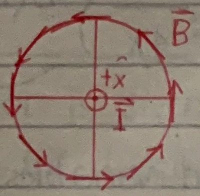
## Applications of Biot_Savart Law:

### Magnetic Field, $\vec{B}$ due to an infinitely long straight current carrying conductor: (wire)

The range of problems to which 'Biot_Savart Law' can be applied is limited primarily by the difficulty experienced in performing the integrations. Some of the tractable situations are considered in this section; in later sections, other techniques for obtaining $\vec{B}$ will be considered.

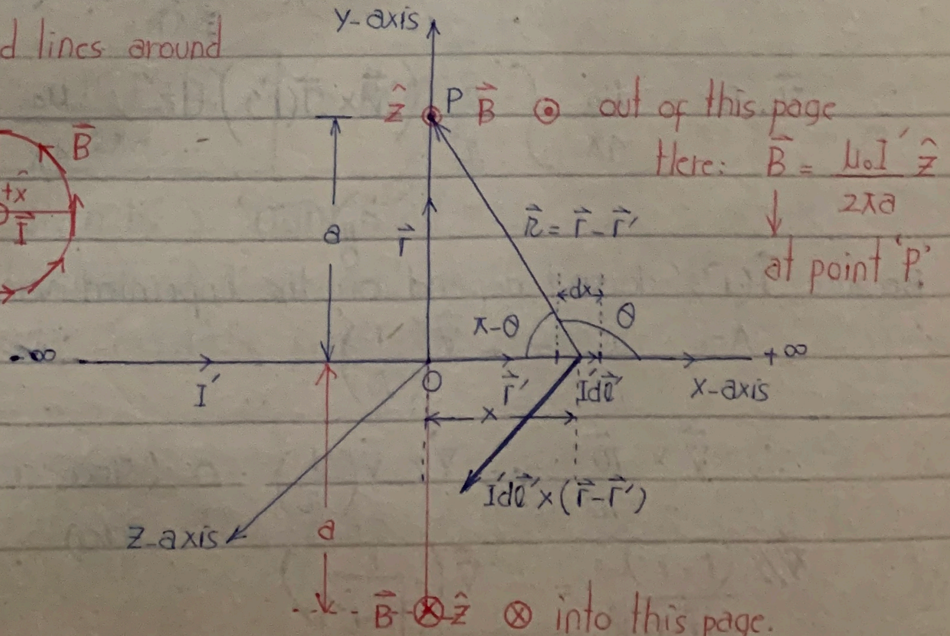
As a first example, the magnetic field due to an infinitely long, straight current carrying wire will be considered. The wire is imagined to lie along the x_axis from minus infinity to plus infinity and to carry a current $I'$.

circular magnetic field lines around straight current carrying wire.




In general,

$$\vec{B} = \frac{\mu_0 I'}{2\pi a}\hat{\phi}$$



According to Biot-Savart Law, the magnetic field at point 'P' due to the current carrying element $I'd\vec{l}'$ is

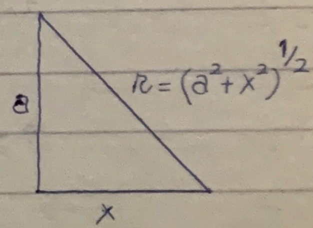
$$d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I'd\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\Rightarrow \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{-\infty}^{+\infty} \frac{I'd\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \vec{B}(P) \quad \text{—— (1)}$$

Magnetic Field at point 'P' due to the whole wire

As,

$$d\vec{l}'\times(\vec{r}-\vec{r}') = dl'|\vec{r}-\vec{r}'|\sin\theta\,\hat{k}$$



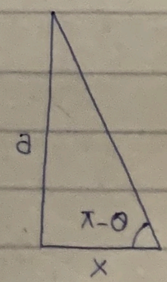
From the Fig., it is clear that

$$|\vec{r}-\vec{r}'| = R = (a^2+x^2)^{1/2}$$

$$dl' = dx$$

$$\therefore \vec{B}(\vec{r}) = \frac{\mu_0 I'\hat{k}}{4\pi}\int_{-\infty}^{+\infty} \frac{dx\,(a^2+x^2)^{1/2}\sin\theta}{(a^2+x^2)^{3/2}}$$

$$= \frac{\mu_0 I'}{4\pi}\hat{k}\int_{-\infty}^{+\infty} \frac{\sin\theta}{(a^2+x^2)}dx \quad \text{—— (2)}$$



From Fig.,

$$\frac{a}{x} = \tan(\pi-\theta)$$

$$= \frac{\sin(\pi-\theta)}{\cos(\pi-\theta)}$$

$$= \frac{\underset{0}{\sin\pi}\cos\theta - \overset{-1}{\cos\pi}\sin\theta}{\underset{-1}{\cos\pi}\cos\theta + \underset{0}{\sin\pi}\sin\theta}$$

$$= -\frac{\sin\theta}{\cos\theta} = -\tan\theta$$

$$\Rightarrow \frac{x}{a} = -\frac{1}{\tan\theta}$$

$$\Rightarrow \frac{x}{a} = -\cot\theta$$

$$\Rightarrow x = -a\cot\theta$$

$$\Rightarrow dx = -a(-\operatorname{cosec}^2\theta)\, d\theta$$
$$= a \operatorname{cosec}^2\theta\, d\theta$$

when $x \to -\infty$

$$-\infty = -a\cot\theta \quad \Rightarrow \quad \infty = \frac{a}{\tan\theta} \quad \Rightarrow \quad \tan\theta = \frac{a}{\infty}$$

$$\Rightarrow \quad \tan\theta = 0 \quad \Rightarrow \quad \theta = \tan^{-1}(0) \quad \Rightarrow \quad \theta = 0^\circ$$

$\boxed{\text{when } x \to -\infty, \quad \theta \to 0}$

when $x \to +\infty$

$$+\infty = -a\cot\theta \quad \Rightarrow \quad -a\cot\theta = +\infty \quad \Rightarrow \quad -\frac{a}{\tan\theta} = +\infty$$

$$\Rightarrow -\tan\theta = \frac{a}{\infty} \quad \Rightarrow \quad -\tan\theta = 0 \quad \Rightarrow \quad \tan(\pi - \theta) = 0$$

$$\Rightarrow \quad \pi - \theta = \tan^{-1}(0) \quad \Rightarrow \quad \pi - \theta = 0^\circ \quad \Rightarrow \quad \theta = \pi$$

$\boxed{\text{when } x \to +\infty, \quad \theta \to \pi}$

$$\therefore \quad \vec{B}(\vec{r}) = \frac{\mu_0 I'}{4\pi}\hat{k}\int_0^{\pi} \frac{\sin\theta}{\underbrace{(a^2 + a^2\cot^2\theta)}_{a^2(1+\cot^2\theta) \,=\, a^2\operatorname{cosec}^2\theta}}\, a\operatorname{cosec}^2\theta\, d\theta$$

$$= \frac{\mu_0 I'}{4\pi a}\hat{k}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{\mu_0 I'}{4\pi a}\hat{k}\,(-\cos\theta)\Big|_0^{\pi}$$

$$= \frac{\mu_0 I'}{2\cancel{4}\pi a}\hat{k}\,\underbrace{(\cos 0 - \cos\pi)}_{2}$$

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0 I'}{2\pi a}\hat{k}} \equiv \vec{B}(P)$$

Its direction is given by a 'right hand rule': point the thumb of your right hand in the direction of the current, and your fingers indicate the direction of the circular magnetic field lines around the wire.