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Electric Currents :

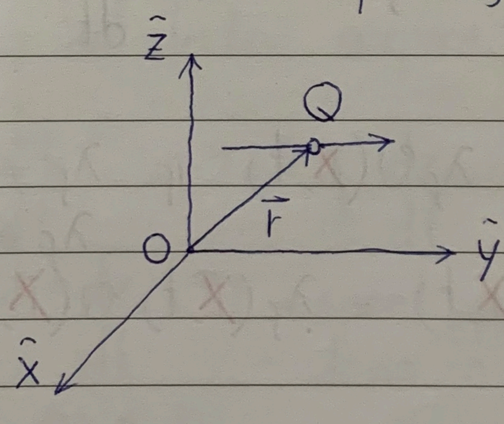
The free electric current I_f at a given point in space (call this point $\vec{r}$, relative to an origin of some coordinate system) is defined as the time-rate of change of free charge Q_f at that point in space.

$$I_f = \frac{dQ_f}{dt}$$

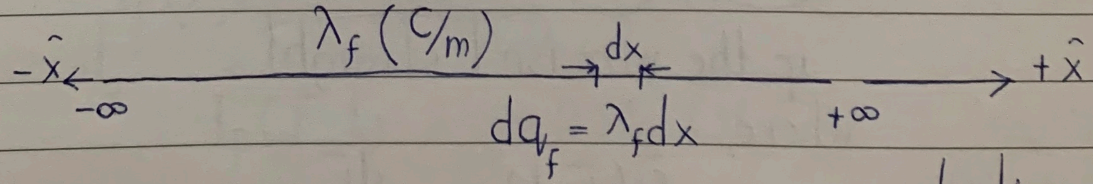
More generally, $I_f(\vec{r},t) = \frac{dQ_f(\vec{r},t)}{dt}$ = instantaneous electric current at point $\vec{r}$ at time t.

SI Unit: Ampere (in honor of André Marie-Ampere, for his 1820 work on understanding the nature of electric currents)

1 Ampere = 1 Coulomb of charge per second



Line Currents :



Consider an infinitely long, thin straight conducting wire. Imagine the wire is electrically charged with line charge density λ_f.

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If the charge is static, i.e., not moving then there is no electric current flowing in the wire $\Rightarrow I_f = 0$
However, if a potential difference is imposed across the ends of the wire, then a line current will begin to flow by an amount:

$$I_f = \lambda_f v \qquad \frac{C}{m} * ms^{-1} = Cs^{-1}$$

v = (relative) speed of charge moving down the wire

relative to the wire itself.

$$I_f = \frac{dQ_f}{dt} \text{ or } \frac{dq_f}{dt} = \frac{dq_f}{dt} = \frac{\lambda_f dx}{dt}$$

$$= \lambda_f \frac{dx}{dt} = \lambda_f v \quad ; \quad \frac{dx}{dt} = v$$

or

$$I_f(\cancel{x},t) = \lambda_f v(\cancel{x},t) \text{ if } \lambda_f \neq \lambda_f(\cancel{x},t) \text{ or } \lambda_f = \text{constant}$$

otherwise, $I_f(\cancel{x},t) = \lambda_f(\cancel{x},t)\, v(\cancel{x},t)$
or more generally,

$$\underbrace{\vec{I}_f(\vec{r},t)}_{\text{if the wire isn't straight}} = \lambda_f(\vec{r},t)\,\vec{v}(\vec{r},t)$$

where,

$$\vec{v}(\vec{r},t) = \frac{d\vec{r}}{dt}$$

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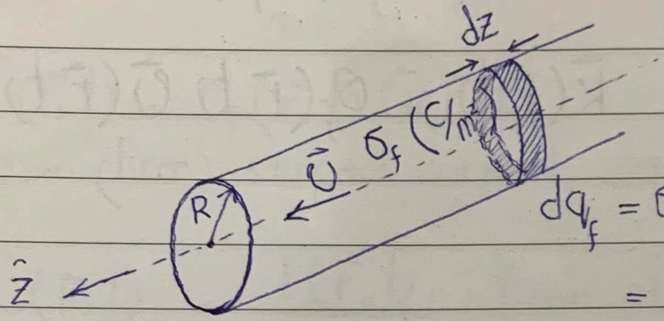
Surface Currents:



where, $d\vec{a} = R\,d\phi\,dz\,\hat{r}$

$da = R\,d\phi\,dz$

$$dq_f = \sigma_f\,da$$
$$= 2\pi R \sigma_f\,dz$$

Imagine an infinitely long, straight, hollow conducting cylindrical tube of radius R with infinitesimally thin walls carrying σ_f of initially stationary electric charges. A potential difference is placed across the length of this hollow tube, causing an electric surface current to flow.

$$I_f = \frac{dq_f}{dt} = 2\pi R \sigma_f \frac{dz}{dt}$$

But $\frac{dz}{dt} = v$ = speed of the charge flowing down in z-direction.

$$I_f = (2\pi R)\,\sigma_f\,v$$

$$I_f(z,t) = (2\pi R)\,\sigma_f\,v(z,t)$$

In general,

$$\vec{I}_f(\vec{r},t) = (2\pi R)\,\sigma_f\,\vec{v}(\vec{r},t)$$

$$\vec{K}(\vec{r},t) \equiv \frac{\vec{I}(\vec{r},t)}{2\pi R} = \sigma_f\,\vec{v}(\vec{r},t)$$

(Free) Surface current density ; SI Unit: Amperes/meter.

Note: not Amperes/m² !!!

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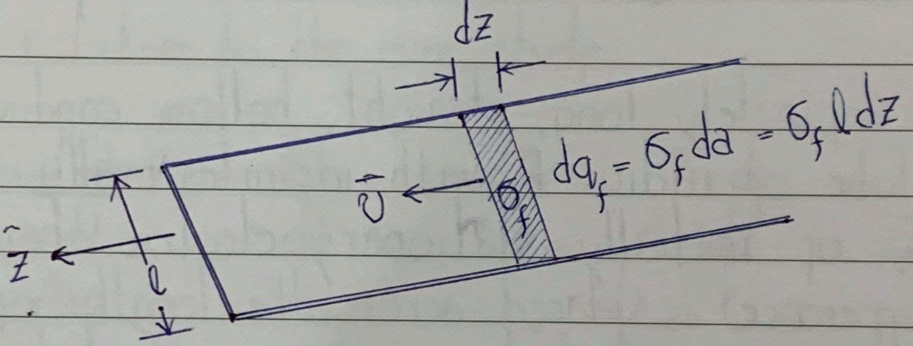
where, $\sigma_f \neq \sigma_f(\vec{r}, t) \Rightarrow \sigma_f = \text{constant}$

but in general,

$$\sigma_f = \sigma_f(\vec{r}, t)$$

More generally,

$$\vec{K}(\vec{r}, t) = \sigma_f(\vec{r}, t)\, \vec{v}(\vec{r}, t)$$



Instead of a surface current flowing on a long, hollow conducting tube of radius R, suppose we had a surface current flowing on a flat conductor of width 'l'. This is simply equivalent to, e.g., cutting the long hollow conducting tube and unrolling it out into a flat plane. Then the width 'l' of the flat sheet = circumference of the original tube, i.e, $l = 2\pi R$.

$$I_f = \frac{dq_f}{dt} = \sigma_f\, l\, \frac{dz}{dt} = \sigma_f\, l\, v \qquad \because \frac{dz}{dt} = v$$

$$I(z, t) = l\, \sigma_f\, v(z, t)$$

In general,

$$\vec{I}(\vec{r}, t) = l\, \sigma_f\, \vec{v}(\vec{r}, t)$$

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$$\vec{K}(\vec{r},t) \equiv \frac{\vec{I}(\vec{r},t)}{l} = \sigma_f \vec{v}(\vec{r},t)$$ assuming, $\sigma_f \neq \sigma_f(\vec{r},t)$

$\Rightarrow \sigma_f \equiv$ constant

If $\sigma_f = \sigma_f(\vec{r},t)$, then

$$\vec{K}(\vec{r},t) = \sigma_f(\vec{r},t)\,\vec{v}(\vec{r},t)$$

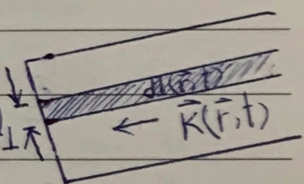
$\vec{K}(\vec{r},t)$ = (Free) surface current density.

$$\vec{K}(\vec{r},t) = \frac{\vec{I}(\vec{r},t)}{l_\perp}$$ (can also be written in this form)

$l_\perp$ tells us that l is perpendicular to the flow of current.

The differential form of above expression is

$$\vec{K}(\vec{r},t) \equiv \frac{d\vec{I}(\vec{r},t)}{dl_\perp} = \sigma_f \vec{v}(\vec{r},t)$$



$$\vec{I}(\cancel{\vec{r}},t) = \int_0^{l_\perp} \vec{K}(\vec{r},t)\,dl_\perp \Rightarrow \underbrace{\vec{I}(\cancel{\vec{r}},t) = \int_{C_\perp} \vec{K}(\vec{r},t)\cdot d\vec{l}}_{\text{In general}}$$