CHAPTER 3
Laser Beam Characteristics

3.1 INTRODUCTION

Light from a laser differs from light from conventional sources in a number of ways. The most striking features of a laser beam are (*i*) directionality, (*ii*) high intensity (*iii*) high degree of coherence and (*iv*) extraordinary monochromaticity. Laser light can also be produced as polarized light. It can be generated in the form of very short pulses, at high powers.

3.2 DIRECTIONALITY

The conventional light sources emit light in all directions. When we need a narrow beam, we obtain it by placing an aperture in front of the source. In case of a laser, the active material is in a cylindrical resonant cavity. Any light that is travelling in a direction other than parallel to the cavity axis is eliminated and only the light that is travelling parallel to the axis emerges from the cavity and becomes the laser beam. Hence, the light is emitted by a laser only in one direction. The directionality of a laser beam is expressed in terms of *beam divergence*.

3.2.1. Divergence

Light from a laser diverges very little. Upto a certain distance, the beam shows little spreading and remains essentially a bundle of parallel light rays. The distance from the laser over which the light rays remain parallel is known as *Rayleigh range*. The laser beam diverges beyond the Rayleigh range as shown in Fig. 3.1. There are two parameters which cause beam divergence. They are (*i*) the size of the beam waist and (*ii*) diffraction. The *divergence angle* is measured from the centre of the beam to the edge of the beam, where the edge is defined as the location in the beam where the intensity decreases to $1/e$ of that at the centre.

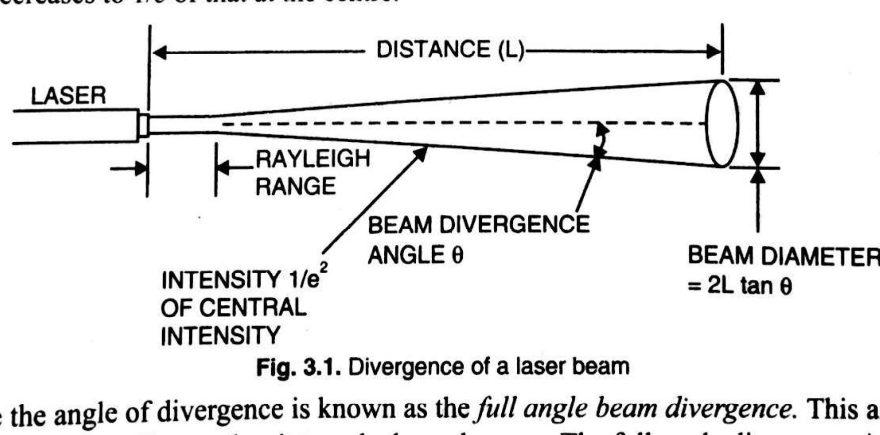


Fig. 3.1. Divergence of a laser beam

Twice the angle of divergence is known as the *full angle beam divergence*. This angle tells us how much the beam will spread as it travels through space. The full angle divergence is given by

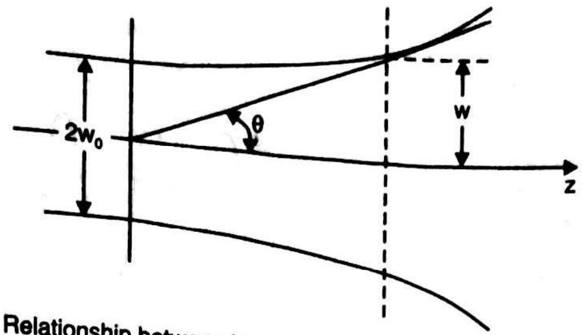


Fig. 3.2. Relationship between beam waist and divergence of a laser beam.

$$2\theta = \frac{4\lambda}{\pi d_0} \quad ...(3.1)$$

where $d_0 = 2W_0$ is the diameter of the beam waist (Fig. 3.2). It is seen from Eq. (3.1) that the divergence is inversely proportional to d_0. Thus, divergence is large for a beam of small waist.

The beam divergence due to diffraction is determined from Rayleigh's criterion

$$\theta = 1.22\frac{\lambda}{D} \quad ...(3.2)$$

where D is the diameter of the laser's aperture.

In case of gas lasers, the diffraction divergence is about twice as large as the beam-waist divergence. A typical value of divergence for a He-Ne laser is 10^{-3} rad. It means that the diameter of the laser beam increases by about 1 mm for every metre it travels. Beam divergence of large lasers may be as small as a micrp-degree. A laser beam of 5 cm. diameter (divergence = 10^{-6} degree) when focussed from Earth will have spread to a diameter of only about 10 m on reaching the surface of the moon. This extreme collimation of the beam makes lasers a very useful tool for surveying.

3.3 INTENSITY

The power output of a laser may vary from a few milliwatts to a few kilowatts. But this energy is concentrated in a beam of very small cross-section. The intensity of a laser beam is approximately given by

$$I = \left(\frac{10}{\lambda}\right)^2 P \qquad W/m^2 \quad ...(3.3)$$

where P is the power radiated by the laser. In case of 1 mW He-Ne laser,

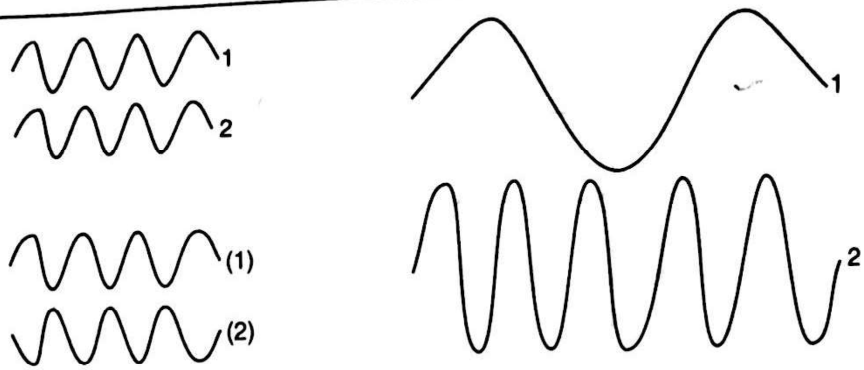
$\lambda = 6328 \times 10^{-10}$ m and

$$I = \frac{100 \times 10^{-3} W}{\left(6328 \times 10^{-10}\right)^2 m^2} = 2.5 \times 10^{11} W/m^2$$

To obtain light of same intensity from a tungsten bulb, it would have to be raised to a temperature of 4.6×10^6 K. The normal operating temperature of a bulb is about 2000 K.

3.4 COHERENCE

Light waves are said to be coherent if they are in phase with each other – for example, if they maintain crest-to-crest and trough-to-trough correspondence, as in Fig.3.3(a). Two things are necessary for light waves to be coherent. First, they must start with the same phase at the same position. Second, their wavelengths must be the same or they will drift out of phase because the crests of the higher frequency will arrive ahead of the crests of the lower frequency wave.



(*a*) Coherent waves (*b*) Incoherent waves

Fig. 3.3.

The light that emerges from a conventional light source is a jumble of short waves which combine with each other in a random manner. The resultant light is incoherent and the wave front varies from point to point and changes from instant to instant. On the other hand, the light from a laser is a resultant of a large number of identical photons which are in phase and is therefore exhibits a high degree of coherence.

Coherence requires that there is a connection between the amplitude and phase of the light at one point and time, and the amplitude and phase of the light at another point and time. Accordingly, we distinguish two classes of coherence, namely *temporal coherence* and *spatial coherence.* They are also known as *longitudinal coherence* and *transverse coherence* respectively. Temporal coherence refers to the constancy and predictability of phase as a function of time when the waves travel along the same path at slightly different times. Spatial coherence refers to the phase relationship between waves travelling side by side at the same time but at some distance from one another.

3.4.1 Temporal Coherence

The concept of temporal coherence can be easily understood with the help of the following example. Let us consider a single wave propagating along x-direction. Let us note the electric field at one point in space at two different times t_1 and t_2, as in Fig. 3.4. Let the phase difference between the field E_1 at t_1 and the field E_2 at t_2 be ϕ_1. Let us again note the electric fields at later times t_3 and t_4, where $(t_4 - t_3) = (t_2 - t_1)$. Let the phase difference be now ϕ_2.

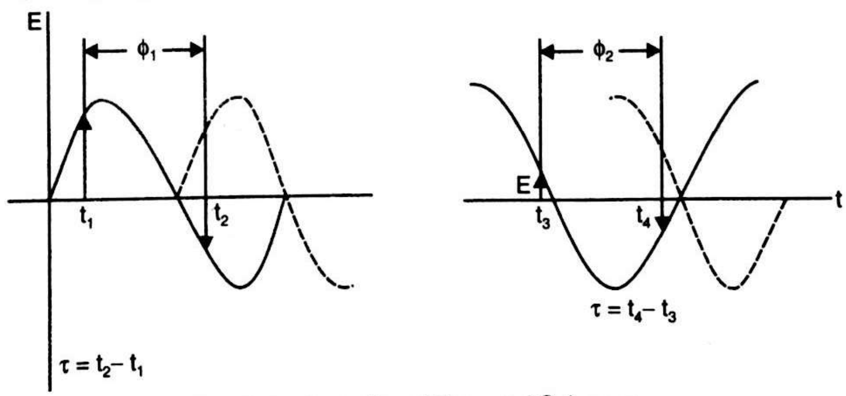


Fig. 3.4. Illustration of Temporal Coherence

If $\phi_2 = \phi_1$ and it is true for any time interval of same duration, then the wave is said to be *temporally coherent.* If the phase difference $\phi_2 \neq \phi_1$ and changes from interval to interval and in an irregular fashion, then the wave is said to the incoherent.

Temporal coherence or incoherence is a characteristic, essentially, of a single beam of light. We know that light is emitted by excited atoms of the source. An excited atom in the process of passing to a lower energy state, gives up the excess energy. The transition lasts for a short time of 10^{-8} sec. A light wave is emitted during this duration, which is not a continuous sine wave of infinite extension but is a wave packet, such as the one shown in Fig 3.5. The light from a source is a jumble of such wave packets emanating from different atoms. Each wave packet has a sustained phase for only about 10^{-8} s after which there is a random phase. On an average, a light beam undergoes random phase changes about 10^8 times per second. Temporal coherence may be expected only for a short distance in space and for a brief time. In view of this, temporal coherence is characterised by two parameters, namely coherence length, l_{coh} and coherencetime, t_{coh}. Both the coherence length and coherence time measure how long light waves remain in phase as they travel in space. The coherence length depends on the central wavelength λ and the bandwidth, Δλ, of the wave packet. It is given by

$$l_{coh} = \frac{\lambda^2}{2\Delta\lambda} \quad ...(3.4)$$

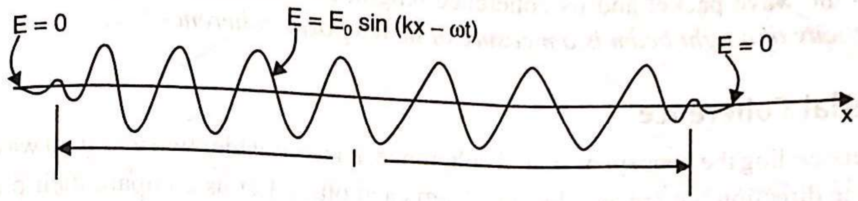


Fig. 3.5. A wave train

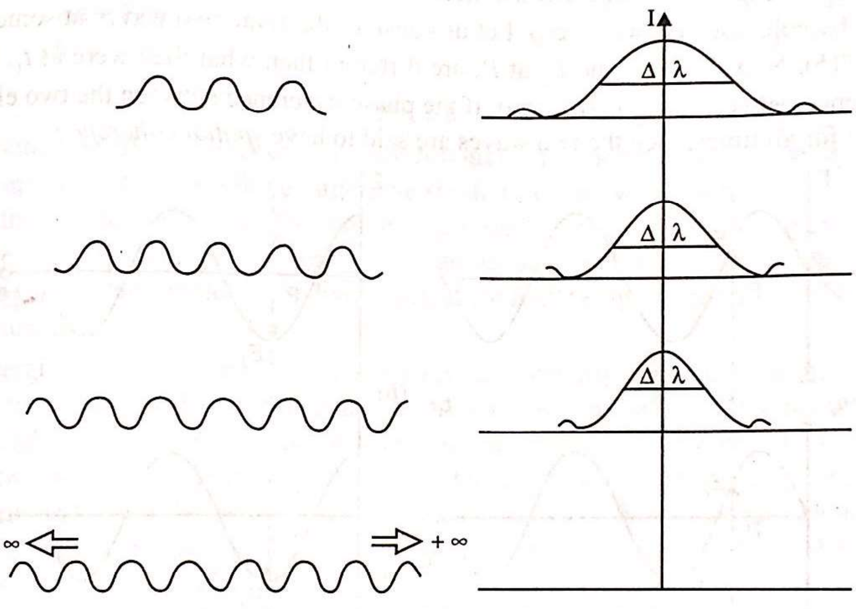


Fig. 3.6. Diagram showing dependence of line width on the length of wave train.

It can be expressed in terms of frequency as

$$l_{coh} = \frac{c}{\Delta\nu} \quad ...(3.5)$$

The coherence length is a very useful measure of temporal coherence because it tells us how far apart two points along the light beam can be, and remain coherent with each other.

Fluorescent tube lights emit visible light of wavelengths in the range, say 4000 Å to 7000 Å, with an average wavelength of 5500 Å. The coherence length

$$l_{coh} = \frac{\lambda^2}{2\Delta\lambda} = \frac{(5000Å)^2}{2 \times 3000\ Å} = 5040\ Å$$

Light from a sodium lamp which is the traditional monochomatic source has a coherence length of about 0.3 mm.

$$l_{coh} = \frac{(5893\text{ Å})^2}{2 \times 6\text{ Å}} = 0.29\text{ mm}$$

A He-Ne laser having a bandwidth of about 2×10^{-5} Å has a coherence length of about 100 m.

$$l_{coh} = \frac{(6328\text{ Å})^2}{2 \times 10^{-5}\text{ Å}} = 100\text{m}$$

It is readily seen from Eqs. (3.4) and (3.5) that temporal coherence is an effect due to the finite line width $\Delta\nu$ of the source. A strictly monochromatic wave ($\Delta\nu = 0$) is an ideal harmonic wave of infinite extension, and of infinite coherence length. A real light wave consists of a train of wave packets because of which its temporal coherence decreases. The broader the line width, $\Delta\nu$, the shorter is the wave packet and its coherence length, as illustrated in Fig. 3.6. In other words, *monochromaticity of a light beam is a measure of its temporal coherence.*

3.4.2. Spatial Coherence

For understanding the concept of spatial coherence, let us consider two identical waves travelling along the same direction but are at a distance from each other. Let us compare their phases at some time, say t_1, as in Fig. 3.7(*a*). The electric field at P_1 is E_1 and at P_2 is E_2. The phase difference between the two electric vectors is zero. Let us consider the same two waves at some later time t_2, as in Fig. 3.7(*b*). Now E_1 at P_1 and E_2 at P_2 are different than what they were at t_1. However, the phase difference between them is still zero. If the phase difference between the two electric vectors remains zero for all times, then the two waves are said to have *spatial coherence.*

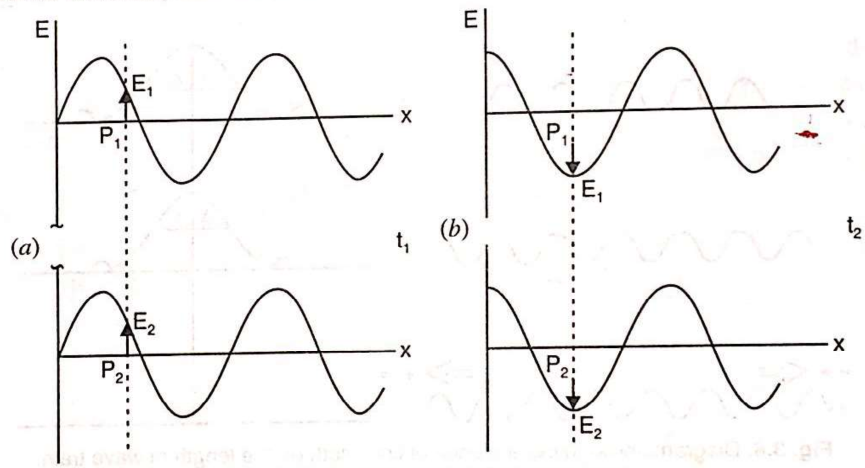
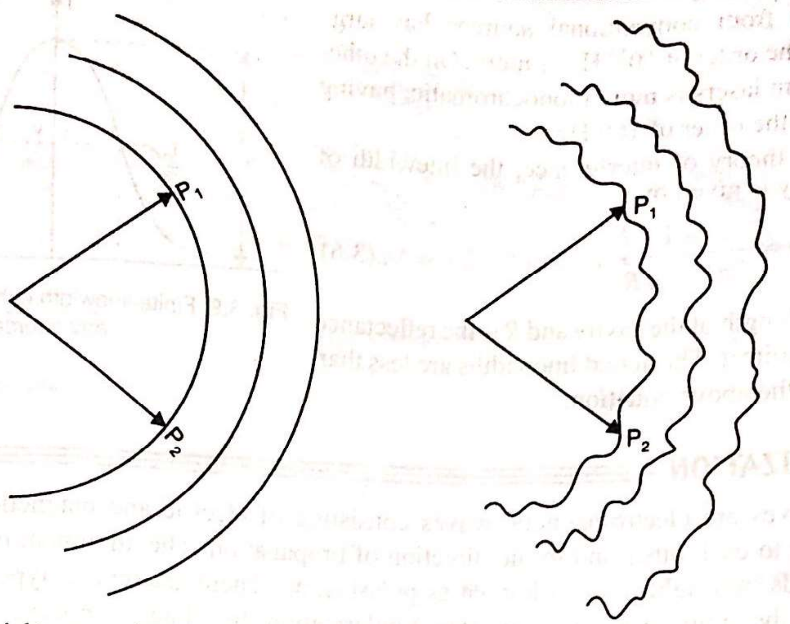


Fig. 3.7. Illustration of spatial coherence.

It implies that spatial coherence measures the area over which light is coherent. If we consider a point light source and look at two points, P_1 and P_2 as in Fig. 3.8 (*a*) that are at equal optical path distances from the source, the waves reaching these points will be exactly in the same phase.

As the source is made larger, it can no longer be claimed that the points P_1 and P_2 are in the same phase (Fig. 3.8(*b*)). Thus, spatial incoherence arises due to the size of the light source. As it characterizes the spatial variation in coherence across the wave front in the direction perpendicular to the light beam, spatial coherence is also known as *transverse coherence* of the beam.



(*a*) Light waves from a point source, producing ideally spherical wave fronts, maintain spatial coherence. Points P_1 and P_2 have the same phase.

(*b*) Extended source produces wave fronts far from ideal. Points P_1 and P_2 do not have the same phase, leading to spatial incoherence.

Fig. 3.8.

The phenomenon of interference is a manifestation of coherence. It can be shown that the *number* of interference fringes will be larger, the smaller the line width or the longer the temporal coherence of the interfering waves. *The degree of contrast of the interference fringes obtained is a measure of the degree of spatial coherence*. It is also deduced that the higher the degree of spatial coherence of a source, the smaller is the divergence of the light beam and consequently, accounts for its high directionality.

Light emerging from a laser is both temporally and spatially coherent to a high degree. The reason is that each stimulated transition is a sort of forced oscillation of the excited atomic system and the atom radiates a photon in phase with the incoming photon. These two photons eventually interact with two other excited atoms, each causing the emission of a photon in phase with it. The process goes on and as all the photons are produced in phase with the original photon, the laser beam exhibits a high degree of coherence. Theoretically, laser light would be completely coherent. In practice, coherence is not ideal because all photons do not originate from the same original photon. So they do not all start out in phase. A laser that oscillates in only one longitudinal mode is more coherent than that oscillates in multiple longitudinal modes, because the different modes have slightly different wavelengths. CW laser beams are more coherent than the pulsed beams. Lasers with low gain are more coherent than high gain lasers. Thus, CW lasers of low gain operating in TEM_{00} mode give the most coherent light.

3.5 MONOCHROMATICITY

If light coming from a source has only one frequency of oscillation, the light is said to be *monochromatic* and the source a *monochromatic source.* In practice, it is not possible to produce light with only one frequency. Light coming out of any source consists of a band of frequencies closely spaced around a central frequency, ν_0 (Fig. 3.9). The band of frequencies, $\Delta\nu$,is called the *linewidth* or *bandwidth.*

The light from conventional sources has large linewidths of the order of 10^{10} Hz or more. On the other hand, light from lasers is more monochromatic, having line-widths of the order of 100 Hz.

From the theory of interference, the linewidth of mirrored cavity is given by

$$\Delta\nu = \frac{c}{2\pi L}\left(\frac{1-R}{\sqrt{R}}\right) \quad ...(3.6)$$

where L is the length of the cavity and R is the reflectance of the out put mirror. The actual linewidths are less than that given by the above equation.

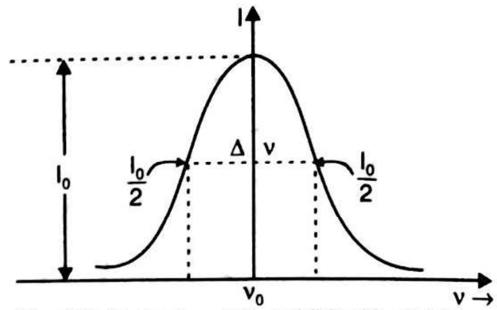


Fig. 3.9. Finite linewidth exhibited by light from any source.

3.6 POLARIZATION

Light waves are electromagnetic waves consisting of electric and magnetic fields vibrating perpendicular to each other and to the direction of propagation. The alignment of the electric and magnetic fields in a light wave is known as polarization. There are several types of polarization, out of which the simplest is linear (or plane) polarization. In a linearly polarized light beam, the orientation of the electric field remains in one plane while its magnitude changes with time. Any other type of polarized light can be viewed as a result of superposition of two linearly polarized waves that are having electric fields perpendicular to each other. For example, unpolarized light can be divided into two components with linear polarization, one with a vertical field and one with a horizontal field.

The conventional light sources produce unpolarized light. External devices such as polarizers and crystal plates are used to convert the unpolarized light into light of desired polarization. In case of lasers, most of them also emit unpolarized light. Laser output can be made linearly polarized light by adding a suitable device. Normally, Brewster window is used to obtain linearly polarized light from a laser. It is a surface arranged at an angle, called Brewster angle, with the resonator axis. Light incident on a surface at the Brewster angle will be reflected or transmitted depending on its polarization. Light polarized parallel to the plane of incidence is transmitted through the Brewster window while light polaized *perpendicular* to the plane of incidence is partly transmitted and partly reflected. The component that undergoes reflection will be lost to the resonator and will not contribute to the oscillation of the laser. The result is that the output beam obtained is plane polarized having polarization parallel to the plane of incidence.

3.7 SPECKLES

If a surface, such as of a wall or of a metal, is illuminated by a laser beam it appears to be granular. The granules are randomly distributed over the surface and appear to be shifting. Such grainy patterns are called laser *speckles.* The shifting speckles is an interference pattern created by slight differences in the paths that light rays travel. Ordinary surfaces are not optically flat and have random in homogeneties which are larger than the wavelength of the laser light. The light reflected diffusely from the inhomogeneties is coherent. The interference of these coherent reflected waves at the retina gives the appearance of alternate dark and bright speckles. If laser radiation is reflected from an optically flat surface, speckles are not observed. Thus, the speckles provide information on the quality of the surface. However, for practical purposes it is undesirable and is merely a background noise.

Example 3.1. *Find the coherent length for white light, the wavelength of white light ranges from 400 nm to 700 nm.*

Solution : Here $\delta\lambda = 700\text{ nm} - 400\text{ nm} = 300\text{ nm}$

$$\text{Average wavelength } \lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{700 + 400}{2} = 550\text{ nm}$$

$$\therefore \quad \text{Coherent length } L = \frac{\lambda^2}{\delta\lambda} = \frac{(550 \times 10^{-9})^2}{300 \times 10^{-9}} = \mathbf{10^{-6}\,nm.}$$

Example 3.2. *If the light of wavelength 660 nm has wave trains 13.2 × 10^{-6} m long, calculate coherent time.*

Solution : Coherent length L = Length of one wave train

$$= 13.2 \times 10^{-6}\text{ m}$$

Let T_c be the coherent time.

Now, $$L = cT_c$$

$$\therefore \quad T_c = \frac{L}{c} = \frac{13.2 \times 10^{-6}}{3 \times 10^8} = 4.4 \times 10^{-14}\,s = \mathbf{44 \times 10^{-15}\,s.}$$

Example 3.3. *The Doppler width for orange line of krypton is 550 × 10^{-15} m. If the wavelength of light is 605.8 nm, calculate the coherent length.*

Solution : Doppler width = line width $\delta\lambda = 550 \times 10^{-15}$ m

$$\lambda = 605.8\text{ nm} = 605.8 \times 10^{-9}\text{ m}$$

$$\therefore \quad \text{Coherent length } L = \frac{\lambda^2}{\delta\lambda} = \frac{605.8 \times 605.8 \times 10^{-18}}{550 \times 10^{-15}} = 667.3 \times 10^{-3}\,m = \mathbf{66.73\ cm.}$$

Example 3.4. *Find the coherence length of a laser source of monochromatic light with frequency width 10,000 Hz.* **(Nagpur U. 2011)**

Solution : Frequency width = band width $\delta\nu$ = 10,000 Hz.

$$\therefore \quad \text{Coherence time } T_c = \frac{1}{\delta\nu} = \frac{1}{10{,}000}\text{ sec.}$$

and $$\text{Coherence length } L = cT_c = 3 \times 10^8 \times \frac{1}{10{,}000} = 3 \times 10^4\text{ m} = \mathbf{30\ km.}$$

Example 3.5. *For an ordinary source coherence time is nearly 10^{-10} sec. Calculate the degree of non-chromocity for λ = 5400 Å.*

Solution : Degree of non-chromacity = Purity factor = frequency stability = $\dfrac{\delta\nu}{\nu}$ = $\dfrac{1/T_c}{c/\lambda} = \dfrac{\lambda}{T_c \times c}$

Again, $$\Delta\nu = \frac{1}{T_c} \text{ and } \nu = \frac{c}{\lambda}$$

$$\therefore \quad \frac{d\nu}{\nu} = \frac{1}{T_c} = \frac{5400 \times 10^{-10}}{10^{-10} \times 3 \times 10^8} = \mathbf{18 \times 10^{-6}}$$

Example 3.6. *The coherence length of sodium D_2 line is 2.5 cm. Deduce the (i) coherence time (ii) spectral width of line and purity factor. Give λ = 5890 Å.*

Solution : $$\text{Coherence time } T_c = \frac{L}{c} = \frac{2.5 \times 10^{-2}}{3 \times 10^8} = 0.83 \times 10^{-10}\text{ sec.}$$

$$\therefore \text{Spectral width = line width } \delta\lambda = \frac{\lambda^2}{L} = \frac{5890 \times 5890 \times 10^{-20}}{2.5 \times 10^{-2}} = 0.1387 \times 10^{-10}\text{m}$$

Spectral purity factor = frequency stability

$$= \frac{\delta\nu}{\nu} = \frac{c}{L} \times \frac{\lambda}{c} = \frac{\lambda}{L} = \frac{5890 \times 10^{-10}}{2.5 \times 10^{-2}} = \mathbf{2.356 \times 10^{-5}}.$$

Example 3.7. *A laser has an organic dye solution with a refractive index 1.4 as the active medium. The length of the resonator is 10 mm. The laser operating at 630 nm with a gain of 100. Then calculate : (i) The variation in the refractive index that can produce the required oscillation*

(ii) The line width of oscillation.

Solution : (*i*) Refractive index, $\mu_r = \frac{\lambda}{L} \times \frac{ln(G)}{\sqrt{\pi}.(G)}$

$$= \frac{633 \times 10^{-9}}{10^{-2}} m \times \frac{ln\,(100)}{\sqrt{\pi}\,(100)} = 9.3 \times 10^{-6} = \mathbf{10^{-5}}$$

(*ii*) Line width of the output in the resonator.

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_0}{4\pi\mu L} ln\,(G)$$

$$\therefore \quad \Delta\lambda = \frac{\lambda^2}{4\pi\mu L} ln\,(G) = \frac{(633 \times 10^{-9})^2}{4\pi \times 1.4 \times 0.01} ln\,(100)$$

$$= 2.27 \times 10^{-12}\text{ m} = \mathbf{2.27 \times 10^{-3}\text{ nm}}$$

Example 3.8. *A laser beam has a wavelength of 7.2 × 10^{-7} m and aperture 5 × 10^{-3} m. The laser beam is focused towards moon the distance of which from earth is 4 × 10^8 m. Calculate (i) the angular spread and (ii) axial spread when the beam reaches to moon.*

Solution : Here $\lambda = 7.2 \times 10^{-7}$ m; $d = 5 \times 10^{-3}$ m, Angular spread $d\theta$ = ?

As the aperture is circular, the angular spread

(*i*) $$d\theta = \frac{1.22\,\lambda}{d} = \frac{1.22 \times 7.2 \times 10^{-7}}{5 \times 10^{-3}} = \mathbf{1.75 \times 10^{-4}\text{ radian}}$$

(*ii*) Distance of moon from the earth $D = 4 \times 10^8$ m

$\therefore$ Axial (aerial) spread $= (D.d\theta)^2$

$$= (4 \times 10^8 \times 1.757 \times 10^{-4})^2 = \mathbf{49.39 \times 10^8\text{ m}^2}$$

Example 3.9. *A laser beam has a power of 100 mW. It has an apertrure of 5 × 10^{-3} m and emits a light of wavelength 6943 Å. The beam is focused with a lens of focal length 0.1 m. Calculate the area and the intensity of the image.*

Solution : Diameter of aperture $d = 5 \times 10^{-3}$ m

Wavelength $\lambda = 6943 \times 10^{-10}$ m; $f = 0.1$ m

Power of laser beam = 100 mW = 10^{-1} watt

As the aperture is circular,

$\therefore$ Angular spread $$d\theta = \frac{1.22\lambda}{d} = \frac{1.22 \times 6943 \times 10^{-10}}{5 \times 10^{-3}} = 1.694 \times 10^{-4}\text{ radian}$$

(*i*) Aerial spread $= (d\theta.f)^2 = (1.694 \times 10^{-4} \times 0.1)^2 = \mathbf{2.89 \times 10^{-10}\text{ m}^2}$

(*ii*) Intensity $$= \frac{\text{Power}}{\text{Area}} = \frac{10^{-1}}{2.89 \times 10^{-10}} = \mathbf{3.46 \times 10^8\text{ watt / m}^2}$$

Example 3.10. *A laser beam has a power of 0.2 watt and has an aperture of 1 mm. It emits light of wavelength 6000 Å. If it is focused by lens of F.L. 80 cm, Calculate the area and intensity of the image.*