6.3 Truth Tables for Propositions

1. Truth Tables for 2-Letter Propositions: In section 6.2, we learned about truth tables for simple statements. For instance, the truth table for "A Ↄ B" is the following:

Conditional

A	B	A Ↄ B
T	T	T
T	F	F
F	T	T
F	F	T

So, if I had told you that, "If you come over and help me move my couch on Saturday, then I will buy you pizza", the ONLY way you might call me a liar is if you DID come over and help me, and I DID NOT buy you pizza.

(To see this, imagine that "p"="you come over and help me move my couch" and "q"="I buy you pizza" and then examine the truth table above)

In this section, we will learn how to make truth tables for more complicated statements. Let's start with a statement which still has ONLY TWO statements as components ("p" and "q").

Example #1: For instance, imagine that I told you: "I know you really well. If you buy cookies, then you will buy milk AND cookies." Let "C"="You buy cookies" and "M"="You buy milk". In that case, my claim can be written as the following:

C Ↄ (M • C)

The truth table for C and M still looks like this:

C	M	C Ↄ (M • C)
T	T	?
T	F	?
F	T	?
F	F	?

Our job now is to figure out what goes in place of the question marks. We figure them out one line at a time. Let's start with the first line, where C and M are both true. In that case, we get the following:

Line 1: C=T, M=T

C Ↄ (M ● C)	becomes:
T Ↄ (T ● T)	(we’ve replaced the letters with “T” for “True” and “F” for “False”)
T Ↄ T	(the conjunction of two true statements is also true)
T	(a conditional where both the antecedent and consequent are true is also true)

So, the right-hand column should have a “T” on the first line, like this:

C	**M**	**C Ↄ (M ● C)**
T	T	T
T	F	?
F	T	?
F	F	?

Let’s do the other 3 lines:

Line 2: C=T, M=F

C Ↄ (M ● C)
T Ↄ (F ● T)
T Ↄ F
F

Line 3: C=F, M=T

C Ↄ (M ● C)
F Ↄ (T ● F)
F Ↄ F
T

Line 4: C=F, M=F

C Ↄ (M ● C)
F Ↄ (F ● F)
F Ↄ F
T

So, now that we have our truth values for lines 1-4, we can fill in the entire truth table:

C	**M**	**C ⊃ (M • C)**
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

The bold values on the far right represents the truth table for the proposition. As we can see, my claim ONLY comes out false if you go to the store and buy cookies, but do NOT buy milk (i.e., C=true and M=false).

<u>Example #2:</u> But, typically, to save space we figure out the T's and F's directly underneath the formula, all together. For instance, someone might say: "If we either exercise or eat anything other than cookies, then we'll eat cookies." Substituting "E" for "We exerise" and "C" for "We eat cookies", we can formalize this as follows:

(E ∨ ~C) ⊃ C

These are the four possible combinations of truth and falsehood for E and C:

E	**C**
T	T
T	F
F	T
F	F

So, we can just write those directly underneath the formula, like this:

(E ∨ ~C) ⊃ C
T T T
T F F
F T T
F F F

The first thing we need to do is take care of the negation in front of the C. To do that, we just reverse the truth values underneath the "~C", like this:

(E ∨ ~C) ⊃ C
T **F**T T
T **T**F F
F **F**T T
F **T**F F

Above, the original truth values for the statements E and C are in black, and the truth values for the negation are in green. Next, we should solve the stuff inside of the parenthesis (the disjunction, "∨"), like this:

(E ∨ ~C) Ɔ C
T **T** **F**T T
T **T** **T**F F
F **F** **F**T T
F **T** **T**F F

The solution for the truth values of the disjunction is in blue. Note that the disjunction is only false on the third line, since disjunctions are only ever false when BOTH of their disjuncts are false. Now, let's solve the main operator, the conditional, "Ɔ":

(E ∨ ~C) Ɔ C
T **T** **F**T **T** T
T **T** **T**F **F** F
F **F** **F**T **T** T
F **T** **T**F **F** F

The bold, red letters represent **the truth function for the whole proposition**. Notice that only the first and third lines are true. So, looking at the original truth values for the statements E and C (in black), we see that the whole statement is false ONLY WHEN "E" is true and "C" is false, or else when "E" is false and "C" is false.

In other words, the statement, "If we either exercise or eat anything other than cookies, then we'll eat cookies" is a lie ONLY when: (1) We DO exercise, but we do NOT eat cookies. Or (2) We do NOT exercise and we DO NOT eat cookies.

On the other hand, that statement is TRUE when: (1) We DO exercise AND eat cookies, or (2) We do NOT exercise, but DO eat cookies.

2. Truth Tables for 3-Letter Propositions: Next, we will look at propositions with THREE distinct statements in them. For instance, let's try one from exercise 6.3, section II, #10 (on page 342 of your textbook). It says:

W ≡ (B ● T)

Now, we've seen that when there is only ONE letter, we get a truth table with TWO lines, and when there are TWO letters, we get a truth table with FOUR lines. For instance:

Negation: 1 Letter

A	~A
T	**F**
F	**T**

Conjunction: 2 Letters

A	B	A • B
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**F**

When there are THREE letters, there will be EIGHT lines. The basic formula is that there will be **2^n lines**, where **n=the number of statement letters**. So, the truth table for THREE letter propositions will look like this:

W	B	T	Proposition
T	T	T	**?**
T	T	F	**?**
T	F	T	**?**
T	F	F	**?**
F	T	T	**?**
F	T	F	**?**
F	F	T	**?**
F	F	F	**?**

Let's determine the truth values for the entire proposition which go in the right-hand column. We begin by writing out all 8 lines with the truth values for the three different statements filled in, as follows:

W	≡	(B	•	T)
T		T		T
T		T		F
T		F		T
T		F		F
F		T		T
F		T		F
F		F		T
F		F		F

Next, let's fill in the truth values for the disjunction. The disjunction will ONLY be true when BOTH conjuncts are true. So, our table should look like this:

W	≡	(B	•	T)
T		T	**T**	T
T		T	**F**	F
T		F	**F**	T
T		F	**F**	F
F		T	**T**	T
F		T	**F**	F
F		F	**F**	T
F		F	**F**	F

Next, let's fill in the final truth values for the bi-conditional. Bi-conditionals are ONLY true whenever the statements on either side of the bi-conditional have the SAME truth value. So, here, we should be comparing the letters underneath the "W" with the green letters underneath the "•". Like this:

W	≡	(B	•	T)
T	**T**	T	**T**	T
T	**F**	T	**F**	F
T	**F**	F	**F**	T
T	**F**	F	**F**	F
F	**F**	T	**T**	T
F	**T**	T	**F**	F
F	**T**	F	**F**	T
F	**T**	F	**F**	F

This statement is true on the first line, and the last 3 lines. In other words, the entire proposition is true if:

<u>Line 1:</u> W=true, B=true, T=true
<u>Line 6:</u> W=false, B=true, T=false
<u>Line 7:</u> W=false, B=false, T=true
<u>Line 8:</u> W=false, B=false, T=false

(See the answers in the back of your textbook for this truth tables, as well as more examples of truth tables for 3-letter propositions)

<u>3. Classifying Statements:</u> In the truth table we just did, the red letters are an assortment of true and false. When this happens, we say that the proposition is "**contingent**". But, sometimes, the column will be ALL **T**'s, or ALL **F**'s. When the truth table for the proposition is ALL **T**'s, we say that the proposition is "**tautologous**". When the truth table is ALL **F**'s, we say that it is "**self-contradictory**". So:

Column Under the Main Operator	**Statement Classification**
All True	Tautologous (Logically True)
All False	Self-Contradictory (Logically False)
At Least One True & At Least One False	Contingent

Tautologous statements are true, but this truth depends ONLY on the FORM of the statement, and not at all on the content—so they are true in a really uninteresting and uninformative way. They are "trivially true".

Self-Contradictory statements are always false, and their falsehood depends upon their FORM. Since their FORM is what makes them false, they can NEVER be true, no matter WHAT the content is.

Contingent statements have a form that allows for them to be EITHER true OR false, depending on the content, and whether the statements that make up the content are true or false.

All of the statements we have examined so far have been contingent ones. (To see this, look at their truth tables, and notice that there are both T's and F's in the column under the main operator). Let's try one that is NOT contingent:

"I have a car that is red all over and not red all over." Symbolize this as follows:

R ● ~R

This only has ONE distinct letter, so there will only be TWO lines, like this:

R ● ~R
T T
F F

First, let's do the negation:

R ● ~R
T **F**T
F **T**F

The truth values for the negation are in green. Now, let's do the conjunction "●". To do that, we should compare the letters under the "R" and the green letters under the "~":

R ● ~R
T **F** **F**T
F **F** **T**F

Since conjunctions are ONLY true when BOTH conjuncts are true, there is no way that the conjunction can ever be true. So, if someone tells you that they have a car that is both red all over, and not red all over, they are claiming something impossible. Since all of the letters under the main operator (in red) are FALSE, this statement is **self-contradictory**.

4. Comparing Statements: In the previous section, we looked a SINGLE propositions and noticed something about them based on the truth values listed under the main operator. In this section, we're going to compare the truth tables of TWO propositions with EACH OTHER. To do this, we take the columns of truth vales under the main operators for EACH of the two propositions, and COMPARE them.

Sometimes, when we compare the truth tables for two different propositions, the truth values under the main operator will be exactly the same (when this happens, the two propositions are said to be "**logically equivalent**"). Sometimes they will be exactly opposite (when this happens, the two propositions are said to be "**contradictory**"). Sometimes, there will be one or more lines where both propositions come out true (in this case, the propositions are "**consistent**"). Sometimes, there will NOT be any lines where both propositions are true (in this case, the propositions are "**inconsistent**").

Columns Under the Main Operators	**Relation Between**
SAME Truth Values on EVERY Line	Logically Equivalent
DIFFERENT Truth Values on EVERY Line	Contradictory
At Least One Line Where Both Are True	Consistent
No Line Where Both Are True	Inconsistent

Let's do Exercise 6.3. Section III. #1 as an example. Harold Carson claims:

"The balance of payments will decrease if and only if interest rates remain steady; however, it is not the case that either interest rates will not remain steady or the balance of payments will decrease."

There are two claims here. (The second claim begins with the word "however") Let "D"="The balance of payments will decrease, and let "S"="interest rates remain steady". Then, we get the following two statements:

$D \equiv S$ $\quad\quad\quad\quad$ $\sim(\sim S \vee D)$

To compare the two propositions, we will need to do TWO truth tables—one for each statement. Since there are two letters, our truth table will have four lines. Like this:

D ≡ S	~(~S v D)
T T	T T
T F	F T
F T	T F
F F	F F

Next, we can completely finish the first statement on the left. For the one on the right, let's start by solving for the "~" in front of the "S":

D ≡ S	~(~S v D)
T **T** T	**F**T T
T **F** F	**T**F T
F **F** T	**F**T F
F **T** F	**T**F F

The negations are in green. Next, let's solve the disjunction by comparing the green letters with the black letters underneath the "D". Remember that a disjunction is only false when BOTH disjuncts are false. So, we get the following:

D ≡ S	~(~S v D)
T **T** T	**F**T **T** T
T **F** F	**T**F **T** T
F **F** T	**F**T **F** F
F **T** F	**T**F **T** F

Finally, we should solve for the final negation, OUTSIDE of the parenthesis. We do this by taking the blue letters and writing down the OPPOSITE of what is written in blue:

D ≡ S	~(~S v D)
T **T** T	**F** **F**T **T** T
T **F** F	**F** **T**F **T** T
F **F** T	**T** **F**T **F** F
F **T** F	**F** **T**F **T** F

Now that we have the truth tables completed for both propositions, we can compare them. To do this, we simply compare the columns in red with one another. Like this:

D ≡ S	~(~S v D)
T	**F**
F	**F**
F	**T**
T	**F**

Are the two statements **logically equivalent**? No. To be logically equivalent, the truth values in red would need to be IDENTICAL. Are they **contradictory**? No. To be contradictory, the truth values in red would need to be EXACTLY OPPOSITE. Though lines 1, 3, and 4, are exactly opposite, on line 2 they are BOTH FALSE (**F**). Are they **consistent**? No. To be consistent, there would need to be at least one line where BOTH statements are true. But there are NO lines with two **T**'s.

Answer: The two statements are **inconsistent**. That is, there is NO circumstance in which both propositions could be true; i.e., there is NO line with two **T**'s.

(See your textbook, section 6.3, as well as starred questions for exercise 6.3 for more examples comparing the truth tables of two propositions).

Note: Do homework for section 6.3 at this time.