

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

In the name of **ALLAH**
the most Beneficent and the most merciful

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In the name of ALLAH



MEASURES OF DISPERSION

BY

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MEASURES OF DISPERSION

The measures of central tendency are not sufficient to describe all the characteristics of the data or distribution.

It is quite possible that two or more distributions may have the same average, but the observations may differ from each other.

MEASURES OF DISPERSION

(i) 45, 45, 45, 45, $\bar{x} = 45$

(ii) 44, 45, 45, 46, $\bar{x} = 45$

(iii) 42, 45, 50, 43, $\bar{x} = 45$

(iv) 35, 40, 85, 20, $\bar{x} = 45$

MEASURES OF DISPERSION

(i) 5, 5 = $10/2 = \bar{x} = 5$ (Zero Dispersion)

(ii) 4, 6 = $10/2 = \bar{x} = 5$ (Small Dispersion)

(iii) 1, 9 = $10/2 = \bar{x} = 5$ (Very High Dispersion)

The means of all the three distributions are same **but** dispersion varies.

MEASURES OF DISPERSION

- By dispersion, we mean how far the values are scattered from each other or from the average.

MEASURES OF DISPERSION

- The daily calorie requirement of a normal adult doing sedentary work is laid down as **2,400 calories**. This clearly is not universally true.
- There must be individual variations. If we examine the data of **blood pressure** or **heights** or **weights** of a large group of individuals, we will find that the values vary from person to person. Even within the same subject, there may be variation from time to time

MEASURES OF DISPERSION

- There are several measures of variation (or "dispersion" as it is technically called) of which the following are widely known:
 - (a) The Range
 - (b) The Mean or Average Deviation
 - (c) The Standard Deviation
 - (d) Coefficient of Variation (CV)

The Range

- The range is by far the simplest measure of dispersion. It is defined as the **difference between the highest and lowest figures** in a given sample. For example, from the following record of diastolic blood pressure of 10 individuals 83, 75, 81, 79, **71**, 90, 75, **95**, 77, 94.
- It can be seen that the highest value was 95 and the lowest 71. The range is expressed as 71 to 95

The Range

- If we have grouped data, the range is taken as the difference between the **mid-points** of the extreme categories. The range is not of much practical importance, because it indicates only the extreme values between the two values and nothing about the dispersion of values between the two extreme values.

The Mean Deviation

- It is the average of the deviations from the arithmetic mean. It is given by the formula:

- M.D. =
$$\frac{\sum |x_i - \bar{x}|}{n}$$

Σ = Summation n = No. of observations

| | = Refers to absolute value ignoring + or - sign

x_i = individual value of observation

\bar{x} = mean of observations

Mean deviation (MD)

- The “mean” of the observations is calculated.
- Then the mean is subtracted from each of the observation to calculate the deviation.
- The mean (or average) of these deviations is then calculated by totaling the differences from the mean and divide by the number of observations without considering the sign of the deviation, which gives mean deviation.

Mean deviation (MD)

The systolic blood pressure in mm of Hg of **10** students is as follows

115, 117, 121, 120, 118, 122, 123, 116, 118,
120

Calculate the “**MEAN DEVIATION**”

x_i	$x_i - \bar{x}$	Deviation
115	$115 - 119$	- 4
117	$117 - 119$	- 2
121	$121 - 119$	+ 2
120	$120 - 119$	+ 1
118	$118 - 119$	- 1
122	$122 - 119$	- 3
123	$123 - 119$	+ 4
116	$116 - 119$	- 3
118	$118 - 119$	- 1
120	$120 - 119$	+ 1

Mean deviation (MD)

$$\bar{x} = \frac{1190}{10} = 119$$

$$\sum = |x_i - \bar{x}| = 22$$

$$MD = \frac{\sum = |x_i - \bar{x}|}{n} = \frac{22}{10} = 2.2$$

so mean deviation is = 2.2

The Standard Deviation

- The standard deviation is the most frequently used measure of deviation.
- In simple terms, it is defined as "**Root Means Square Deviation.**" It is denoted by the Greek letter sigma σ or by the initials S.D.

The Standard Deviation

- The standard deviation is calculated from the basic formula :

- S.D.=
$$\sqrt{\frac{\sum (X - \bar{X})^2}{n}}$$

When the sample size is more than 30, the above basic formula may be used without modification.

The Standard Deviation

- For smaller samples, the above formula tends to underestimate the standard deviation, and therefore needs correction, which is done by substituting the denominator $n-1$ for n .

The Standard Deviation

- The modified formula is as follows :

$$S.D = \sqrt{\frac{\sum (X - X^-)^2}{n-1}}$$

The Standard Deviation

- The steps involved in calculating the standard deviation are as follows :
- (a) First of all, take the **deviation** of each value from the arithmetic mean, $(x_i - \bar{x})$
- (b) Then, **square** each deviation --
 $(x_i - \bar{x})^2$
- (c) **Add** up the squared deviations~
 $\sum (x_i - \bar{x})^2$

The Standard Deviation

- (d) **Divide** the result by the number of observations n (or $(n-1)$ in case the sample size is less than 30)
- (e) Then take the **square root**, which gives the standard deviation.
- The meaning of standard deviation can only be appreciated fully when we study it with reference to what is described as normal curve.

The Standard Deviation

- It gives us an idea of the 'spread' of the dispersion; that the **larger** the standard deviation, the **greater** the dispersion of values about the mean.

The Standard Deviation

- 1) A standard deviation(SD) is the **universally** accepted unit of dispersion of values, from the mean value.
- 2) SD summarizes the variation of a large distribution in **one figure**
- 3) SD measures the position or distance of observation from the mean
- 4) SD indicates whether variation of difference of an individual from the mean, is by **chance(natural)** or **real** due to some special reasons

The Standard Deviation

- 5) SD helps in finding the **size** of the sample
- 6) SD is used to calculate **Standard Error(SE)** of mean & SE of difference between two mean
- 7) SD is used for calculation of “relative deviate” or “**Z score**”
- 8) SD is used in the calculation of “**Coefficient of Variation**”(CV)

COEFFICIENT OF VARIATION (CV)

The CV is the standard deviation expressed as the “percentage of the mean”

CV is a unit less number, therefore CV is well suited for all types of dissimilar measurements such as Height and Weight, or Hemoglobin and Weight, or pulse rate and mid-arm circumference

COEFFICIENT OF VARIATION (CV)

$$CV = \left(\frac{S}{\bar{X}} \right) \cdot 100\%$$

COEFFICIENT OF VARIATION (CV)

- Measure of Relative Variation
- Always a %
- Shows Variation Relative to Mean
- Used to Compare 2 or More Groups

SD

$$CV = \frac{\text{SD}}{\text{Mean}} \times 100$$

Mean

COEFFICIENT OF VARIATION (CV)

EXERCISE

The mean and SD of **Hb** level of a group is **12.6 gm %** & **1.5 gm%** respectively while the mean and SD of **body weight** of the same group is **50 kg** & **2.2 kg** respectively

Compare the deviations of these two sets of observations.

COEFFICIENT OF VARIATION

ANSWER (CV)

$$\text{CV of Hb level} = \frac{1.5}{12.6} \times 100 = 11.9\%$$

$$\text{CV of body Wt} = \frac{2.2}{50} \times 100 = 4.4\%$$

Variation is greater for Hb level than for body Wt

COEFFICIENT OF VARIATION

EXERCISE

In two series of boys & girls of same age of 20 years, following values of height were obtained. Find which sex shows greater variation

Sex	Mean (Ht)cm	SD(cm)
Boys	163.25	6.25
Girls	150.35	5.25

COEFFICIENT OF VARIATION

6.25

$$\text{CV of boys} = \frac{\quad}{163.26} \times 100 = 3.83\%$$

5.25

$$\text{CV of girls} = \frac{\quad}{150.35} \times 100 = 3.49\%$$

Heights in boys shows slightly greater variation than in girls in the ratio of 3.83:3.49 = 1.1:1.0

Thank you

