It is also seen from Eq. (1.99) that the laser condition becomes more difficult to be satisfied as the laser frequency increases. It will require that the lifetime t_c of the photons is made as large as possible.

1.39 CONDITION FOR STEADY STATE OSCILLATION

According to wave picture of light, light amplification implies a continuous and marked increase in the amplitude of the light wave. For this to occur, it is necessary that a wave making a complete round trip inside the resonator has to fulfil a phase condition in addition to the amplitude condition given by eq. (1.84). It is necessary that the wave returning to some point in the medium must have the same phase as that of the original wave with any number of reflections from the mirrors. It means that the phase delay between the waves must be some multiple of 2π. Thus, if light starts out at a wave peak when it is reflected from the output mirror, it should be again at the wave peak after one round trip or any number of round trips. This imposes a certain constraint on the relationship between the wavelength λ and the length of the laser rod, L. It is required that the optical path length travelled by a wave between two consecutive reflections at the same end mirror should be an integral multiple of the wavelength. For example, if light starts at a wave peak when it is reflected from the output mirror, it will travel an integral number of wavelengths before it reaches the output mirror again, where it will be again at peak. The waves stimulated by that wave will be at peak at the output mirror as shown in Fig. 1.28. Therefore, they all add up in amplitude by constructive interference and add up in amplitude. Thus, the condition for amplification is

$$\boxed{2\mu L = m\lambda} \quad (m = 1, 2, 3, ...) \qquad ...(1.100)$$

where μ is the refractive index of active medium and μL is the optical path. When the condition (1.100) is not satisfied, each wave will reach the output mirror in a different phase, and undergo destructive interference and cancel out each other. The cavity length L thus imposes a severe restriction that only those light waves which can fit an integral number of wavelengths within twice the cavity length are amplified strongly. Waves of all other wavelengths are attenuated.

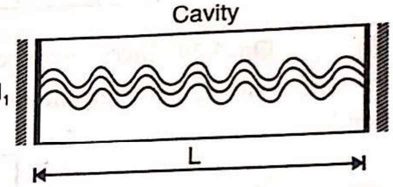


Fig. 1. 28. Condition of resonance

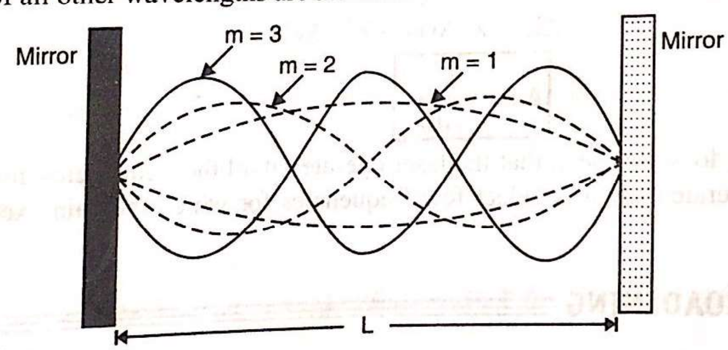


Fig. 1.29. Standing wave pattern and axial modes in optical resonator.

If the space bounded by the two end mirrors in a laser is regarded as a mirror cavity resonator, then the length L of the resonator should accommodate an integral number of standing half-waves as shown in Fig. 1.29. Thus, Eq. (1.100) which may be rewritten as

$$L = \frac{m\lambda}{2\mu} \qquad ...(1.101)$$

expresses the *condition of resonance* between the mirror cavity and the light waves.

1.40 CAVITY RESONANCE FREQUENCIES

It is seen in the previous section that the cavity will be resonant for those waves which fit an integral number of half-wavelength between the mirrors. The wavelengths of such waves are given by Eq.(1.101) as

$$\lambda_m = \frac{2\mu L}{m} \quad ...(1.102)$$

We can express Eq. (1.102) in terms of frequency as

$$\nu_m = \frac{mc}{2\mu L} \quad ...(1.103)$$

Theoretically, the cavity can resonate at a very large number of frequencies that satisfy the condition (1.103). For example, if we take $L = 0.5$ m, $\lambda = 5000$Å and $\mu = 1.5$, we obtain $m = 3 \times 10^6$. It means that the resonator supports 3×10^6 frequencies.

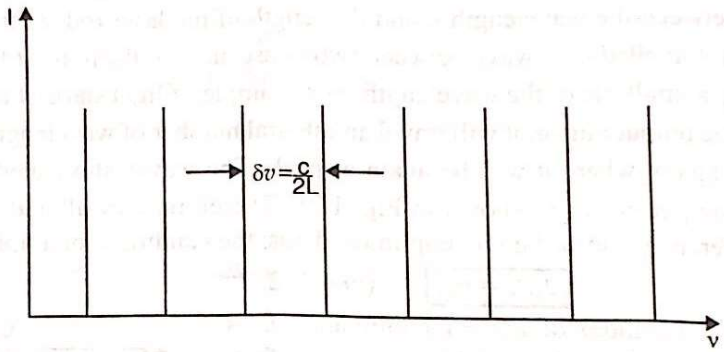


Fig. 1.30. Cavity resonance frequencies representing the possible longitudinal modes.

The spacing between the neighbouring frequencies is constant as shown in Fig. 1.30 and is given by,

$$\Delta\nu = \nu_{m+1} - \nu_m = \frac{c}{2\mu L} \quad ...(1.104)$$

By using $\Delta\nu/\nu = \Delta\lambda/\lambda$, Eq. (1.104) can be written in terms of $\Delta\lambda$ as

$$\Delta\lambda = \lambda . \Delta\nu/\nu = \lambda^2 . \Delta\nu/c$$

$$\therefore \quad \Delta\lambda = \frac{\lambda^2}{2\mu L} \quad ...(1.104)$$

However, it does not mean that the laser operates at all the frequencies given by Eq. (1.103). The laser will operate only at a select few frequencies for which the gain exceeds all the cavity losses.

1.41 LINE BROADENING

Whenever an atom absorbs or emits light it is generally mentioned that the frequency at which absorption or emission takes place is given by

$$\nu_0 = \frac{E_2 - E_1}{h} \quad ...(1.105)$$

The above equation is based on the tacit assumption that the discrete energy levels E_2 and E_1 are infinitely narrow and the atomic transition from level E_1 to level E_2 (or from level E_2 to E_1) occurs at a single definite frequency ν_0. However, in actual practice, we find that the light emitted by atoms is not strictly monochromatic. It becomes therefore necessary that we have to modify our visualisation of energy levels. In order to understand the experimental facts, we assume that the energy levels E_2 and E_1 are not sharp but are spread over a range of energies dE_2 and dE_1 respectively, as shown in Fig. 1.31(*a*). Since transitions can occur between dE_2 and dE_1, the light emitted by the atoms will have a frequency spread $d\upsilon$, with the most intense part appearing at the central frequency ν_0. The emitted light is in the form of wave trains with an exponential damping of their amplitudes. If the intensity of light is plotted against the frequency, we obtain a bell shaped curve as shown in Fig. 1.31(*b*). We have already noted, in an earlier section, that the *line broadening* shown in Fig. 1.31(*b*) can be described by the *lineshape function* g(ν).

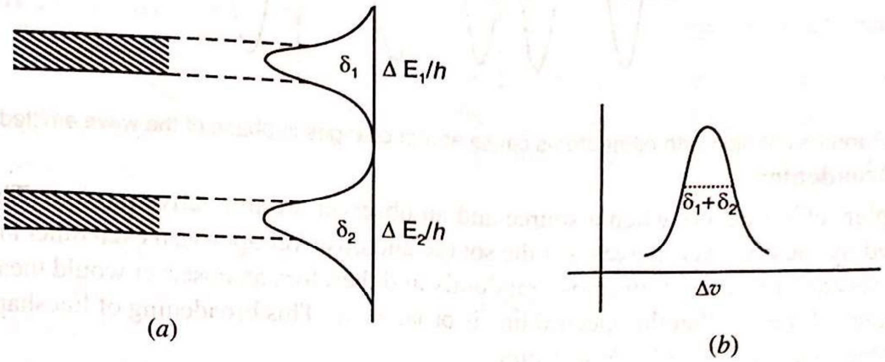


Fig. 1.31

There are a number of mechanisms which lead to line broadening. They are broadly classified into two categories : (*i*) homogeneous and (*ii*) inhomogeneous broadening. If the broadening mechanism effects each individual atom in the sample to the same extent, then the broadening is said to be *homogeneous.* In such a case all of the atoms in the sample will have the same centre frequency ν_0 and the same lineshape. Natural broadening and collision broadening belong to this category. On the other hand, if different atoms in the sample have slightly different frequencies for the same transition, the overall response of the sample broadens out and the broadening is said to be *inhomogeneous.* Doppler broadening and broadening due to crystal defects belong to this category.

Natural Broadening :

The energy levels in an atom are in reality not sharp. If the energy levels are sharp ($dE = 0$), the Heisenberg uncertainty principle

$$\Delta E . \Delta t \sim \hbar$$

leads to the impossible conclusion that the atom can stay in the excited state for an infinite length of time. ($\Delta t \to \infty$, if $\Delta E \to 0$). Hence, the uncertainty principle requires that each energy level has a spread, in terms of frequency,

$$\Delta\nu = \frac{1}{2\pi\tau} \qquad ...(1.106)$$

where $\tau = \Delta t = \hbar/\Delta E$ is known as the *mean lifetime* of the atom in the energy state *E*.

The upper state have a mean lifetime of the order of 10^{-8} s. A spectral line arising from a transition from an excited state to the ground state will have a frequency spread of the order of

$$\Delta\nu = \frac{1}{2 \times 3.14 \times 10^{-8} s} \cong 16\ MHz$$

The natural broadening is relatively small in magnitude and is often masked by other mechanisms.

Collision Broadening:

In a gas random collisions occur between the atoms. If an atom which is emitting a wave train undergoes a collision, then the phase of the wave train is suddenly altered. Thus each collision leads to random phase changes and the effect of collisions may be viewed as shortening of the wave train as shown in Fig. 1.32. It is equivalent to broadening of the spectral line. This broadening is known as *collision broadening*.

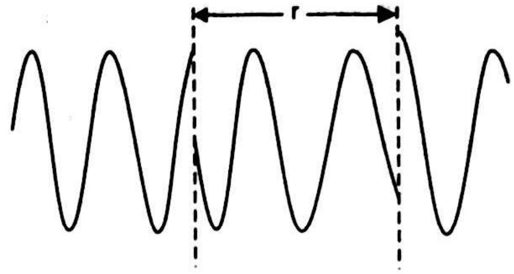


Fig. 1.32. Random collision with other atoms cause abrupt changes in phase of the wave emitted by an atom.

Doppler Broadening :

Doppler effect occurs when a source and an observer are in relative motion. The frequency as measured by the observer increases if the source and observer approach each other and decreases when they recede. In a gas, atoms move randomly and therefore an observer would measure a range of frequencies. It means that the spectral line is broadened. This broadening of lineshape caused by Doppler effect is called *Doppler broadening.*

In actual practice, all the broadening mechanisms will be present simultaneously and are to be taken into consideration.

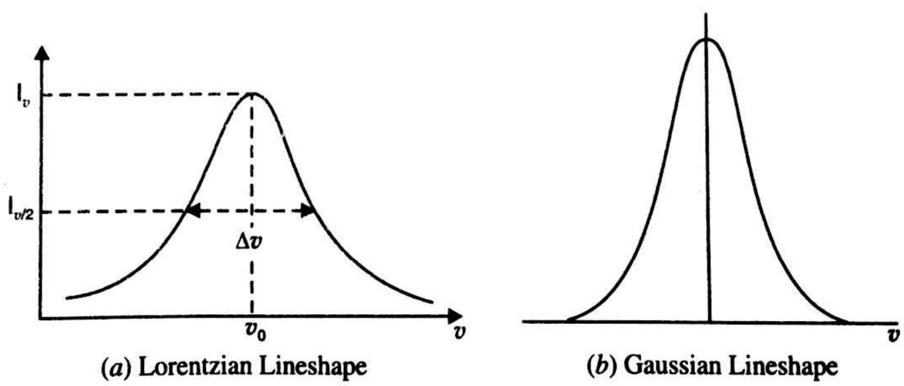
Homogeneous broadening mechanisms will result in a *Lorentzian lineshape*. In that case the lineshape function is given by

$$g_L(\nu) = \frac{\Delta\nu}{2\pi\left[(\nu-\nu_0)^2 + (\Delta\nu/2)^2\right]} \quad ...(1.107)$$

where $\Delta\nu$ is the line width. *Line width* is measured as the separation between the two points on the frequency curve (Fig. 1.19) where the value of $g(\nu)$ falls to half of its peak value.

Inhomogeneous broadening mechanisms will result in a *Gaussian lineshape* in cases of gases. The lineshape function is given by

$$g_G(\nu) = \frac{2}{\Delta\nu}\left(\frac{\ell n2}{\pi}\right)^{1/2} \exp\left[-\ell n2\left(\frac{\nu-\nu_0}{\Delta\nu/2}\right)^2\right] \quad ...(1.108)$$



(*a*) Lorentzian Lineshape (*b*) Gaussian Lineshape

Fig. 1.33

Putting $\nu = \nu_0$ in Eq. (1.107), we get

$$g_L(\nu_0) = \frac{2}{\pi\Delta\nu}$$

and from Eq. (1.108), we obtain

$$g_G(v_0) \approx \frac{2}{\Delta\nu}\left(\frac{ln2}{\pi}\right)^{1/2}$$

The above results may be approximated in general to

$$\boxed{g(\nu_0) \approx \frac{1}{\Delta\nu}} \quad ...(1.109)$$

1.42 GAIN SATURATION

Initially, population inversion condition is created in the lasing medium by the pumping agent by pumping at a fixed average rate. Now that the population of upper lasing level is more than the lower level, and the rates of induced transitions B_{21} and B_{12} are equal, light of suitable frequency induces more transitions from level E_2 to level E_1 than in the opposite direction. To begin with, the gain of the medium may be well above the threshold value and amplification takes place. Lasing will begin and the strength of the light field within the active medium increases exponentially. The rate at which stimulated emissions take place is proportional to the strength of the light field present. Therefore as the intensity of the light builds up in the medium, the rate at which atoms are removed from the excited state increases. With each stimulated emission of a photon, the population inversion decreases by 2. As the intensity of light due to stimulated emission increases, the degree of population inversion decreases. Consequently, the gain will decrease. The gain ultimately settles down at a value where the rate of production of the excess inverted population is balanced by the rate of decrease through stimulated emission. It happens when the gain just balances the losses in the medium. In terms of population inversion, there is a threshold value N_{th} corresponding to this situation. In the steady state condition $(N_2 - N_1)$ remains equal to N_{th} even though the pumping rate is greater than the threshold pumping rate. To sum up, light amplification in a laser medium cannot increase without limit. As the amplification increases, there is a companion decrease in the population at the upper level. As a result, the population inversion is reduced, the number of stimulated emmision events decrease and the amplification goes down. The reduction in the population inversion and consequent self-adjustment of gain caused by the presence of light field is called *gain saturation*. The gain saturation is the mechanism which adjusts the gain to a value where it just balances the losses in the cavity so that steady oscillations can result.

1.43 GAIN BANDWIDTH

Ideally, a group of identical atoms radiate at the same frequency. However, in practice, because of the various broadening mechanisms there will be a small spread of frequencies about the central value. As a result we have to consider a certain frequency interval called bandwidth corresponding to a (stimulated) transition.

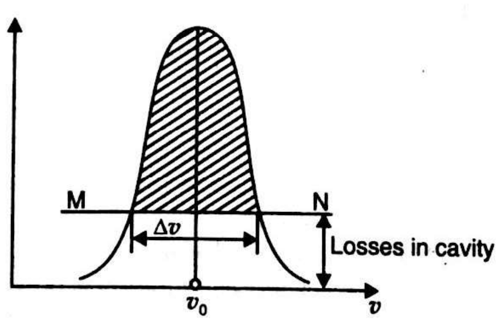


Fig. 1.30. Gain profile with loss level superposed on it.