Markov Chain:

A markov chain is a mathematical system that experiences transition from one state to another according to some probabilistic rules.

The defining characteristic of a Markov chain is that no matter how the process arrived at its present state, the possible future states are fixed. In other words, the probability of transitioning to any particular state is dependent solely on the current state and time elapsed.

Transition Matrics:-

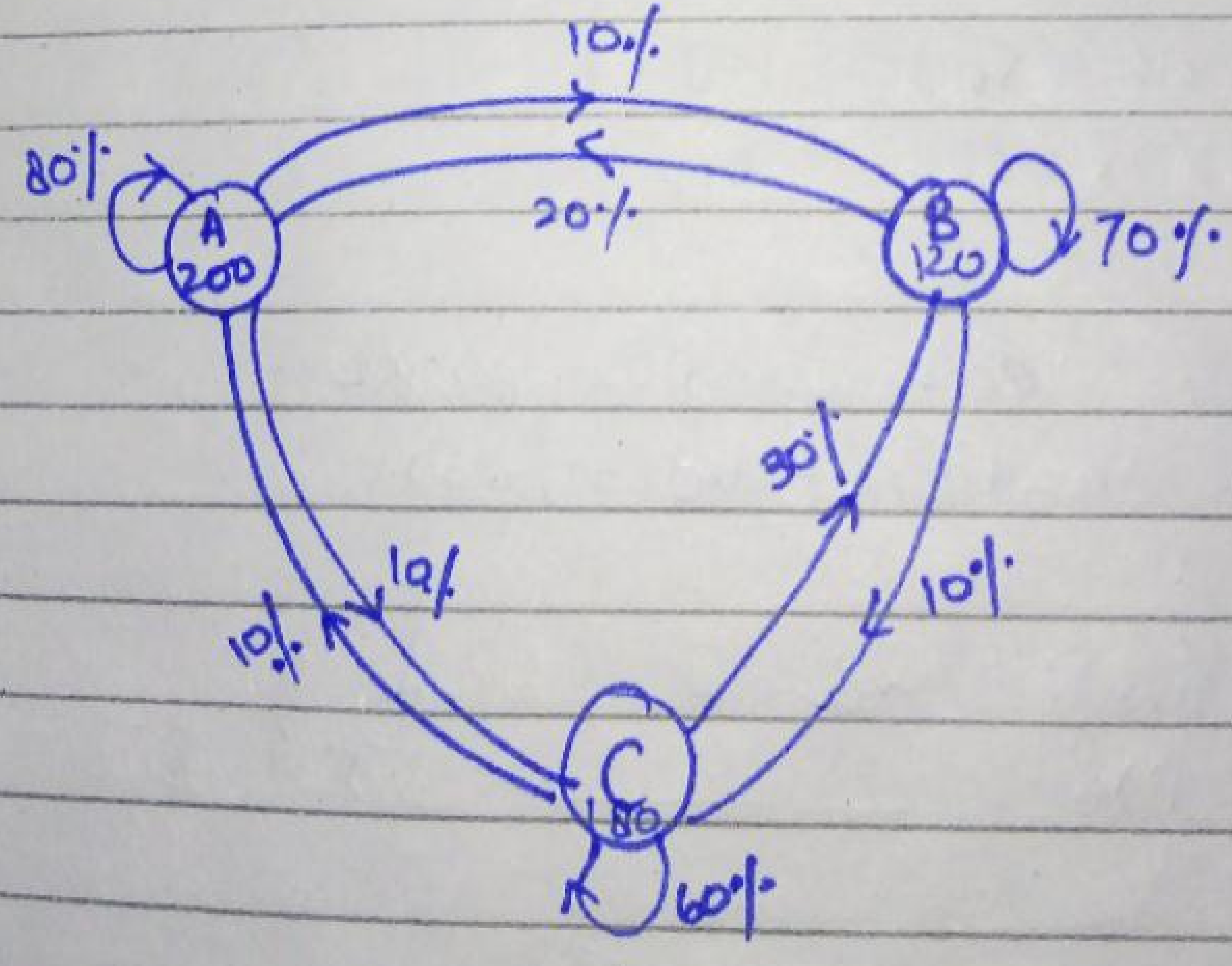
A transition matrix P_t for Markov chain at time t is a matrix containing information on the probability of transitioning between states. In particular, given an ordering of a matrix's rows and columns by state spaces. This means each row of a matrix is a probability vector and the sum of its entries is 1.

Most of our study of probability has dealt with independent trail process. when we study a sequence of chance experiments from an independent trails process, the possible outcomes of each experiment are the same and occur with the same probability. Further, knowledge of the outcomes of the previous experiments does not influence our predictions for the outcomes of the next experiment.

Modren probability theory studies chance process for which knowledge of previous outcomes influences predictions for future experiments. In principle, when we observe a sequence of chance experiments, all of the past outcomes could influence our predictions for the next experiments. For example, this should be the case in predicting a student's grades on a sequence of exams in a course. But to allow this much generally would make it very difficult to prove general results.

In 1907, Markov began the study of an important new type of chance process. In this process, the outcome of a given experiment can effect the outcome of the next experiment. This type of process is called a Markov chain.

Question: There are three shopping malls in a certain colony. A sample of 500 people are collected from these malls on the basis of their choices about these shopping malls. From the data given below in the form of chain or transition diagram calculated the next state to illustrate the condition of these shopping malls.



[Next state] = [Matrix of transition probabilities] [Current state]

$$
\begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \end{array} \begin{bmatrix} 200 \\ 120 \\ 180 \end{bmatrix} \begin{bmatrix} 200/500 \\ 120/500 \\ 180/500 \end{bmatrix}
$$

OR

$$
= \begin{bmatrix} (0.8)(0.4) + (0.2)(0.4) + (0.1)(0.36) \\ (0.1)(0.4) + (0.7)(0.24) + (0.3)(0.36) \\ (0.1)(0.4) + (0.1)(0.24) + (0.6)(0.36) \end{bmatrix}
$$

$$
= \begin{bmatrix} 0.404 \\ 0.316 \\ 0.280 \end{bmatrix} \rightarrow \text{sum should be one.}
$$

$0.404 + 0.316 + 0.280 = 1$

Now the next state is as follow

$$
\begin{bmatrix} 0.404 \times 500 = 202 \\ 0.316 \times 500 = 158 \\ 0.28 \times 500 = 140 \end{bmatrix}
$$

Total sample people

$202 + 158 + 140 = 500$

Why they call it chain

$X_1 = P X_0 \longrightarrow$ Initial state

Next or Future state

Probability Matrix

$X_2 = P X_1$, $X_3 = P X_2$ and so on.

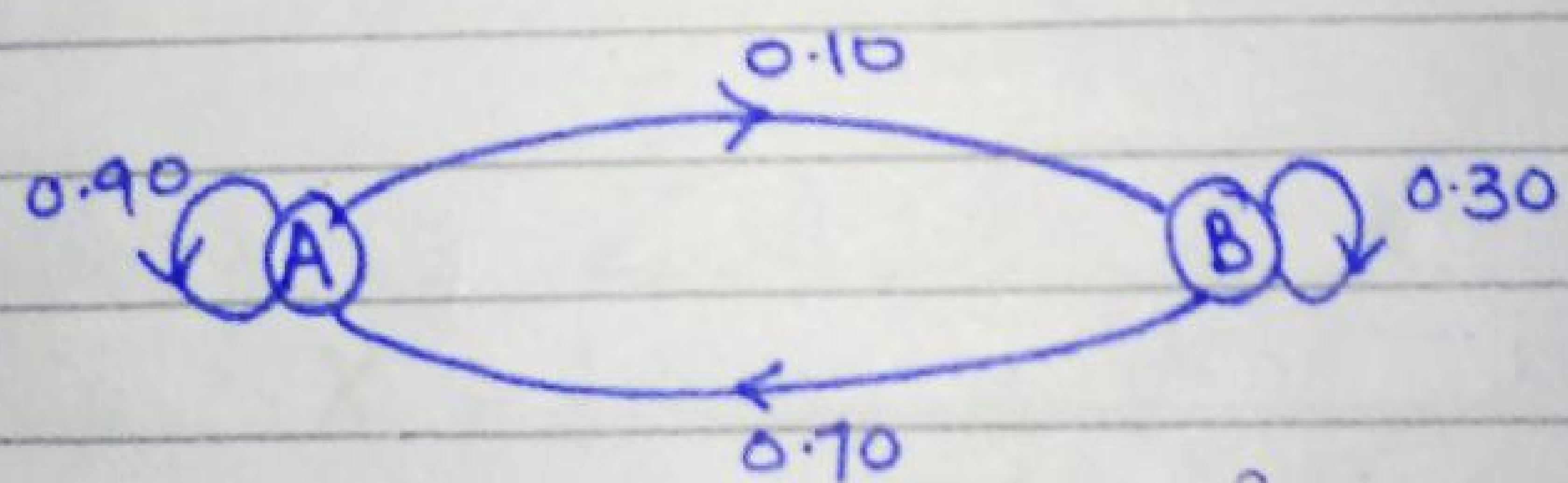
Question :-

Suppose that an orange juice company controls 20% of the organe juice market. They hire a market research company to predict the effect on an aggressive ad compaign. Suppose they conclude

→ Someone using Brand A will stay with Brand A with 90% probability, after 1 week.

→ Someone not using Brand A will switch to Brand A with 70% probability after 1 week.

Transition Diagram



$$[\text{Next state}] = \begin{array}{c|cc} & A & B \\ \hline A & 0.90 & 0.70 \\ B & 0.10 & 0.30 \end{array} \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$$

$$= \begin{bmatrix} (0.9)(0.2) + (0.7)(0.8) \\ (0.1)(0.2) + (0.3)(0.8) \end{bmatrix}$$

$$= \begin{bmatrix} 0.74 \\ 0.26 \end{bmatrix} \quad \text{Next state.}$$