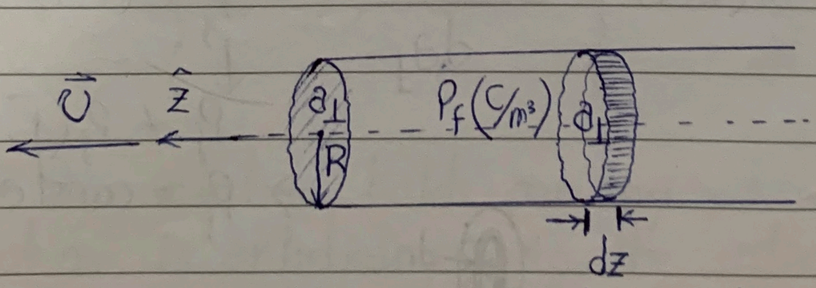
in general.

Volume Currents:



$$d\vec{a}_\perp = r\,dr\,d\phi\,\hat{z}$$

$$da_\perp = r\,dr\,d\phi$$

$$a_\perp = \int_0^R r\,dr \int_0^{2\pi} d\phi$$

$$= \pi R^2$$

$$dq_f = \rho_f\,d\tau = \rho_f\,a_\perp\,dz$$

$$= \rho_f\,\pi R^2\,dz$$

Date:

Consider an infinitely long circular conducting rod of radius R, with cross-sectional area $a_\perp = \pi R^2$. For simplicity, let us assume the rod is initially charged with constant free volume charge density ρ_f, and $\rho_f \neq \rho_f(\vec{r}, t)$. We again place a potential difference ΔV across the ends of the rod, and a volume current I flows in the rod.

$$(I \text{ or}) \ I_f = \frac{dq_f}{dt} = \frac{\rho_f \, dz}{dt} = \rho_f (\pi R^2) \frac{dz}{dt} = \rho_f (\pi R^2) v$$

$$I(z,t) = \rho_f (\pi R^2) v(z,t)$$

In general,

$$\vec{I}(\vec{r},t) = \rho_f (\pi R^2) \vec{v}(\vec{r},t)$$

$$\underbrace{\vec{J}(\vec{r},t)}_{\downarrow} \equiv \frac{\vec{I}(\vec{r},t)}{\pi R^2} = \rho_f \vec{v}(\vec{r},t)$$

(Free) Volume current density ; SI Unit: Amperes/m^2

Note: not Amperes/m^3 !!!

or $$\vec{J}(\vec{r},t) \equiv \frac{\vec{I}(\vec{r},t)}{a_\perp} = \rho_f \vec{v}(\vec{r},t)$$

The differential form is

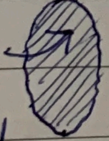
$$\vec{J}(\vec{r},t) = \frac{d\vec{I}(\vec{r},t)}{da_\perp} = \rho_f \vec{v}(\vec{r},t)$$

$\downarrow$

$\rho_f \neq \rho_f(\vec{r},t)$

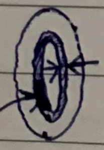
$\rightarrow \rho_f = \text{constant}$

$a_\perp$



$\downarrow$

cross-sectional area of conductor



dr

$da_\perp$

$\downarrow$

infinitesimal cross-sectional area element of $a_\perp$

Date:

$$\vec{I}(\cancel{\vec{r}},t) = \int_{S_\perp} \vec{J}(\vec{r},t)\, da_\perp$$

In general, $\vec{I}(\cancel{\vec{r}},t) = \int_{S_\perp} \vec{J}(\vec{r},t)\cdot d\vec{a}$

Current through any closed surface, S is

$$\vec{I}(\cancel{\vec{r}},t) = \oint_S \vec{J}(\vec{r},t)\cdot d\vec{a}$$

the rate at which the charge is decreasing in volume, V.

$$\underbrace{\oint_S \vec{J}(\vec{r},t)\cdot d\vec{a}}_{\text{current flowing out through surface, S (bounding the volume, V)}} = -\frac{dq_f}{dt} = \underbrace{-\frac{d}{dt}\int_V \rho_f(\vec{r},t)\,d\tau}_{\text{decrease in the charge density in volume, V.}}$$

Electric charge conservation: The electric charge can neither be created, nor destroyed.
The electric charge is conserved, whatever electric charge flows out of or flows into the surface S must come from or go into the volume V respectively.

Applying Divergence theorem,

$$\int_V (\vec{\nabla}\cdot\vec{J})\,d\tau = \int_V -\frac{\partial \rho_f(\vec{r},t)}{\partial t}\,d\tau$$

Since, this relation holds for any volume 'V' (i.e., the choice of volume is arbitrary) so the integrands on the LHS and RHS must be equal.

DALMATIAN

Date:

$$\vec{\nabla}\cdot\vec{J}(\vec{r},t) = -\frac{\partial\rho(\vec{r},t)}{\partial t}$$

= Equation of continuity
(Electric charge conservation)

Under static conditions, (For steady currents)
No time dependence.

$\rho \neq \rho(\vec{r},t)$ $\quad \vec{\nabla}\cdot\vec{J}(\vec{r}) + \cancelto{=0}{\frac{\partial\rho(\vec{r})}{\partial t}} = 0$

If ρ = constant $\Rightarrow$ either $\rho \neq \rho(\vec{r},t)$ or $\rho = \rho(\vec{r})$

$\therefore \frac{\partial\rho}{\partial t} = 0$

$\Rightarrow \vec{\nabla}\cdot\vec{J}(\vec{r}) = 0$

$\int_V \vec{\nabla}\cdot\vec{J}(\vec{r},\cancel{t})\,d\tau = 0$

Applying DT,

$\oint_S \vec{J}(\vec{r},\cancel{t})\cdot d\vec{a} = 0$

$\vec{\nabla}\cdot\vec{J}(\vec{r},\cancel{t}) = 0$

volume current flux through any closed surface, S

Summary:

line charge density	Surface charge density	Volume charge density
$\lambda(\vec{r})$ (C/m)	$\sigma(\vec{r})$ (C/m²)	$\rho(\vec{r})$ (C/m³)
$Q = \int_C \lambda(\vec{r})\,dl$	$Q = \int_S \sigma(\vec{r})\,da$	$Q = \int_V \rho(\vec{r})\,d\tau$

line current	Surface current density	Volume current density
$\vec{I}$ (A)	$\vec{K}(\vec{r})$ (A/m)	$\vec{J}(\vec{r})$ (A/m²)
	$I = \int_C \vec{K}(\vec{r})\,d\vec{l}$	$I = \int_S \vec{J}(\vec{r})\,d\vec{a}$

Date:

Example 5.4:

(a)

Since, I is uniformly distributed over cross_sectional area A of the wire $\Rightarrow \vec{J}$ must also be uniform.

$$J = \frac{I}{A} = \frac{I}{\pi a^2}$$

(b) $J(s) = ks$

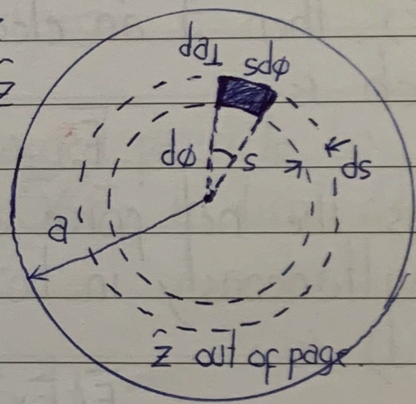
$s \equiv$ radial distance from cylindrical symmetry axis

$k \equiv$ constant (SI Unit: A/m^3)

$\vec{J}(s) = ks\,\hat{z}$

$d\vec{a} = da_\perp \hat{z}$

$= s\,ds\,d\phi\,\hat{z}$



cross_sectional view of the wire.

$$I = \int_{S_\perp} \vec{J}(s) \cdot d\vec{a}$$

$$= \int_{S_\perp} ks\,(s\,ds\,d\phi)\,\underbrace{\hat{z}\cdot\hat{z}}_{\equiv 1}$$

$$= k\int_0^a s^2 ds \int_0^{2\pi} d\phi$$

$$= k\,\frac{a^3}{3}\,2\pi = \frac{2\pi k a^3}{3}$$