When the number of factors is small it is usually necessary to replicate the experiment to obtain an estimate of error.

Types of Confounding:

There are two types of confounding:

→ **Complete Confounding:**
When the same effect is confounded in all the replicates, then the confounding is known as complete confounding.

→ **Partial confounding:**
When the different effects are confounded in different replicate, then the confounding is known a partial confounding.

Analysis of Complete confounding:

Suppose 2^2 factorial design is confounded in two blocks when AB is confounded in all the three replicates then the layout will be.

Replicate I (AB confounded)

Block 1	Block 2
1	a
ab	b

Replicate II (AB confounded)

Block 1	Block 2
1	a
ab	b

Replicate III (AB confounded)

Block 1	Block 2
1	a
ab	b

Now the general ANOVA table for this design.

SoV	df	SS	MS	F
Block	$P(\gamma)-1$	$\frac{\sum_k B_k^2}{n_B} - \frac{(T_{\cdots})^2}{p^n(\gamma)}$		
A	$P-1$	$(\text{effect of } A)^2/p^n(\gamma)$	$SS(A)/p-1$	$MS(A)/MSE = F_1$
B	$P-1$	$(\text{effect of } B)^2/p^n(\gamma)$	$SS(B)/p-1$	$MS(B)/MSE = F_2$
Error	By subtraction	By subtraction	$SSE/\text{it d.f}$	
total	$P^n(\gamma)-1$	$\sum_i\sum_j\sum_k Y_{ijk}^2 - \frac{(T_{\cdots})^2}{p^n(\gamma)}$		

where n_B is Block size.

Confounding term will not include in SoV it is a drawback of complete confounding.

P(Y)

Example of Complete Confounding:

An engineer is interested in the effects of cutting speed (A), tool geometry (B) and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen and three replicates of a 2^3 factorial design are run. These results are as follows:

Treatment Combination	Replicate I	Replicate II	Replicate III	Total
(1)	22	31	25	78
a	32	43	29	104
b	35	34	50	119
ab	55	47	46	148
c	44	45	38	127
ac	40	37	36	113
bc	60	50	54	164
abc	39	41	47	127
				980

Suppose that these observations could not all be run using the same bar stock. Set up a design to run these observations in two blocks of four observations each with ABC confounded. Analyze the data.

Solution

$$L = x_1 + x_2 + x_3$$

1: $L = 0+0+0 = 0 = 0 \pmod 2$
a: $L = 1+0+0 = 1 = 1 \pmod 2$
b: $L = 0+1+0 = 1 = 1 \pmod 2$
ab: $L = 1+1+0 = 2 = 0 \pmod 2$
c: $L = 0+0+1 = 1 = 1 \pmod 2$
ac: $L = 1+0+1 = 2 = 0 \pmod 2$
bc: $L = 0+1+1 = 2 = 0 \pmod 2$
abc: $L = 1+1+1 = 3 = 1 \pmod 2$

Replicate I — ABC is confounded

Block 1	Block 2
(1) = 22	a = 32
ab = 55	b = 35
ac = 40	c = 44
bc = 60	abc = 39
Total 177	150

Replicate II — ABC is confounded

Block 1	Block 2
(1) = 31	a = 43
ab = 47	b = 34
ac = 37	c = 45
bc = 50	abc = 41
165	163

Replicate III — ABC is confounded

Block 1	Block 2
(1) = 25	a = 29
ab = 46	b = 50
ac = 36	c = 38
bc = 54	abc = 47
161	164

1- Formulation of Hypothesis: $2^k - 1 \Rightarrow 2^3 - 1 = 7$ one is confounding term so $7 - 1 = 6$

(i) H_0: estimated effect of cutting speed (A) is insignificant
H_1: estimated effect of cutting speed (A) is significant

(ii) H_0: estimated effect of tool geometry (B) is insignificant
H_1: estimated effect of tool geometry (B) is significant

(iii) H_0: there is no interaction between A & B
H_1: there is interaction between A & B

(iv) H_0: estimated effect of cutting angle (c) is insignificant
H_1: estimated " " " is significant

(v) H_0: there is no interaction between A & C
H_1: there is interaction " "

(vi) H_0: there is no interaction between B & C
H_1: there is interaction " "

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② Level of Significance:

$\alpha = 0.05$

③ Test statistic:

(i) $F_1 = \frac{MS(A)}{MSE}$ (ii) $F_2 = \frac{MS(B)}{MSE}$ (iii) $F_3 = \frac{MS(AB)}{MSE}$

(iv) $F_4 = \frac{MSC}{MSE}$ (v) $F_5 = \frac{MS(AC)}{MSE}$ (vi) $F_6 = \frac{MS(BC)}{MSE}$

④ Calculation.

$$\text{effect of } A = -1 + a - b + ab - c + ac - bc + abc$$
$$= -78 + 104 - 119 + 148 - 127 + 113 - 164 + 127$$
$$= 4$$

$$SS(A) = \frac{(\text{effect of } A)^2}{2^n(r)} = \frac{(4)^2}{2^3(3)} = 0.67$$

$$\text{effect of } B = -78 - 104 + 119 + 148 - 127 - 113 + 164 + 127$$
$$= 136$$

$$SS(B) = \frac{(\text{effect of } B)^2}{2^n(r)} = \frac{(136)^2}{2^3(3)} = 770.67$$

$$\text{effect of } AB = +78 - 104 - 119 + 148 + 127 - 113 - 164 + 127$$
$$= -20,$$

$$SS(AB) = \frac{(-20)^2}{2^3(3)} = 16.67$$

$$\text{effect of } C = -78 - 104 - 119 - 148 + 127 + 113 + 164 + 127$$
$$= 82$$

$$SS(C) = \frac{(82)^2}{2^3(3)} = 280.17$$

$$\text{effect of } AC = +78 - 104 + 119 - 148 - 127 + 113 - 164 + 127$$
$$= -106$$

$$SS(AC) = \frac{(-106)^2}{2^3(3)} = 468.17$$

$$\text{effect of } BC = +78 + 104 - 119 - 148 - 127 - 113 + 164 + 127$$
$$= -34$$

$$SS(BC) = \frac{(-34)^2}{2^3(3)} = 48.17$$

$$SS(Blocks) = \frac{\sum B_k^2}{n_B} - \frac{(T_{...})^2}{P^n(r)}$$

$$= \frac{(177)^2+(150)^2+(165)^2+(163)^2+(161)^2+(164)^2}{4} - \frac{(980)^2}{2^3(3)}$$

$$= \frac{160440}{4} - 40016.67$$

$$= 40110 - 40016.67$$

$$= 93.33$$

$$SS\ Total = \sum\sum\sum Y_{ijk}^2 - \frac{(T_{...})^2}{P^n(r)}$$

$$= (22)^2+(32)^2+(35)^2+\cdots+(47)^2 - 40016.67$$

$$= 42112 - 40016.67$$

$$= 2095.33$$

$$SSE = SS\ Total - SS\ Block - SS(A) - SS(B) - SS(AB) - SS(C) - SS(AC) - SS(BC).$$

$$= 2095.33 - 93.33 - 0.67 - 770.67 - 16.67 - 280.17 - 468.17 - 48.17$$

$$= 417.48$$

ANOVA Table

SoV	d.f	SS	MS	F	
Block	2(3)-1 = 5	93.33			
A	2-1 = 1	0.67	0.67	0.0002 = F_1	
B	2-1 = 1	770.67	770.67	22.15 = F_2	> 4.75
AB	2-1 = 1	16.67	16.67	0.48 = F_3	
C	2-1 = 1	280.17	280.17	8.05 = F_4	> 4.75
AC	2-1 = 1	468.17	468.17	13.46 = F_5	> 4.75
BC	2-1 = 1	48.17	48.17	1.38 = F_6	
Error	23-5-6 = 12	417.48	34.79		
Total	$2^3(3)-1 = 23$	2095.33			

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⑤ Critical region:

$$F_{\alpha(v_1,v_2)} = F_{0.05(1,12)} = 4.75$$

If $F_{cal} \geq 4.75$ then reject H_0 otherwise don't reject H_0.

⑥ Conclusion:

By analyzing the data we conclude that the tool geometry (B) and cutting angle (C) have significant effect and there is in interaction between cutting speed (A) and cutting angle (C).

Analysis of Partial Confounding:

Suppose, there are two replicates of the 2^2 design and different interaction has been confounded in each replicate. That is AB is confounded in replicate I and A is confounded in replicate II.

Replicate I
AB is confounded

Block I	Block II
1	a
ab	b

Replicate II
A is confounded

Block I	Block II
1	a
b	ab

As a result, information on AB can be obtained from the data in replicate II and information on A can be obtained from the data in replicate I. We say that one-half information can be obtained on the interactions because they are unconfounded in one replicate. Yates (1937) calls the ratio (1/2), the relative information for the confounded effects. This design is said to be partially confounded.

There are 3 degree of freedom among the four blocks. This is usually partitioned into 1 for replicates and 2 for blocks within replicates.

The analysis of variance for this design is:

SoV	d.f	SS	MS	F
Replicates	$r-1$	$\sum_h R_h^2/2^k - (T_{..})^2/p^n(r)$		
Blocks within replicate	r	$\sum_k B_k^2/n_B - (T_{..})^2/p^n(r)$		
A′(from replicate I)	$P-1$	$(\text{effect of } A')^2/p^n(r-1)$	$SS(A')/P-1$	$MS(A')/MSE = F_1$
B	$P-1$	$(\text{effect of } B)^2/p^n(r)$	$SS(B)/P-1$	$MS(B)/MSE = F_2$
AB′(from replicate II)	$P-1$	$(\text{effect of } AB')^2/p^n(r-1)$	$SS(AB')/P-1$	$MS(AB')/MSE = F_3$
Error	By Subtraction	By Subtraction	SSE/its d.f	
total	$P^n(r)-1$	$\sum\sum\sum Y_{ijk}^2 - (T_{..})^2/p^n(r)$		

where R_h is the total of observations in the h^{th} replicate. There are four replicates of the 2^3 design, but a different interaction has been confounded in each replicat. That is ABC is confounded in replicate I. AB is confounded in replicate II, BC is confounded in replicate III and AC is confounded in replicate IV.

Replicate I ABC is confounded		Replicate II AB is confounded		Replicate III BC is confounded		Replicate IV AC confounded	
Block I	Block II	Block I	Block II	Block I	Block II	Block I	Block II
1	a	1	a	1	b	1	a
ab	b	c	b	a	c	b	c
ac	c	ab	ac	bc	ab	ac	ab
bc	abc	abc	bc	abc	ac	abc	bc

→ Page No: 286 (the ratio will be 3/4)

The analysis of variance for this design is:

SoV	d.f	SS	MS	
Replicates	$Y-1$	$\sum R_h^2/2^k - (T_{..})^2/p^n(r)$		
Blocks	r	$\sum_k B_k^2/n_B - (T_{..})^2/p^n(r)$		
A	$P-1$	$(\text{effect of } A)^2/p^n(r)$	$SS(A)/P-1$	$MS(A)/MSE = F_1$
B	$P-1$	$(\text{effect of } C)^2/p^n(r)$	$SS(B)/P-1$	$MS(B)/MSE = F_2$
AB′	$P-1$	$(\text{effect of } AB')^2/p^n(r-1)$	$SS(AB')/P-1$	$MS(AB')/MSE = F_3$
C	$P-1$	$(\text{effect of } C)^2/p^n(r)$	$SS(C)/P-1$	$MS(C)/MSE = F_4$
AC′	$P-1$	$(\text{effect of } AC')^2/p^n(r-1)$	$SS(AC')/P-1$	$MS(AC')/MSE = F_5$
BC′	$P-1$	$(\text{effect of } BC')^2/p^n(r-1)$	$SS(BC')/P-1$	$MS(BC')/MSE = F_6$
ABC′	$P-1$	$(\text{effect of } ABC')^2/p^n(r-1)$	$SS(ABC')/P-1$	$MS(ABC')/MSE = F_7$
Error	By subtraction	By subtraction	SSE/its d.f	
Total	$P^n(r)-1$	$\sum\sum\sum Y_{ijk}^2 - (T_{..})^2/p^n(r)$		

Example of Partial Confounding:

An engineer is interested in the effects of cutting speed (A), tool geometry (B) and cutting angle (C) on the life (in hours) of a machine tool. Suppose that only four treatment combinations can be tested by using same bar stock. Thus each replicate of the 2^3 design must be run in two blocks. Three replicates are run with ABC confounded in replicate I, AB in replicate II and BC in replicate III. Calculate the factor effect estimates. Construct the analysis of variance table by using the following results.

Treatment combination	Replicate I	Replicate II	Replicate III	Total	Total of I & II	Total of I & III	Total of II & III
(1)	22	31	25	78	53	47	56
a	32	43	29	104	75	61	72
b	35	34	50	119	69	85	84
ab	55	47	46	148	102	101	93
c	44	45	38	127	89	82	83
ac	40	37	36	113	77	76	73
bc	60	50	54	164	110	114	104
abc	39	41	47	127	80	86	88
				980			

Solution:

ABC in replicate I	AB in replicate II	BC in replicate III
$L = X_1 + X_2 + X_3$	$L = X_1 + X_2$	$L = X_2 + X_3$
$1: L = 0+0+0 = 0 = 0 \pmod 2$	$1: L = 0+0 = 0 = 0 \pmod 2$	$1: L = 0+0 = 0 = 0 \pmod 2$
$a: L = 1+0+0 = 1 = 1 \pmod 2$	$a: L = 1+0 = 1 = 1 \pmod 2$	$a: L = 0+0 = 0 = 0 \pmod 2$
$b: L = 0+1+0 = 1 = 1 \pmod 2$	$b: L = 0+1 = 1 = 1 \pmod 2$	$b: L = 1+0 = 1 = 1 \pmod 2$
$ab: L = 1+1+0 = 2 = 0 \pmod 2$	$ab: L = 1+1 = 2 = 0 \pmod 2$	$ab: L = 1+0 = 1 = 1 \pmod 2$
$c: L = 0+0+1 = 1 = 1 \pmod 2$	$c: L = 0+0 = 0 = 0 \pmod 2$	$c: L = 0+1 = 1 = 1 \pmod 2$
$ac: L = 1+0+1 = 2 = 0 \pmod 2$	$ac: L = 1+0 = 1 = 1 \pmod 2$	$ac: L = 0+1 = 1 = 1 \pmod 2$
$bc: L = 0+1+1 = 2 = 0 \pmod 2$	$bc: L = 0+1 = 1 = 1 \pmod 2$	$bc: L = 1+1 = 2 = 0 \pmod 2$
$abc: L = 1+1+1 = 3 = 1 \pmod 2$	$abc: L = 1+1 = 2 = 0 \pmod 2$	$abc: L = 1+1 = 2 = 0 \pmod 2$

Replicate I
ABC is confounded

Block I	Block II
1 = 22	a = 32
ab = 55	b = 35
ac = 40	c = 44
bc = 60	abc = 39
Total 177	150

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Replicate II
AB is confounded

Block I	Block II
1 = 31	a = 43
ab = 47	b = 34
c = 45	ac = 37
abc = 41	bc = 50
164	164

328

Replicate III
BC is confounded

Block I	Block II
1 = 25	b = 50
a = 29	ab = 46
bc = 54	c = 38
abc = 47	ac = 36
155	170

325

① Formulation of Hypothesis: $2^k - 1 = 2^3 - 1 = 7$

(i) H_0: estimated effect of cutting speed (A) is insignificant
H_1: " " " " is significant

(ii) H_0: estimated effect of tool geometry (B) is insignificant
H_1: " " " is significant

(iii) H_0: There is no interaction between cutting speed (A) and tool geometry (B)
H_1: There is interaction " " ".

(iv) H_0: estimated effect of cutting angle (c) is insignificant
H_1: " " " " is significant.

(v) H_0: There is no interaction between A and C
H_1: there is interaction " ".

(vi) H_0: There is no interaction between B and C
H_1: there is interaction " ".

(vii) H_0: There is no interaction between A, B and C.
H_1: there is interaction " ".

② Level of Significance:

$\alpha = 0.05$

③ Test Statistics

$F_1 = \dfrac{MS(A)}{MSE}$ $\quad F_2 = \dfrac{MS(B)}{MSE}$ $\quad F_3 = \dfrac{MS(AB)}{MSE}$ $\quad F_4 = \dfrac{MS(C)}{MSE}$

$F_5 = \dfrac{MS(AC)}{MSE}$ $\quad F_6 = \dfrac{MS(BC)}{MSE}$ $\quad F_7 = \dfrac{MS(ABC)}{MSE}$

④ Calculation:

effect of $A = -78+104-119+148-127+113-164+127$
$= 4$

$$SS(A) = \frac{(\text{effect of } A)^2}{p^n(r)} = \frac{(4)^2}{2^3(3)} = 0.06$$

effect of $B = -78-104+119+148-127-113+164+127$
$= 136$

$$SS(B) = \frac{(\text{effect of } B)^2}{p^n(r)} = \frac{(136)^2}{2^3(3)} = 770.67$$

effect of $C = -78-104-119-148+127+113+164+127$
$= 82$

$$SS(C) = \frac{(82)^2}{2^3(3)} = 280.17$$

effect of $AC = +78-104+119-148-127+113-164+127$
$= -106$

$$SS(AC) = \frac{(-106)^2}{2^3(3)} = 468.17$$

effect of $AB' = +47-61-85+101+82-76-114+127$
$= 21$

$$SS(AB') = \frac{(\text{effect of } AB')^2}{p^n(r-1)} = \frac{(21)^2}{2^3(3-1)} = \frac{(21)^2}{16} = 27.56$$

effect of $BC' = +53+75-69-102-89-77+110+80$
$= -19$

$$SS(BC') = \frac{(-19)^2}{2^3(2)} = 22.56$$

effect of $ABC' = -56+72+84-93+83-73-104+88$
$= 1$

$$SS(ABC') = \frac{(1)^2}{2^3(2)} = 0.063$$

$$SS(\text{Blocks}) = \frac{\sum B_i^2}{n_B} - \frac{(T_{...})^2}{p^n(r)}$$

$$= \frac{(177)^2+(150)^2+(164)^2+(164)^2+(155)^2+(170)^2}{4} - 40016.67$$

$$= \frac{160546}{4} - 40016.67 = 119.83$$

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$$SS\ Total = \sum\sum\sum Y_{ijk}^2 - \frac{(T_{..})^2}{p^n(r)}$$

$$= (22)^2 + (32)^2 + (35)^2 + \cdots + (47)^2 - 40016.67$$

$$= 2095.33$$

$$SS\ Error = SS\ Total - SS(A) - SS(B) - SS(C) - SS(AC) - SS(AB') - SS(BC') - SS(ABC') - SS(Blocks) - SS(replicates)$$

$$= 2095.33 - 0.06 - 770.67 - 280.17 - 468.17 - 27.56 - 22.56 - 0.063 - 119.83 - 0.58$$

$$= 405.67$$

$$SS(replicates) = \frac{\sum_h R_h^2}{2^k} - \frac{(T_{..})^2}{p^n(r)}$$

$$= \frac{(327)^2 + (328)^2 + (325)^2}{2^3} - 40016.67$$

$$= 40017.25 - 40016.67$$

$$= 0.58$$

ANOVA Table

SoV	d.f	SS	MS	F	
Replicates	2	0.58			
Blocks	3	119.83			
A	2-1=1	0.06	0.06	0.06/36.88 = 0.002	
B	2-1=1	770.67	770.67	770.67/36.88 = 20.897	> 4.84
AB'	2-1=1	27.56	27.56	27.56/36.88 = 0.747	
C	2-1=1	280.17	280.17	280.17/36.88 = 7.597	> 4.84
AC	2-1=1	468.17	468.17	468.17/36.88 = 12.69	> 4.84
BC'	2-1=1	22.56	22.56	22.56/36.88 = 0.611	
ABC'	2-1=1	0.063	0.063	0.063/36.88 = 0.002	
Error	11	405.67	36.88		
Total	$2^3(3)-1=23$	2095.33			

(5) Critical Region

$$F_{\alpha(v_1, v_2)} = F_{0.05(1, 11)} = 4.84$$

If $F_{cal} \geq 4.84$ then reject H_0 otherwise do not reject H_0.

⑥ Conclusion:

By analyzing the data we conclude that the tool geometry (B) and cutting angle (C) have significant effect and there is interaction between cutting speed (A) and cutting angle (C).

⇒ Example 7.3:

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