**PROPERTIES OF ESTIMATORS**

UNBIASEDNESS:

An estimator is said to be unbiased if in the long run it takes on the value of the population parameter. That is, if you were to draw a sample, compute the statistic, repeat this many, many times, then the average over all of the sample statistics would equal the population parameter.

Example:

Let $X\_{1},X\_{2},…………………X\_{n}$ be a random sample of size n from a population having mean $μ$ So, we have to prove $E\left[\overbar{X}\right]=μ$.

$$E\left[\overbar{X}\right]=E\frac{(\sum\_{i=1}^{n}X\_{i})}{n}=\frac{1}{n}E(X\_{1}+X\_{2}+…+X\_{n})$$

 $=\frac{1}{n}[E\left(X\_{1}\right)+E\left(X\_{2}\right)+…+E\left(X\_{n}\right)]$

 =$\frac{1}{n}\left[μ+μ+…μ\right]=\frac{1}{n}nμ$

$$E\left[\overbar{X}\right]=μ$$

So $\overbar{X}$ is an unbiased estimator of $μ$.

CONSISTENCY:

The statistics $t\_{n}$ is a consistent estimator of $θ$ if and only if for each arbitrarily small positive number or constant $\in $

$\lim\_{n\to \infty }P(|$ $t\_{n}-θ\left|\geq \in \right)=0$

Or equivalently if and only if

$\lim\_{n\to \infty }P(|$ $t\_{n}-θ\left|<\in \right)=1$

Example:

Show that $\overbar{X}$ is a consistent estimator of parameter $μ$ when random sample is taken from N($μ,σ^{2})$.

 $E\left[\overbar{X}\right]=μ$ is an unbiased estimator

$$Var\left[\overbar{X}\right]=Var\left(\frac{\sum\_{i=1}^{n}X\_{i}}{n}\right)=\frac{1}{n^{2}}var\left(\sum\_{i=1}^{n}X\_{i}\right)=\frac{1}{n^{2}}\sum\_{i=1}^{n}\left(varX\_{i}\right)$$

$$=\frac{1}{n^{2}}nσ^{2}=\frac{σ^{2}}{n}$$

Put $\lim\_{n\to \infty }.$on both sides of the above equation

$$\lim\_{n\to \infty }Var\left[\overbar{X}\right]=\lim\_{n\to \infty }\frac{σ^{2}}{n}$$

 = 0 $\overbar{X}$ is a consistent estimator.

EFFICIENCY:

An unbiased estimator is said to be efficient if the variance of x sampling distribution smaller than that of the sampling distribution of any other unbiased estimator of the same parameter.

Example:

Suppose we have some prior knowledge that the population from which we are about to sample is normal. The mean of this population is however unknown to us. Because it is normal we know that $\overbar{X}$ and median sample is unbiased.

$$E\left[\overbar{X}\right]=μ$$

$$E\left[md\right]=μ$$

However, consider their variances

$$Var\left[md\right]=\frac{πσ^{2}}{2n}$$

$$Var\left[\overbar{X}\right]=\frac{σ^{2}}{n}$$

Clearly, $\overbar{X}$ is the more efficient since it has the smaller variance.

SUFFICIENCY:

We say that an estimator is sufficient if it uses all the sample information. The median, because it considers only rank, is not sufficient. The sample mean considers each member of the sample as well as its size, so is a sufficient statistic. Or, given the sample mean, the distribution of no other statistic can contribute more information about the population mean. We use the factorization theorem to prove sufficiency. If the likelihood function of a random variable can be factored into a part which has as its arguments only the statistic and the population parameter and a part which involves only the sample data, the statistic is sufficient.