Exercises

6.39 Use the gamma function with $y = \sqrt{2x}$ to show that $\Gamma(1/2) = \sqrt{\pi}$.

6.40 In a certain city, the daily consumption of water (in millions of liters) follows approximately a gamma distribution with $\alpha = 2$ and $\beta = 3$. If the daily capacity of that city is 9 million liters of water, what is the probability that on any given day the water supply is inadequate?

6.41 If a random variable X has the gamma distribution with $\alpha = 2$ and $\beta = 1$, find $P(1.8 < X < 2.4)$.

6.42 Suppose that the time, in hours, required to repair a heat pump is a random variable X having a gamma distribution with parameters $\alpha = 2$ and $\beta = 1/2$. What is the probability that on the next service call

(a) at most 1 hour will be required to repair the heat pump?

(b) at least 2 hours will be required to repair the heat pump?

6.43 (a) Find the mean and variance of the daily water consumption in Exercise 6.40.

(b) According to Chebyshev's theorem, there is a probability of at least 3/4 that the water consumption on any given day will fall within what interval?

6.44 In a certain city, the daily consumption of electric power, in millions of kilowatt-hours, is a random variable X having a gamma distribution with mean $\mu = 6$ and variance $\sigma^2 = 12$.

(a) Find the values of α and β.

(b) Find the probability that on any given day the daily power consumption will exceed 12 million kilowatt-hours.

6.45 The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?

6.46 The life, in years, of a certain type of electrical switch has an exponential distribution with an average life $\beta = 2$. If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year?

6.47 Suppose that the service life, in years, of a hearing aid battery is a random variable having a Weibull distribution with $\alpha = 1/2$ and $\beta = 2$.

(a) How long can such a battery be expected to last?

(b) What is the probability that such a battery will be operating after 2 years?

6.48 Derive the mean and variance of the beta distribution.

6.49 Suppose the random variable X follows a beta distribution with $\alpha = 1$ and $\beta = 3$.

(a) Determine the mean and median of X.

(b) Determine the variance of X.

(c) Find the probability that $X > 1/3$.

6.50 If the proportion of a brand of television set requiring service during the first year of operation is a random variable having a beta distribution with $\alpha = 3$ and $\beta = 2$, what is the probability that at least 80% of the new models of this brand sold this year will require service during their first year of operation?

6.51 The lives of a certain automobile seal have the Weibull distribution with failure rate $Z(t) = 1/\sqrt{t}$. Find the probability that such a seal is still intact after 4 years.

6.52 Derive the mean and variance of the Weibull distribution.

6.53 In a biomedical research study, it was determined that the survival time, in weeks, of an animal subjected to a certain exposure of gamma radiation has a gamma distribution with $\alpha = 5$ and $\beta = 10$.

(a) What is the mean survival time of a randomly selected animal of the type used in the experiment?

(b) What is the standard deviation of survival time?

(c) What is the probability that an animal survives more than 30 weeks?

6.54 The lifetime, in weeks, of a certain type of transistor is known to follow a gamma distribution with mean 10 weeks and standard deviation $\sqrt{50}$ weeks.

(a) What is the probability that a transistor of this type will last at most 50 weeks?

(b) What is the probability that a transistor of this type will not survive the first 10 weeks?

6.55 Computer response time is an important application of the gamma and exponential distributions. Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds.