

Exercises

- 6.39** Use the gamma function with $y = \sqrt{2x}$ to show that $\Gamma(1/2) = \sqrt{\pi}$.
- 6.40** In a certain city, the daily consumption of water (in millions of liters) follows approximately a gamma distribution with $\alpha = 2$ and $\beta = 3$. If the daily capacity of that city is 9 million liters of water, what is the probability that on any given day the water supply is inadequate?
- 6.41** If a random variable X has the gamma distribution with $\alpha = 2$ and $\beta = 1$, find $P(1.8 < X < 2.4)$.
- 6.42** Suppose that the time, in hours, required to repair a heat pump is a random variable X having a gamma distribution with parameters $\alpha = 2$ and $\beta = 1/2$. What is the probability that on the next service call
- at most 1 hour will be required to repair the heat pump?
 - at least 2 hours will be required to repair the heat pump?
- 6.43** (a) Find the mean and variance of the daily water consumption in Exercise 6.40.
 (b) According to Chebyshev's theorem, there is a probability of at least $3/4$ that the water consumption on any given day will fall within what interval?
- 6.44** In a certain city, the daily consumption of electric power, in millions of kilowatt-hours, is a random variable X having a gamma distribution with mean $\mu = 6$ and variance $\sigma^2 = 12$.
- Find the values of α and β .
 - Find the probability that on any given day the daily power consumption will exceed 12 million kilowatt-hours.
- 6.45** The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?
- 6.46** The life, in years, of a certain type of electrical switch has an exponential distribution with an average life $\beta = 2$. If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year?
- 6.47** Suppose that the service life, in years, of a hearing aid battery is a random variable having a Weibull distribution with $\alpha = 1/2$ and $\beta = 2$.
- How long can such a battery be expected to last?
 - What is the probability that such a battery will be operating after 2 years?
- 6.48** Derive the mean and variance of the beta distribution.
- 6.49** Suppose the random variable X follows a beta distribution with $\alpha = 1$ and $\beta = 3$.
- Determine the mean and median of X .
 - Determine the variance of X .
 - Find the probability that $X > 1/3$.
- 6.50** If the proportion of a brand of television set requiring service during the first year of operation is a random variable having a beta distribution with $\alpha = 3$ and $\beta = 2$, what is the probability that at least 80% of the new models of this brand sold this year will require service during their first year of operation?
- 6.51** The lives of a certain automobile seal have the Weibull distribution with failure rate $Z(t) = 1/\sqrt{t}$. Find the probability that such a seal is still intact after 4 years.
- 6.52** Derive the mean and variance of the Weibull distribution.
- 6.53** In a biomedical research study, it was determined that the survival time, in weeks, of an animal subjected to a certain exposure of gamma radiation has a gamma distribution with $\alpha = 5$ and $\beta = 10$.
- What is the mean survival time of a randomly selected animal of the type used in the experiment?
 - What is the standard deviation of survival time?
 - What is the probability that an animal survives more than 30 weeks?
- 6.54** The lifetime, in weeks, of a certain type of transistor is known to follow a gamma distribution with mean 10 weeks and standard deviation $\sqrt{50}$ weeks.
- What is the probability that a transistor of this type will last at most 50 weeks?
 - What is the probability that a transistor of this type will not survive the first 10 weeks?
- 6.55** Computer response time is an important application of the gamma and exponential distributions. Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds.