Convective Heat Transfer:

Mathematical and Computational Modelling of Viscous Fluids and Porous Media

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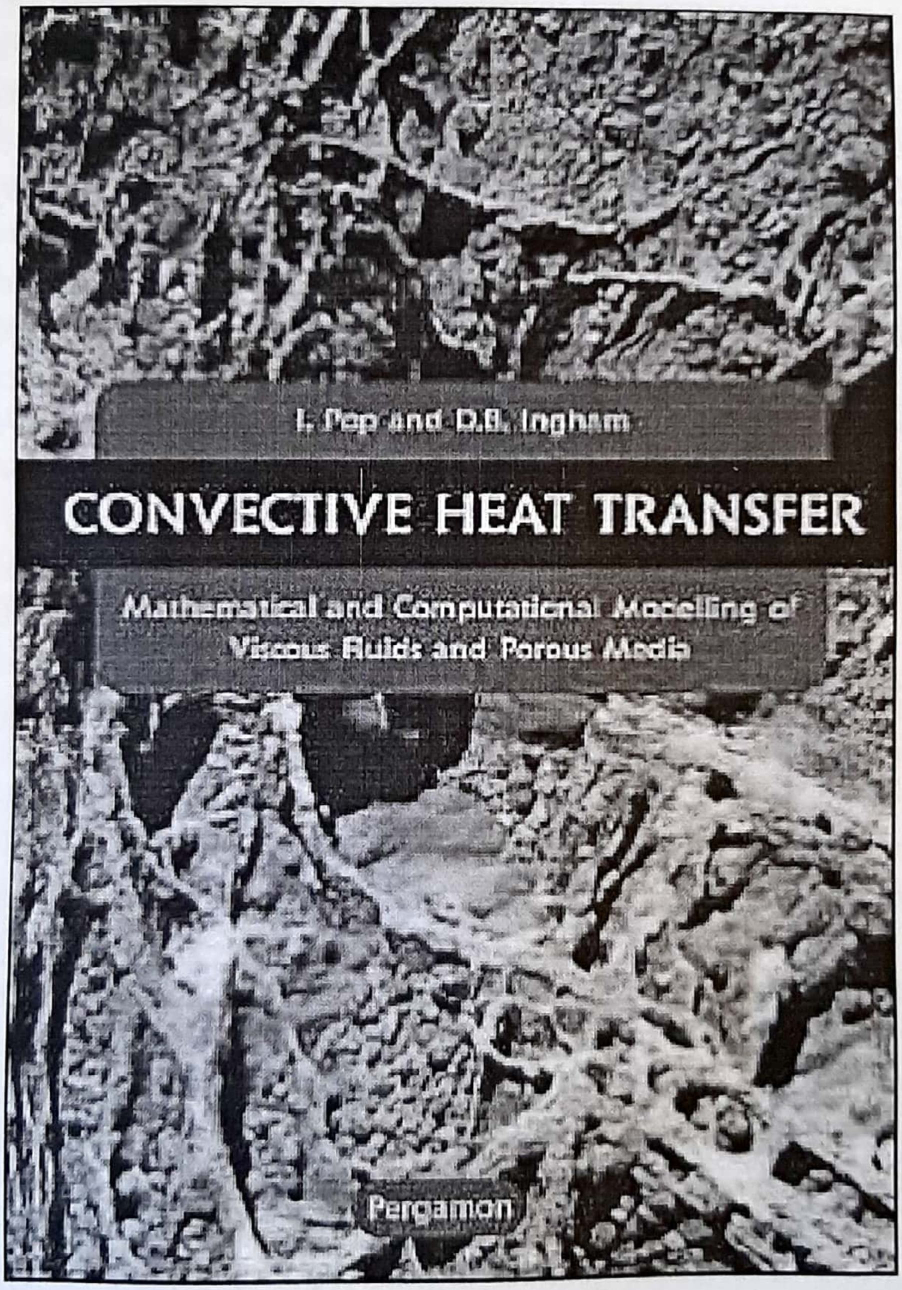

- ISBN: 0080438784
- Pub. Date: February 2001
- Publisher: Elsevier Science & Technology Books

Heat Transfer ①

Radiation Effects on Mixed Convection along Vertical Plate:

Introduction to thermal radiation:

Thermal radiation is electromagnetic radiation generated by the thermal motion of charged particle in matter. All matter with Temperature greater than absolute Zero emits the thermal radiation. All substances continuously emit radiation in form of electromagnetic waves because of the molecular and atomic agitation which is associated with the internal energy of the material. Here we consider radiation that is detected as heat or light. It is the mode of heat transfer in which heat is transferred through electromagnetic waves. Hence, in the absence of an intervening medium, there is net heat transfer by radiation between two surfaces at different temperatures. Usually, we consider the radiation emitted by solids, but emission may occur from liquid and gases. Now matter what the form of the matter, i.e emission is attributed to changes in electron configurations of the constituent atoms or molecules. In thermal radiation field ergy is transmitted by ② electromagnetic waves or ~~prob~~ Photons. Medium is not required like conduction or convection to transfer of energy in radiation.

A new dimension is added to the study of mixed convection flow past vertical plate by considering the effect of thermal radiation. Thermal radiation effect plays a significant role in controlling heat transfer processes polymer processing industry. Also, the effect of thermal radiation on flow and heat transfer processes is of major important in the design of many advanced energy convection systems which operate at high temperature. Another important effect of considering thermal radiation is enhance the thermal diffusivity of the cooling liquid in the ~~test~~ mixed convection flow over vertical plate. When radiation is included in flow problem fluid ~~is~~ called thermal conducting fluid, known as optical dense gray fluid. In many new engineering process (such as fuel combustion energy process, solar power technology, astrophysical flows, and space vehicle re-entry) occur at high temperatures so knowledge of radiation heat transfer beside the convective heat transfer play very important role and can not be neglected.

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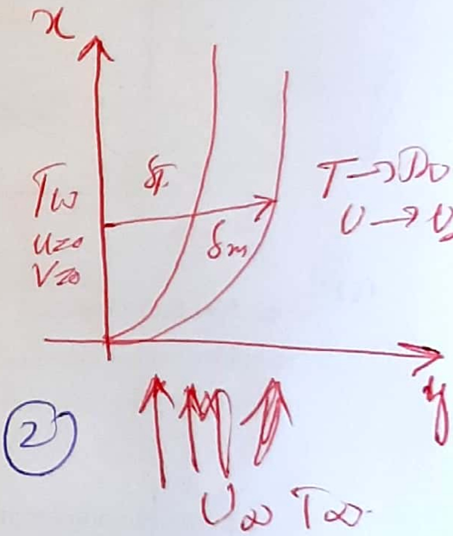
The Rosseland approximation is used to describe the radiative heat flux in the energy equation.

Mathematical Problem:-

Let us consider two dimensional steady, incompressible flow of viscous fluids along the vertical plate. Here, T_w, T_∞ are the temperatures of surface and ambient fluid where $T_w > T_\infty$. Taking into account the thermal radiation term in the energy equation, the governing equations of motion and heat transfer.



Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \longrightarrow (1)$$

Momentum Equation Bouyancy term

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \boxed{g\beta(T - T_\infty)} \longrightarrow (2)$$

Energy Equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \boxed{\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}} \longrightarrow (3)$$

↳ Inclusion of radiative heat flux

Boundary Conditions

$$u = v = 0, \quad T = T_w \quad \text{at } y = 0 \longrightarrow (4)$$

$$u \to U_\infty, \quad T \to T_\infty \quad \text{as } y \to \infty$$

Here u and v are the velocity components along the flow direction (x-direction) and normal to flow direction (y-direction), ν is kinematic viscosity, K is thermal conductivity

c_p is the specific heat of the fluid at a constant pressure, ρ is density of fluid, q_r is the radiative heat flux, T is the temperature across the thermal boundary layer, T_w is a constant temperature of the wall, T_∞ is constant temperature of the ~~ab~~ ambient fluid and U_∞ is a constant free stream velocity.

Using the Rosseland approximation for radiation, the radiative heat flux is simplified as

$$\boxed{q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}} \longrightarrow (5)$$

where σ^* and k^* are the Stefan-Boltzman constant and the mean absorption coefficient, respectively. We assume that the temperature differences with in the flow such that the term T^4 may be expressed as linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting high-order terms, we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \longrightarrow (6)$$

In view of equations (5) and (6), Eq (3) reduces

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(4T_\infty^3 T - 3T_\infty^4\right)$$

$$= \alpha\frac{\partial^2 T}{\partial y^2} + \frac{4T_\infty^3}{\rho c_p}$$

(5)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}\right)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{4\sigma^*}{3k^*\rho c_p}\frac{\partial^2 T^4}{\partial y^2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{4\sigma^*}{3k^*\rho c_p}\frac{\partial^2}{\partial y^2}\left(4T_\infty^3 T-3T_\infty^4\right)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{4\sigma^* T_\infty^3 4}{3k^*\rho c_p}\frac{\partial^2 T}{\partial y^2}$$

$$\boxed{-3\frac{\partial^2 T_\infty^4}{\partial y^2}=0}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{4.\,4\sigma^* T_\infty^3 k}{3k^*\rho c_p k}\frac{\partial^2 T}{\partial y^2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{k}{\rho c_p}\frac{4}{3}\cdot\frac{4\sigma^* T_\infty^3}{k^* k}\frac{\partial^2 T}{\partial y^2}\quad ;\ \alpha=\frac{k}{\rho c_p}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\alpha\cdot\frac{4}{3}\frac{1}{Rd}\frac{\partial^2 T}{\partial y^2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(1+\frac{4}{3}Rd\right)\frac{\partial^2 T}{\partial y^2}\longrightarrow (7)$$

where $\alpha=\frac{k}{\rho c_p}$ is thermal diffusivity.

$$\boxed{;\ Rd=\frac{kk^*}{4\sigma^* T_\infty^3}}$$

Similarity Variable formulation: 6

System To ~~[illegible]~~ transform the partial differential equations (1) — (2), (7) with boundary conditions given in (4) ~~state into~~ into system of ordinary differential equations, we introduce similarity variables

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}, \quad \psi(x,y) = \sqrt{\nu x U_\infty}\, f(\eta)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \Rightarrow (8)$$

Where η, $f(\eta)$ ~~and a dimensionless stream~~ are similarity variable η and dimensionless stream function $f(\eta)$ as.

We use

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

$$u = \frac{\partial}{\partial y}\left[\sqrt{\nu x U_\infty}\, f(\eta)\right] = \sqrt{\nu x U_\infty} \cdot \frac{\partial f}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}$$

$$u = \sqrt{\nu x U_\infty}\, f' \cdot \sqrt{\frac{U_\infty}{\nu x}}$$

$$\boxed{u = U_\infty f'}$$

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}$$

$$\frac{\partial \eta}{\partial y} = \sqrt{\frac{U_\infty}{\nu x}}$$

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(7)

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(U_\infty f') = U_\infty \frac{\partial f'}{\partial \eta}\cdot\frac{\partial \eta}{\partial x}$$

$$\frac{\partial u}{\partial x} = U_\infty f''\left(\frac{-\eta}{2x}\right)$$

$$\boxed{\frac{\partial u}{\partial x} = -\frac{\eta U_\infty}{2x} f''}$$

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}$$

$$\eta = y\sqrt{\frac{U_\infty}{\nu}}\, x^{-\frac{1}{2}}$$

$$\frac{\partial \eta}{\partial x} = -\frac{x^{-\frac{1}{2}-1}}{2}\, y\sqrt{\frac{U_\infty}{\nu}}$$

$$\frac{\partial \eta}{\partial x} = -\frac{x^{-\frac{1}{2}}\, y\sqrt{\frac{U_\infty}{\nu}}}{2x}$$

$$\frac{\partial \eta}{\partial x} = -\frac{y\sqrt{\frac{U_\infty}{\nu x}}}{2x}$$

$$\boxed{\frac{\partial \eta}{\partial x} = -\frac{\eta}{2x}}$$

$$u\frac{\partial u}{\partial x} = U_\infty f' \times -\frac{\eta U_\infty}{2x} f''$$

$$\boxed{u\frac{\partial u}{\partial x} = -\frac{U_\infty^2 \eta}{2x} f' f''}$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$v = -\frac{\partial}{\partial x}\left(\sqrt{\nu x U_\infty}\, f(\eta)\right)$$

$$v = -\frac{\partial}{\partial x}\left(x^{\frac{1}{2}}\sqrt{\nu U_\infty}\, f(\eta)\right)$$

$$v = -\frac{\sqrt{\nu U_\infty}\,\partial}{\partial x}\left(x^{\frac{1}{2}} f(\eta)\right)$$

$$v = -\sqrt{\nu U_\infty}\left(\frac{1}{2x^{1/2}} f + x^{\frac{1}{2}}\frac{\partial f}{\partial \eta}\cdot\frac{\partial \eta}{\partial x}\right)$$

$$v = -\sqrt{\nu U_\infty}\left(\frac{f}{2x^{1/2}} - x^{\frac{1}{2}}\frac{\eta}{2x} f'\right)$$

$$v = -\sqrt{\nu U_\infty}\left(\frac{f}{2x^{1/2}} - \frac{\eta}{2x^{1/2}} f'\right)$$

$$v = -\frac{\sqrt{\nu U_\infty}}{2x^{1/2}}(f - \eta f') = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f' - f)$$

$$\boxed{v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f' - f)}$$

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$$\frac{\partial v}{\partial y} = \frac{1}{2}\frac{\partial}{\partial y}\sqrt{\frac{\nu U_\infty}{x}}\,(\eta f' - f)$$

$$\frac{\partial v}{\partial y} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left[\frac{\partial}{\partial y}(\eta f') - \frac{\partial}{\partial y}(f)\right]$$

$$\frac{\partial v}{\partial y} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left[\eta\frac{\partial f'}{\partial \eta}\cdot\frac{\partial \eta}{\partial y} + f'\frac{\partial \eta}{\partial y} - \frac{\partial f}{\partial \eta}\cdot\frac{\partial \eta}{\partial y}\right]$$

$$\frac{\partial v}{\partial y} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left[\eta f''\cdot\sqrt{\frac{U_\infty}{\nu x}} + f'\sqrt{\frac{U_\infty}{\nu x}} - f'\sqrt{\frac{U_\infty}{\nu x}}\right]$$

$$\frac{\partial v}{\partial y} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\cdot\sqrt{\frac{U_\infty}{\nu x}}\left[\eta f'' + \cancel{f'} - \cancel{f'}\right]$$

$$\frac{\partial v}{\partial y} = \frac{1}{2}\frac{\eta U_\infty}{x} f'' \Rightarrow \boxed{\frac{\partial v}{\partial y} = \frac{\eta U_\infty}{2x} f''}$$

$$\frac{\partial u}{\partial y} = U_\infty\frac{\partial}{\partial y}(f') = U_\infty\frac{\partial f'}{\partial \eta}\cdot\frac{\partial \eta}{\partial y}$$

$$\boxed{\frac{\partial u}{\partial y} = U_\infty f''\cdot\sqrt{\frac{U_\infty}{\nu x}}}$$

$$v\frac{\partial u}{\partial y} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\,(\eta f' - f)\cdot U_\infty f''\cdot\sqrt{\frac{U_\infty}{\nu x}}$$

$$v\frac{\partial u}{\partial y} = \frac{1}{2}\frac{U_\infty^2}{x}(\eta f' f'' - f f'')$$

$$\boxed{v\frac{\partial u}{\partial y} = \frac{U_\infty^2}{2x}(\eta f' f'' - f f'')}$$

Put

$\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in eq. (1), (9)

$$-\frac{\eta U_\infty}{2x} f'' + \frac{\eta U_\infty}{2x} f'' = 0$$

$$\boxed{0 = 0}$$

Now

$$\frac{\partial^2 u}{\partial y^2} = U_\infty \sqrt{\frac{U_\infty}{\nu x}} \cdot \frac{\partial f''}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = U_\infty \sqrt{\frac{U_\infty}{\nu x}} \cdot f''' \sqrt{\frac{U_\infty}{\nu x}}$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = \frac{U_\infty^2}{\nu x} f'''}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\boxed{\theta \Delta T = T - T_\infty} \Rightarrow$$

put $u\frac{\partial u}{\partial x}$, $v\frac{\partial u}{\partial y}$ and $\frac{\partial^2 u}{\partial y^2}$ = equation

$$-\frac{U_\infty^2}{2x} f' f'' + \frac{U_\infty^2}{2x}(\eta f' f'' - f f'') = \frac{\nu U_\infty^2}{\nu x} f''' + g\beta \Delta T \theta$$

$$\cancel{-\frac{U_\infty^2}{2x}\eta f' f''} + \cancel{\frac{U_\infty^2}{2x}\eta f' f''} - \frac{U_\infty^2}{2x} f f'' = \frac{\nu U_\infty^2}{\nu x} f''' + g\beta \Delta T \theta$$

$$-\frac{U_\infty^2}{2x} f f'' = \frac{U_\infty^2}{x} f''' + g\beta\Delta T \theta$$

$$-\frac{1}{2} f f'' = \frac{x}{U_\infty^2} \times \frac{U_\infty^2}{x} f''' + \frac{g\beta\Delta T \times x}{U_\infty^2} \theta$$

$$-\frac{1}{2} f f'' = f''' + \frac{g\beta \Delta T x^3 \nu^2}{x^2 \nu^2 U_\infty^2} \theta$$

$$0 = f''' + \frac{1}{2} f f'' + \frac{g\beta\Delta T x^3}{\nu^2} \cdot \frac{\nu^2}{x^2 U_\infty^2} \theta$$

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$$0 = f''' + \frac{1}{2} f f'' + \frac{Gr}{Re^2}\theta \rightarrow 9$$

$$f''' + \frac{1}{2} f f'' + \alpha\theta = 0 \rightarrow$$

$$\alpha = \frac{Gr}{Re^2}, \quad Gr = \frac{g\beta\Delta T x^3}{\nu^2}, \quad Re = \frac{U_\infty}{\nu x}$$

α is mixed convection parameter

Gr is Grashoff number

Re is Reynold's number.

Now we find

$$\cancel{\frac{\partial\theta}{\partial x} = \frac{\partial\theta}{\partial\eta}\cdot\frac{\partial\eta}{\partial x} + \frac{\partial\theta}{\partial\eta}\cdot\frac{\partial\eta}{\partial y}}$$

$$T - T_\infty = \Delta T\,\theta$$

$$T = \Delta T\,\theta + T_\infty$$

$$\frac{\partial T}{\partial x} = \Delta T\,\frac{\partial\theta}{\partial\eta}\cdot\frac{\partial\eta}{\partial x} + \frac{\partial T_\infty}{\partial x}$$

$$\frac{\partial T}{\partial x} = \Delta T\,\theta'\left(\frac{-\eta}{2x}\right)$$

$$\boxed{\frac{\partial T}{\partial x} = -\frac{\Delta T\,\eta}{2x}\,\theta'}$$

$$u\frac{\partial T}{\partial x} = -U_\infty f'\,\frac{\Delta T\,\eta}{2x}\,\theta'$$

$$\boxed{u\frac{\partial T}{\partial x} = -\frac{U_\infty\Delta T\,\eta}{2x} f'\theta'}$$

(11)

$$\frac{\partial T}{\partial y} = \cancel{\Delta T}\ \Delta T \frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial y}$$

$$\frac{\partial T}{\partial y} = \Delta T\, \theta' \cdot \sqrt{\frac{U_\infty}{\nu x}} = \Delta T \sqrt{\frac{U_\infty}{\nu x}}\, \theta'$$

$$v\frac{\partial T}{\partial y} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\,(\eta f' - f) \times \Delta T \sqrt{\frac{U_\infty}{\nu x}}\, \theta'$$

$$v\frac{\partial T}{\partial y} = \frac{U_\infty}{2x}(\eta f'\theta' - f\theta')$$

$$v\frac{\partial T}{\partial y} = \frac{\Delta T U_\infty}{2x}(\eta\theta' f' \cancel{\theta} - f\theta')$$

$$\boxed{v\frac{\partial T}{\partial y} = \frac{U_\infty}{2x}(\eta\theta' f' - f\theta')}$$

$$\frac{\partial^2 T}{\partial y^2} = \Delta T\sqrt{\frac{U_\infty}{\nu x}} \cdot \theta'' \cdot \frac{\partial \eta}{\partial y}$$

$$\frac{\partial^2 T}{\partial y^2} = \Delta T\sqrt{\frac{U_\infty}{\nu x}} \cdot \theta'' \cdot \sqrt{\frac{U_\infty}{\nu x}}$$

$$\frac{\partial^2 T}{\partial y^2} = \Delta T\, \frac{U_\infty}{\nu x} \cdot \theta''$$

$$\boxed{\frac{\partial^2 T}{\partial y^2} = \frac{\Delta T U_\infty}{\nu x}\, \theta''}$$

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Put $u\frac{\partial T}{\partial x}$, $v\frac{\partial T}{\partial y}$, $\frac{\partial^2 T}{\partial y^2}$ in eq (7)

$$-\frac{U_\infty \eta \Delta T}{2x} f' \theta' + \frac{U_\infty}{2x}(\eta \theta' f' - f\theta') = \alpha\left(1+\frac{4}{3R_d}\right)\frac{\Delta T U_\infty}{\nu x}\theta''$$

$$-\frac{U_\infty \eta \Delta T}{2x} f'\theta' + \frac{U_\infty \Delta T}{2x}(\eta \theta' f' - f\theta') = \alpha\left(1+\frac{4}{3R_d}\right)\times\frac{\Delta T U_\infty}{\nu x}\theta''$$

$$\cancel{-\frac{U_\infty \eta \Delta T}{2x} f'\theta'} + \cancel{\frac{U_\infty \eta \Delta T}{2x}\theta' f'} - \frac{U_\infty \Delta T}{2x} f\theta' = \frac{\alpha \times \Delta T U_\infty}{\nu x}\left(1+\frac{4}{3R_d}\right)\theta''$$

$$-\frac{U_\infty \Delta T}{2x} f\theta' = \frac{\alpha \Delta T U_\infty}{\nu x}\left(1+\frac{4}{3R_a}\right)\theta''$$

$$-\frac{1}{2} f\theta' = \frac{x}{U_\infty \Delta T}\cdot\frac{\Delta T U_\infty}{\nu x}\alpha\left(1+\frac{4}{3R_d}\right)\theta''$$

$$-\frac{1}{2} f\theta' = \frac{\alpha}{\nu}\left(1+\frac{4}{3R_d}\right)\theta''$$

$$-\frac{1}{2} f\theta' = \frac{1}{Pr}\left(1+\frac{4}{3R_d}\right)\theta''$$

$$\frac{1}{Pr}\left(1+\frac{4}{3R_d}\right)\theta'' + \frac{1}{2} f\theta' = 0$$

$$\boxed{\left(1+\frac{4}{3R_d}\right)\theta'' + \frac{Pr}{2} f\theta' = 0}$$

Boundary Conditions:

$U_\infty f' = 0, \quad \Delta T\theta + T_\infty = T_w, \qquad \text{at } \eta = 0$

$\cancel{f=0}\; f' = 0, \quad \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\,(\eta f'(0) - f) = 0$

$f = 0, \; f' = 0, \quad \Delta T \theta = T_w - T_\infty \qquad \text{at } \eta = 0$

$f = 0, \; f' = 0, \quad \theta \Delta T = \Delta T \qquad \text{at } \eta = 0$

$f = 0, \; f' = 0, \quad \theta = 1, \qquad \text{at } \eta = 0$

or $u \to 0$

$U_\infty f' \to 0, \qquad T \to T_\infty \qquad \eta = y\sqrt{\frac{U_\infty}{2\nu}} \to \infty$

$f' \to 0, \qquad \Delta T\theta + T_\infty \to T_\infty, \qquad \text{as } \eta \to \infty.$

$\theta \to 0$

So B.Cs (Transformed

$f = 0, \; f' = 0, \quad \theta = 1, \qquad \text{at } \eta = 0$

$f' \to 0, \quad \theta \to 0, \quad \text{as } \eta \to \infty.$