatment of the imbalance between total value of the product and total payments to the factors (the product is not 'exhausted' by factor payments) within a general equilibrium approach. However, increasing returns to scale lead to serious difficulties. In this section we will examine the effects of increasing returns to scale (*a*) on the average cost curves of the firm, and (*b*) on the curvature of the production possibility curve.

(a) Increasing returns to scale and the AC curves of the firm

The consequence of increasing returns to scale is that the LAC falls as output increases. From Chapter 4 we know that when the AC curve falls, the MC curve lies below it. This situation is shown in figure 23.20.

The condition for profit maximisation in a perfectly competitive market is that the firm sets its MC equal to the market price

$$MC_x = P_x$$

However, when there are increasing returns to scale, adherence to this rule would

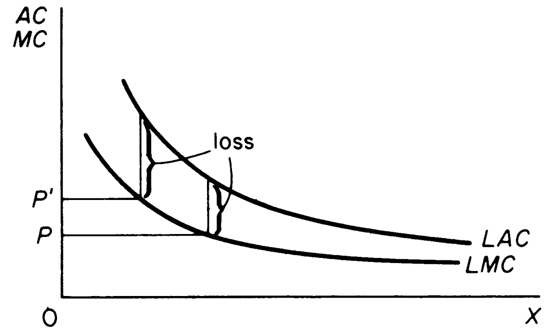

Figure 23.20

lead to losses, since $MC < AC$. In other words the maximisation of social welfare requires $P_x < AC$, that is, perpetual losses. But this situation is incompatible with perfect competition: firms which have losses close down in the long run, and the market system collapses.

Increasing returns to scale create another problem, namely the total value of the product is less than total factor payments

$$(w \cdot L + r \cdot K) > (P_x \cdot X + P_y \cdot Y)$$

(b) Increasing returns to scale and the shape of the production possibility curve

We will continue to assume that the factor intensity (K/L ratio) is different in the two commodities. In particular, the way in which we have been drawing the contract curve of production implies that the $(K/L)_x$ is less than the $(K/L)_y$.[1] This can be seen from figure 23.21. At point a we have $(K/L)_x < (K/L)_y$. Similarly at point b we observe that $(K/L)_x$ is less than $(K/L)_y$. The shape of our contract curve implies that X is less capital intensive than Y.

With different factor intensities, if the increasing returns to scale are not important, the PPC can still be concave to the origin, so that the model is valid. While doubling the inputs in the production of X would more than double the output of X, an increase in X at the expense of Y will, in general, not take place by means of such proportional expansion of factors, because efficient production takes place along the contract curve, and at each point of this curve the K/L ratio of X and Y change; as

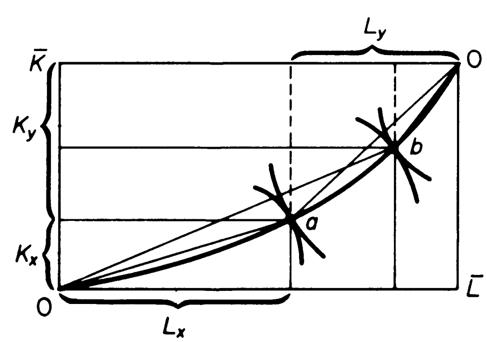

Figure 23.21

[1] If the K/L ratio were the same for both commodities the contract curve of production would coincide with the diagonal. In this case, with the assumption of constant returns to scale, the PPC would be a straight line with negative slope.

we have drawn the contract curve of production, labour is more important relative to capital in producing X, and vice versa for Y. Thus as the production of X expands, the producers of X will be coerced to use capital in greater proportion to labour. In other words, the K/L ratio in X becomes less 'favourable' as the production of X expands. The opposite is true of the K/L ratio in Y as the production of Y declines.

The above argument remains valid if we have unimportant increasing returns to scale in both functions. The *PPC* will become less curved, but so long as it stays below the diagonal of the Edgeworth box the production possibility curve will be concave, as required for the unique solution of the welfare-maximisation problem. In this 'mild' case of increasing returns to scale, with a still concave *PPC*, the previous maximising rules give the correct result for a maximum social welfare. Furthermore, the constants embedded in the system retain their meaning as 'prices', because they still reflect marginal rates of substitution and transformation. In this case also, the total value of maximum-welfare 'national' output ($V^* = P_x \cdot X + P_y \cdot Y$), valued at these 'shadow prices' (constants of the system), is still at a maximum.

If, however, the increasing returns to scale are strong the production possibility curve will be convex to the origin (figure 23.22). In this case two results are possible, depending on the curvature of the community indifference curves.

(i) If the curvature of the community indifference curves is greater than the curvature of the convex production possibility curve, social welfare is maximised at e, where the isovalue–product line AB is tangent to the production possibility curve. The constants implied by the maximum-of-welfare problem retain their meaning as 'shadow prices': they still reflect marginal rates of substitution and transformation. However, maximum social welfare is no longer associated with maximum value-of-output. In fact at point e the value-of-output is at a a minimum. Given P_x/P_y the value of output would be maximised at either F or at F' (the 'corners' of the convex production possibility curve).[1] Thus, if the curvature of the *PPC* is smaller than that of the community indifference curves, social welfare is maximised, but the value of output is at a minimum.

(ii) If, however, the curvature of the *PPC* is greater than the curvature of the community indifference curves, both social welfare and value of output are minimised if we apply the rules of welfare maximisation. In figure 23.23 the production possibility curve FF' is more convex than the indifference curves (U), and the point of tangency z is a point of both minimum welfare and minimum value of output. Welfare maximisation is attained at point F, a 'corner tangency' of the *PPC* and the highest possible community indifference curve (U_2).

In summary: When the *PPC* is convex to the origin, the *relative* curvatures (of the

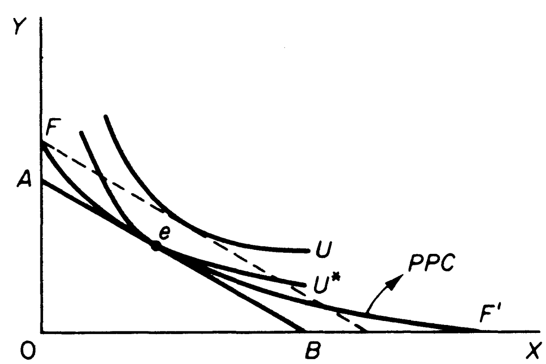

Figure 23.22

[1] In figure 23.22 the slope of AB is such that the value of output would be maximised at F.

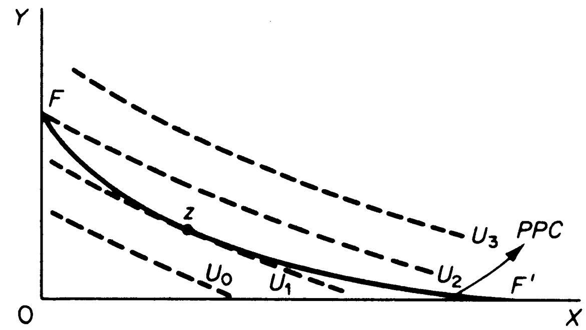

Figure 23.23

PPC and the community indifference curves) are crucial: tangency points may be either maxima or minima of welfare.

11. INDIVISIBILITIES IN THE PRODUCTION FUNCTION

If the technology consists of small-scale and large-scale production processes, with large-scale methods having a lower average cost than the small-scale methods, but large-scale methods are indivisible, then perfect competition does not result in a Pareto-optimal allocation of resources, nor to a maximisation of social welfare.

Assume a situation in which perfect competition prevails with a large number of small firms being in equilibrium. Furthermore, assume that the economy has attained the equality of the *MRPT* and the (equalised) *MRS* of the two commodities among the consumers. Although in this situation (configuration) the three marginal conditions of Pareto optimality are fulfilled, the use of resources is inefficient and social welfare is not maximised if the production methods are indivisible and the small firms cannot take advantage of the lower cost of the large-scale production techniques. Under these conditions it is obvious that a few large firms, using the more efficient large-scale methods, can produce a greater amount of output with the same total quantities of inputs available in the economy. This means that the small firms produce at a point below the *PPC*, because, due to the indivisibilities, they cannot make full use of the available technical knowledge. Although the small firms satisfied the marginal condition for efficient production the use of resources was really inefficient in the initial situation: the small firms produced less output with the same amounts of resources.

In conclusion, the existence of indivisibilities is incompatible with the assumptions of perfect competition and the $2 \times 2 \times 2$ welfare model.

From the above examination of the assumptions of the $2 \times 2 \times 2$ model, we may conclude that the model collapses when:

(i) a welfare function does not exist;
(ii) there are externalities in production;
(iii) there are interdependencies in the utility functions;
(iv) there are strong economies of scale, which render the *PPC* convex to the origin *and* its curvature is greater than the curvature of the community indifference curves;
(v) there are indivisibilities in the production function.