(1)

Law of large number. CODE NO: 835

30/Rs.

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✓ Law of large number is used to established the consistency of results.

Detail:- In probability theory law of large number is a theorem that describe the results of performing same experiment a large number of times. According to this law the average of results obtained from a large number of trails should be closed to expected value of results and will tends to become closer as more trails are performed.

∴ Law of large no. also called law of average.

There are two types of law of large number

1) weak law of large number (WLLN):-

2) ~~strong law of large number (SLLN):-~~

1) Weak law of large number:-

Let $X_1, X_2, \dots, X_n$ are independent trails with expected value of X_i $[E(X_i)] = \mu$ and finite variance σ^2. Let S_n = Sum of indep. trails $= X_1 + X_2 + \dots + X_n$. Then for any arbitrary value $\epsilon > 0$ the WLLN states that the sample average $\left(\frac{S_n}{n}\right)$ converges in probability towards the expected value.

Mathematically:-

$$\boxed{\begin{array}{c} P\left[\left(\frac{S_n}{n} - \mu\right) > \epsilon\right] = 0 \\ \\ P\left[\left(\frac{S_n}{n} - \mu\right) \le \epsilon\right] = 1 \end{array}}$$

as $n \to \infty$

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$\because \frac{S_n}{n} = \bar{x}$

$\because \mu = E(\bar{x})$

$$\bar{x} \xrightarrow{P} \mu$$

Here P means "converges in probability".

2) Strong law of large number:-

Let $X_1, X_2, \dots, X_n$ are the independent trails with $E(X_i) = \mu$ and finite variance σ^2. $S_n = X_1 + X_2 + \dots + X$

than for any no. $\epsilon > 0$ the strong law of large number states that the sample average converges almost surely to the expected value

Mathematically :-

$$\frac{S_n}{n} \xrightarrow{a.s} \mu$$

∵ a.s = almost surely (یقینی)

✓ Q:- State and prove law of large number.

Statement :-

Let S_n be a sequence of independent random variables with finite μ and finite variance σ^2. Then for any number $\epsilon > 0$

$$P\left\{\left[\frac{S_n}{n} - E\left(\frac{S_n}{n}\right)\right] > \epsilon\right\} \xrightarrow{P} 0 \quad n \to \infty$$

$$\lim_{n \to \infty} P\left\{\left[\frac{S_n}{n} - E\left(\frac{S_n}{n}\right)\right] > \epsilon\right\} \xrightarrow{P} 0$$

Proof:-

$$S_n = x_1 + x_2 + \cdots + x_n$$

then $$\frac{S_n}{n} = \bar{x}_n$$

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$$E(\bar{X}_n) = E\left(\frac{S_n}{n}\right)$$

$$= E\left(\frac{X_1 + X_2 + \cdots + X_n}{n}\right)$$

$$= \frac{1}{n}\left(E(X_1) + E(X_2) + \cdots + E(X_n)\right)$$

$$= \frac{1}{n}\left(\mu + \mu + \cdots + \mu\right)$$

$$= \frac{n\mu}{n} = \mu$$

$$V(\bar{X}_n) = V\left(\frac{S_n}{n}\right) = \frac{1}{n^2} V(S_n)$$

$$= \frac{1}{n^2}\left\{V(X_1) + V(X_2) + \cdots + V(X_n)\right\}$$

$$= \frac{1}{n^2}\left\{\sigma^2 + \sigma^2 + \cdots + \sigma^2\right\} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

According to chebyshev's inequality

$$\boxed{P\left\{\left[\bar{X}_n - E(\bar{X}_n)\right] > \epsilon\right\} \leq \frac{V(X_n)}{\epsilon^2}}$$ ✓

$$P\left[(\bar{X}_n - \mu) > \epsilon\right] \leq \frac{\sigma^2}{n\epsilon^2}$$

Applying limit on both sides

$$\lim_{n\to\infty} P\left[(\bar{X}_n - \mu) > \epsilon\right] \leq \lim_{n\to\infty} \frac{\sigma^2}{n\epsilon^2}$$

$$\lim_{n\to\infty} P\left[(\bar{X}_n - \mu) > \epsilon\right] \leq 0.$$