
بسم الله الرحمن الرحيم
In the name of ALLAH
the most Beneficent and the most merciful

Chi Square Test

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Chi Square Test

- ‘ χ’is a Greek letter,
- not equivalent of English letter ‘X’,
- written as “chi, and
- pronounced as “Kye” and
- typed as ‘χ’

Chi Square Test

1. First, a table is prepared out of qualitative data. Actual observed frequencies of 2 sets of events are entered in a two-way table, which is also known as "Contigency table"

   (Latin, con= together: tangere=to touch)

Since, this table also helps to know the association between two sets of events, table is also called as "Association table"

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2. Null Hypothesis is setup stating

- there is no association between the events.
- χ^2- test can also be applied when there are more than two classes or groups, such as social classes I, II, III and IV among smokers and non-smokers

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3. Expected frequency for each cell is calculated on the assumption of no association, using the formula

$$E = \frac{\text{Row total} \times \text{column total}}{\text{Grand total}}$$

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4. Then the difference between the observed and the expected frequencies for each cell is found i.e., $O - E$

5. χ^2- value for each cell is calculated by using the formula

$$\chi^2 = \frac{(O - E)^2}{E}$$

Chi Square Test

6. Then the total of χ^2 for all the four cells is calculated by the formula(summation of all 4 cells χ^2 - values)

$$\text{Total } \chi^2 = \sum \frac{(O - E)^2}{E}$$

$$\text{Alternate formula, } \chi^2 = \frac{(ad - bc)^2 \times G}{(a+b)(c+d)(b+d)(a+c)}$$

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7. Degree of freedom (D.F) is calculated by using the formula

$$D.F = (c - 1)(r - 1)$$

Where

c = no. of columns

r = no. of rows

Chi Square Test

Lastly to know whether the calculated χ^2- value is significant or not, we have to refer to "Fisher's χ^2- table"

If the calculated value is higher than the table- value, it is concluded that it is significant and the Null hypothesis is to be rejected.

If the calculated value is LOWER than the table- value, Null hypothesis is accepted

Chi-Square Distribution

Degrees of Freedom *(df)*	Area in Upper Tail			
	0.10	**0.05**	**0.01**	**0.001**
1	2.706	3.841	6.635	10.828
2	4.605	5.991	9.210	13.816
3	6.251	7.815	11.345	16.267
4	7.779	9.488	13.277	18.467
5	9.236	11.071	15.086	20.515
6	10.645	12.592	16.812	22.458
7	12.017	14.067	18.475	24.322
8	13.362	15.507	20.090	26.125
9	14.684	16.919	21.666	27.877
10	15.987	18.307	23.209	29.588

EXAMPLE

Apply χ^2-test to find efficacy of a drug from the data given below

Outcome(result)of treatment with drug & placebo

Group	Died	Survived	Total
A. Control (on placebo)	(O) 10 (a)	(O) 25 (b)	35 (a +b)
B. Experimental (on Drug)	(O) 5 (c)	(O) 60 (d)	65 (c + d)
Total	15(a + c)	85 (b + d)	G= 100

Chi Square Test

Group	Died	Survived	Total
A. Control(on placebo)	(O) 10 (a) (E) 5.25	(O) 25 (b) (E) 29.25	35 (a +b)
B. Experimental(on) Drug	(O) 5 (c) (E) 9.75	(O) 60 (d) (E) 55.25	65 (c + d)
Total	15(a + c)	85 (b + d)	G= 100

Chi Square Test

The null hypothesis that the drug has no effect

(Drug & placebo are same),(there is no difference between the sample proportions and the population proportion of 100)

The expected(E) value and χ^2-value is calculated for each cell as follows

Chi Square Test

(a) expected number and χ^2-value of "died" in control group

$$E = \frac{\text{Row total} \times \text{Column total}}{\text{Grand total}}$$

$$= \frac{35 \times 15}{100} = 5.25$$

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$$\chi^2 = \frac{(O - E)^2}{E} = \frac{(10 - 5.25)^2}{5.25}$$

$$\chi^2 = \frac{(4.75)^2}{5.25} = \frac{22.5226}{5.25} = 4.29$$

Chi Square Test

b) expected number and χ^2-value of "survived" in control group

$$E = \frac{85 \times 35}{100} = 29.75$$

$$\chi^2 = \frac{(O - E)^2}{E} = \frac{(25 - 29.75)^2}{29.75}$$

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$$\chi^2 = \frac{(-4.75)^2}{29.75} = \frac{22.56}{29.75}$$

$$\chi^2 = 0.76$$

Chi Square Test

c) expected number and χ^2-value of "died" in experimental group

$$E = \frac{15 \times 65}{100} = \frac{39}{4} = 9{\cdot}75$$

Chi Square Test

$$\chi^2 = \frac{(O - E)^2}{E} = \frac{(5 - 9.75)^2}{9.75}$$

$$\chi^2 = \frac{(-4.75)^2}{9.75} = \frac{22.56}{9.75} = 2.31$$

Chi Square Test

d) expected number and χ^2-value of "survived" in experimental group

$$E = \frac{85 \times 65}{100} = 85 \times .65 = 55.25$$

Chi Square Test

$$\chi^2 = \frac{(O - E)^2}{E} = \frac{(60 - 55.25)^2}{55.25}$$

$$\chi^2 = \frac{(5.25)^2}{55.25} = \frac{22.56}{55.25} = 0.408$$

Chi Square Test

$\Sigma \chi^2$ = Total χ^2 value of all 4 cells

= 4.29 + 0.76 + 2.31 + 0.41

= 7.77

DF = (c – 1)(r – 1) = (2 – 1)(2 – 1) = (1x 1)= 1

Where

DF= Degree of freedom

c= no .of columns

r= no. of rows

Chi Square Test

On referring to Fisher's χ^2 - table with 1 df, the tabulated χ^2 - value, corresponding to probability of 0.05(at 95% significance level) is 3.84

Since the calculated value(7.77) is more than table value(3.84),the null hypothesis is rejected ,accepting the alternative hypothesis

YATES' CORRECTION

- When the expected frequency in any cell of the fourfold table is less than 5, Yates' correction, also known as correction for continuity, should be applied as shown below to obtain a more accurate value of chi square.

- by using the formula $\chi 2 = \frac{\{(O-E)-0.5\}^2}{E}$