or $\lambda = 105983$ Å

$= \mathbf{10.5\ \mu m}$

$$\therefore \quad \nu = \frac{c}{\lambda} = \frac{3 \times 10^8\ m/s}{10.5 \times 10^{-6}\ m} = \mathbf{2.9 \times 10^{13}\ Hz}$$

Example 1.3. *A pulsed laser is constructed with a ruby crystal as the active element. The ruby rod contains typically a total of $3 \times 10^{19} Cr^{3+}$ ions. If the laser emits light at 6943 Å wavelength, find (a) the energy of one emitted photon (in eV) and (b) the total energy available per laser pulse (assuming total population inversion).*

Solution : (*a*) The photon energy in eV is given by

$$E = \frac{12400}{\lambda\ (\text{Å})}\ \text{eV}$$

$$\therefore \quad E = \frac{12400}{6943\ \text{Å}} = \mathbf{1.79eV}$$

(*b*) Energy per pulse = (Energy of one photon) × (Total number of photons)

= (Energy of one photon) × (Total number of atoms in the excited state)

$E_T = (1.79\text{eV} \times 1.602 \times 10^{-19}\ \text{J/eV}) \times (3 \times 10^{19})$

$= \mathbf{8.6\ J}$

1.6 PLANCK'S RADIATION LAW

On the basis of quantum theory, Planck obtained the formula for an average energy of an oscillator is

$$E = \frac{h\nu}{e^{h\nu/kT} - 1} \quad \text{...(1.10)}$$

It can be shown that the number of oscillations or degrees of freedom per unit volume in the frequency range ν and $\nu + d\nu$ and in given by

$$N(\nu)\, d\nu = \frac{8\pi\nu^2}{C^3}\, d\nu \quad \text{...(1.11)}$$

where c is the speed of light in vacuum.

Then assuming that the average value of energies of the various modes of oscillations in black body radiation is given by Eq. 1.10, Planck obtained the relation.

$$p_\nu\, d\nu = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/kT} - 1}\, d\nu \quad \text{...(1.12)}$$

Where $p_\nu\, d\nu$ is the energy per unit volume in the frequency range ν and $\nu + d\nu$ and p_ν is the energy per unit volume per unit frequency range at frequency ν.

In terms of wavelength of radiation, the equation is expressed as

$$p_\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}\, d\lambda \quad \text{...(1.13)}$$

Eq. (1.12) and Eq. (1.13) are two forms of Planck's radiation law.

When the values of p_λ as obtained from Eq. (1.13) for different values of λ are plotted against the corresponding values of λ, we get curves as shown in fig. (1.3).

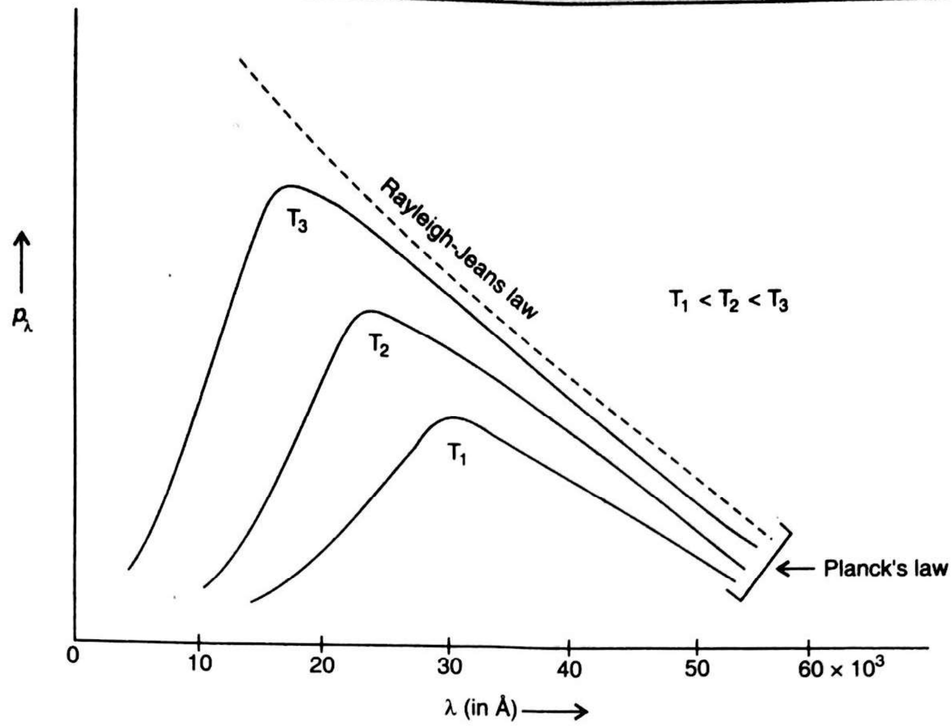

Fig. 1.3

These curves obtained for temperatures T_1, T_2 and T_3 ($T_1 < T_2 < T_3$) agree very well with the experimental results over the whole range of wavelengths.

Further, the Rayleigh-Jeans law, Wien's law and the stefan-Boltzman formula are derived using Planck's law given by Eq. (1.13). Here lies the grand success of quantum theory proposed by Max. Planck.

1.7 PHOTOELECTRIC EFFECT

When an electromagnetic radiation of sufficiently high frequency, as ultraviolet (UV) light and X-rays is incident on a clean metal surface, electrons are emitted out from it. This phenomenon is known as the photoelectric effect and the emitted electron are called as photoelectrons.

The whole range of Electro-magnetic (E-M) spectrum right from Infra-red rays, visible Ultra-violets, X-rays, γ-rays produce this effect depending upon the metal. Most metals give this effect when exposed to UV rays or X-rays. Sodium is a low z material (Here $z = 3$, atomic number) gives this effect when exposed to all radiations of wavelengths smaller than 5455Å. Caesium produces this effect when exposed to all radiations to wavelengths smaller than 6438A°.

Einstein's photoelectric equation. For photoelectric emission of an electron from the metal $h\nu$ must be equal to or greater than W_0 ($h\nu \geq W_0$). Here W_0 is the photoelectric work function of the metal. When an electron at the metal surface absorbs the energy $h\nu$, a certain *mimum* part of this energy is used up by the electron to do work equal to W_0 to overcome the attractive forces of the positive ions of the metal. The remaining *maximum* energy ($h\nu - W_0$) is in the form of maximum kinetic energy $\left(\frac{1}{2}mv_{max}^2\right)$ of the electron emitted from the metal surface.

$$\therefore \qquad \frac{1}{2}mv_{max}^2 = h\nu - W_0 \qquad ...(1.14)$$

This is known as Einstein's photoelectric equation.

Putting $W_0 = h\nu_0$ where ν_0 is called the threshold frequency for a given metal surface. Substituting, we get

$$\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0$$

or $$\frac{1}{2}mv_{max}^2 = h(\nu - \nu_0) \quad ...(1.15)$$

Eq. (1.15) is another form of Einstein's photoelectric equation.

Example 1.4. *Light of wavelength 2000 Å falls on an aluminium surface with work function 4.2 eV. Calculate the threshold wavelength.*

Solution : Given: Wavelength of light, $\lambda = 2000\text{Å}$

Work function, $W_0 = 4.2$ eV

Threshold wavelength, $$\lambda_0 = \frac{hc}{W_0} = \frac{(6.626 \times 10^{-34}\text{ Js}) \times (3 \times 10^8\text{ m/s})}{W_0(\text{eV})}$$

$$= \frac{6.626 \times 3 \times 10^{-26}}{4.2 \times (1.602 \times 10^{-19})}$$

$$= 2.954 \times 10^{-7}\text{ m} \qquad (\because 1\text{eV} = 1.602 \times 10^{-19}\text{ J})$$

$$= \mathbf{2954\ Å}$$

Example 1.5. *The photoelectric threshold wavelength of silver is 2762 Å. Calculate (a) the maximum kinetic energy of the ejected electrons, (b) the maximum velocity of the electrons ejected, when the silver surface is illuminated with wavelength 2000 Å.*

Solution : Given : $\lambda = 2000 \times 10^{-10}\text{ m} = 2 \times 10^{-7}$ m.

and $\lambda_0 = 2762 \times 10^{-10}\text{ m} \Rightarrow 2.762 \times 10^{-7}$ m

(*a*) $E_{max} = h(\nu - \nu_0)$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = 6.63 \times 10^{-34} \times 3 \times 10^8 \left(\frac{10^7}{2} - \frac{10^7}{2.762}\right)$$

$$= 6.63 \times 3 \times 10^{-19}\left(\frac{1}{2} - \frac{1}{2.762}\right)$$

$$= 6.63 \times 3 \times 10^{-19} \times 0.138$$

$$= \mathbf{2.745 \times 10^{-19}\ J}$$

(*b*) The maximum velocity is given by

$$\frac{1}{2}mv_{max}^2 = E_{max}$$

$$\therefore \quad V_{max} = \left(\frac{2 \times E_{max}}{m}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2 \times 2.745 \times 10^{-19}}{9.11 \times 10^{-31}}\right)^{\frac{1}{2}} = 10^5\left(\frac{2 \times 274.5}{9.11}\right)^{\frac{1}{2}}$$

$$= \mathbf{7.76 \times 10^5\ m/sec.}$$

1.8 COMPTON EFFECT

When a beam of monochromatic* X-rays strikes a targets, the X-rays will be dispersed in all possible directions after interacting with the atoms in the target. This phenomenon is known as

* Monochromatic means single wavelength

scattering and the angle between the directions of incident and scattered rays is called scattered angle. Scattering process differs from reflection, refraction and diffraction in that there is no fixed angular relationship between the incident and scattered rays. According to classical theory, there should be no change in wavelength in scattered rays.

However, American physicist, Arthur H. compton in 1923 showed experimentally that for each angle of incidence, two peaks appear corresponding to scattered X-ray photon with two different wavelengths. The change in wavelength was observed.

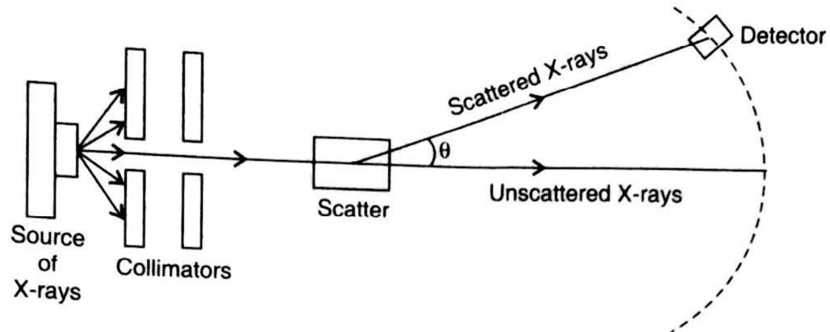

Fig. 1.4. Arrangement for the Study of compton effect.

The wavelength of one peak does not change with the angle of incidence. This is called **primary** or **unmodified component.** We denote it by λ_i. The wavelength of the other peak varies strongly with the angle of incidence and hence it is called **modified component**. It is denoted by λ_i. This effect is known as compton effect. The change in wavelength $\Delta\lambda$ is called the **Compton Shift**. Compton applied Planck-Einstein hypothesis and developed a theory which is consistent with the experimental results. The relation for $\Delta\lambda$, the compton shift is given by

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_0 c}(1 - \cos\theta) \quad ...(1.16)$$

Here, m_0 is the rest mass of the stationary electron in the target. The quantity h/m_0c has a value of 0.02426 Å.

$$\frac{h}{m_0 c} = \frac{(6.626 \times 10^{-34}\text{ Js})}{(9.1 \times 10^{-31}\text{ kg})(3 \times 10^8\text{ m/s})} = 0.02426\text{Å}$$

Example 1.6. *If the incident radiation is 1.372 Å, find the wavelength of scattered radiation at angle 30°.*

Solution : Given : Wavelength of incident, $\lambda_i = 1.372$ Å.

Angle of scattering, $\theta = 30°$

Wavelength of scattered radiation,

$$\lambda_f = \lambda_i + \frac{h}{m_0 c}(1 - \cos\theta) = 1.372 \times 10^{-10} + \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8}(1 - \cos 30)$$

$$= 1.372 \times 10^{-10} + 0.02426 \times 10^{-10} \times (1 - 0.866)$$

$$= [1.372 + (0.02426 \times 0.134)] \times 10^{-10}$$

$$= 1.375 \times 10^{-10}\text{ m} = \mathbf{1.375Å}.$$

Example 1.7. *In Compton scattering, the incident protons have wavelength 3.0×10^{-10} m. Calculate the wavelength of scattered radiation if they are viewed at an angle of 60° to the direction of incidence.*