Exercise 1 Calculate in Example 13.6 the number of radioactive nuclei that remain after 8.00 h.

Answer 1.58×10^{10} nuclei

EXAMPLE 13.7 A Radioactive Isotope of Iodine

A sample of the isotope ^{131}I, which has a half-life of 8.04 days, has an activity of 5 mCi at the time of shipment. Upon receipt of the ^{131}I in a medical laboratory, its activity is 4.2 mCi. How much time has elapsed between the two measurements?

Solution We can make use of Equation 13.10 in the form

$$\frac{R}{R_0} = e^{-\lambda t}$$

Taking the natural logarithm of each side, we get

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

$$(1) \qquad t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

To find λ, we use Equation 13.11:

$$(2) \qquad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{8.04 \text{ days}}$$

Substituting (2) into (1) gives

$$t = -\left(\frac{8.04 \text{ days}}{0.693}\right)\ln\left(\frac{4.2 \text{ mCi}}{5.0 \text{ mCi}}\right) = 2.02 \text{ days}$$

13.5 DECAY PROCESSES

As we stated in the preceding section, a radioactive nucleus spontaneously decays by means of one of three processes: alpha decay, beta decay, or gamma decay. Let us discuss these processes in more detail.

Alpha Decay

If a nucleus emits an α particle ($^4_2$He), it loses two protons and two neutrons. Therefore, the atomic number Z decreases by 2, the mass number A decreases by 4, and the neutron number decreases by 2. The decay can be written as

Alpha decay

$$^A_Z\text{X} \longrightarrow {}^{A-4}_{Z-2}\text{Y} + {}^4_2\text{He} \qquad (13.12)$$

where X is called the **parent nucleus** and Y the **daughter nucleus.** For example, ^{238}U and ^{226}Ra are both alpha emitters and decay according to the schemes

$$^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^4_2\text{He} \qquad (13.13)$$

$$^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^4_2\text{He} \qquad (13.14)$$

The half-life for the ^{238}U decay is 4.47×10^9 years, and that for ^{226}Ra decay is 1.60×10^3 years. In both cases, note that the mass number of the daughter nucleus is 4 less than that of the parent nucleus. Likewise, the atomic number is reduced by 2. The differences are accounted for in the emitted α particle (the ^{4}He nucleus). Observe that alpha decay processes release energy because

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the decay products, especially the alpha particle, are more tightly bound than the parent nucleus (see Fig. 13.10.)

Figure 13.16 depicts the spontaneous decay of ^{226}Ra. As a general rule, (1) the sum of the mass numbers A must be the same on both sides of the equation, and (2) the net charge must be the same on both sides of the equation. In addition, the total relativistic energy and momentum must be conserved. If we call M_X the atomic mass of the parent, M_Y the mass of the daughter, and M_α the mass of the alpha particle, we can define the **disintegration energy** Q:

Disintegration energy Q

$$Q = (M_X - M_Y - M_\alpha)c^2 \tag{13.15}$$

Note that atomic mass rather than nuclear mass can be used here, because the electronic masses cancel in an evaluation of the mass differences. Q is in joules when the masses are in kilograms, and c is the usual 3.00×10^8 m/s. However, when masses are expressed in the more convenient unit u, the value of Q can be calculated in MeV with the expression

$$Q = (M_X - M_Y - M_\alpha) \times 931.494 \text{ MeV/u} \tag{13.16}$$

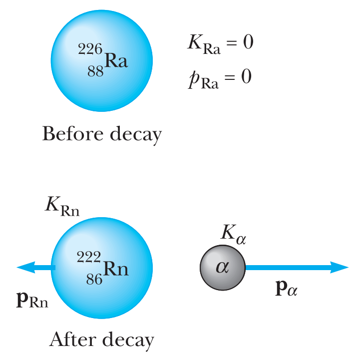

Figure 13.16 Alpha decay of radium. The radium nucleus is initially at rest. After the decay, the radon nucleus has kinetic energy K_{Rn} and momentum $\mathbf{p}_{Rn}$, and the alpha particle has kinetic energy K_α and momentum $\mathbf{p}_\alpha$.

The disintegration energy Q appears in the form of kinetic energy of the daughter nucleus and the α particle. The quantity given by Equation 13.16 is sometimes referred to as the Q value of the nuclear reaction. In the case of the ^{226}Ra decay described in Figure 13.16, if the parent nucleus is at rest when it decays, the residual kinetic energy of the products is 4.87 MeV. Most of the kinetic energy is associated with the alpha particle because this particle is much less massive than the recoiling daughter nucleus, ^{222}Rn. That is, because momentum must be conserved, the lighter α particle recoils with a much higher speed than the daughter nucleus. Generally, light particles carry off most of the energy in nuclear decays.

It is fairly easy to calculate the fraction of the disintegration energy carried off by the α particle by applying conservation of energy and momentum:

$$Q = K_Y + K_\alpha \tag{13.17}$$

$$p_Y = p_\alpha \tag{13.18}$$

where the subscript Y stands for the daughter nucleus. Since the total kinetic energy released in alpha decay (several MeV) is small compared to the rest energies of the α particle (3726 MeV) and the daughter nucleus (206.793 BeV for ^{222}Rn), we can use the classical expressions for momentum and kinetic energy in Equations 13.17 and 13.18 to show that

$$K_\alpha = \frac{M_Y}{M_Y + M_\alpha} Q \tag{13.19}$$

where M_Y and M_α are the atomic masses of the daughter nucleus and the α particle (see Problem 43 at the end of the chapter).

Interestingly, if one assumed that ^{238}U (or other alpha emitters) decayed by emitting a proton or neutron, the mass of the decay products would exceed that of the parent nucleus, corresponding to negative Q values. These negative Q values indicate that such decays do not occur spontaneously.

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EXAMPLE 13.8 The Energy Liberated When Radium Decays

The ^{226}Ra nucleus undergoes alpha decay according to Equation 13.12. Calculate the Q value for this process. Take the atomic masses to be 226.025 406 u for ^{226}Ra, 222.017 574 u for ^{222}Rn, and 4.002 603 u for $^{4}_{2}$He, as found in Appendix B.

Solution Using Equation 13.16, we see that

$$Q = (M_X - M_Y - M_\alpha) \times 931.494\,\frac{\text{MeV}}{\text{u}}$$

$$= (226.025\,406\text{ u} - 222.017\,574\text{ u} - 4.002\,603\text{ u}) \times 931.494\,\frac{\text{MeV}}{\text{u}}$$

$$= (0.005\,229\text{ u}) \times \left(931.494\,\frac{\text{MeV}}{\text{u}}\right) = 4.87\text{ MeV}$$

It is left to Problem 43 to show that the kinetic energy of the α particle is about 4.8 MeV, whereas the recoiling daughter nucleus carries off only about 0.1 MeV of kinetic energy.

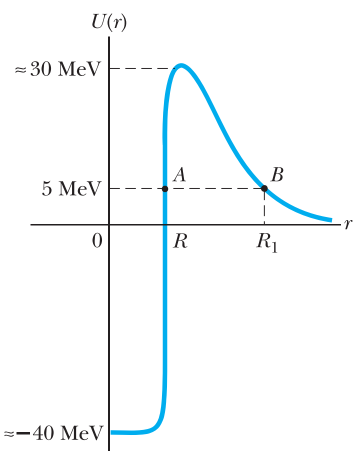

Figure 13.17 Potential energy versus separation for the alpha particle–nucleus system. Classically, the energy of the alpha particle is not great enough to overcome the barrier, and so the particle should not be able to escape the nucleus.

We now turn to the mechanism of α decay. Imagine that an α particle is somehow formed within the nucleus. Figure 13.17 is a plot of potential energy versus distance r from the nucleus for a typical alpha particle–nucleus system, where R is the range of the nuclear force. The curve represents the combined effects of (1) the Coulomb repulsive energy, which gives the positive peak for $r > R$, and (2) the nuclear attractive force, which causes the curve to be negative for $r < R$. As we saw in Example 13.8, a typical disintegration energy is about 5 MeV, which is the approximate kinetic energy of the α particle, represented by the lower dashed line in Figure 13.17. According to classical physics, the α particle is trapped in the potential well. How, then, does it ever escape from the nucleus?

The answer to this question was first provided by George Gamow in 1928 and, independently, by R. W. Gurney and E. U. Condon in 1929, using quantum mechanics. Briefly, the view of quantum mechanics is that there is always some probability that the particle can penetrate (tunnel through) the barrier (Section 7.2). Recall that the probability of locating the particle depends on its wavefunction ψ and that the probability of tunneling is measured by $|\psi|^2$. Figure 13.18 is a sketch of the wavefunction for a particle of energy E meeting a square barrier of finite height, a shape that approximates the nuclear barrier. Note that the wavefunction exists both inside and outside the barrier. Although the amplitude of the wavefunction is greatly reduced on the far side of the barrier, its finite value in this region indicates a small but finite probability that the particle can penetrate the barrier. As the energy E of the particle

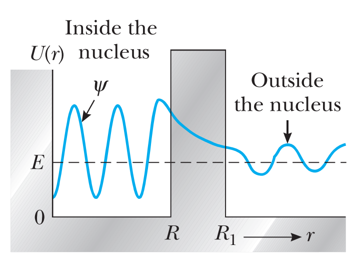

Figure 13.18 The nuclear potential energy is modeled as a square barrier. The energy of the alpha particle is E, which is less than the height of the barrier. According to quantum mechanics, the alpha particle has some chance of tunneling through the barrier, as indicated by the finite size of the wavefunction for $r > R_1$.

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increases, its probability of escaping also increases. Furthermore, the probability increases as the width of the barrier decreases.

EXAMPLE 13.9 Probability for Alpha Decay

Apply the tunneling methods of Chapter 7 to compute the probability of escape from a $^{226}_{88}$Ra nucleus of an α particle with disintegration energy 5 MeV.

Solution The escape probability is none other than the transmission coefficient $T(E)$ for the Coulomb barrier shown in Figure 13.17. Equation 7.10 of Chapter 7 gives $T(E)$ approximately as

$$T(E) \approx \exp^{-(2/\hbar)\sqrt{2m}\int\sqrt{U(r)-E}\,dr}$$

The integral is taken over the classically forbidden region where $E < U$. For alpha decay, this region is bounded below by the nuclear radius R and above by $R_1 = 2Zke^2/E$ [from $E = U(R_1) = 2Zke^2/R_1$] (see Fig. 13.17). In this expression, Z is the atomic number of the *daughter* nucleus.

The tunneling integral to be evaluated is

$$\int \sqrt{U(r) - E}\,dr = \sqrt{E}\int_R^{R_1}\sqrt{\frac{R_1}{r} - 1}\,dr$$
$$= R_1\sqrt{E}\int_{R/R_1}^{1}\sqrt{\frac{1}{z} - 1}\,dz$$

where $z = r/R_1$. An exact value for this integral can be had with some effort, but a useful approximation is found readily by noting that R/R_1 is a small number ($R \sim 10$ fm; $R_1 \sim 50$ fm for $E \sim 5$ MeV). Thus, as a first estimate we set the lower limit to zero and change variables with $z = \cos^2\theta$, obtaining

$$\int_0^1\sqrt{\frac{1}{z} - 1}\,dz = 2\int_0^{\pi/2}\sin^2\theta\,d\theta$$
$$= \int_0^{\pi/2}[1 - \cos 2\theta]\,d\theta = \frac{\pi}{2}$$

To improve upon this, we break the original integral into two and approximate the second, using $1/z \gg 1$ for z small, to get

$$\int_0^1\sqrt{\frac{1}{z} - 1}\,dz - \int_0^{R/R_1}\sqrt{\frac{1}{z} - 1}\,dz = \frac{\pi}{2}$$
$$- \int_0^{R/R_1}\frac{dz}{\sqrt{z}} \approx \frac{\pi}{2} - 2\sqrt{\frac{R}{R_1}}$$

Combining this result with $R_1 = 2Zke^2/E$ gives, for the decay probability,

$$T(E) = \exp\{-4\pi Z\sqrt{(E_0/E)} + 8\sqrt{Z(R/r_0)}\}$$

The parameter $r_0 = \hbar^2/M_\alpha ke^2$ is a kind of Bohr radius for the alpha particle, with the value 7.25 fm, and E_0 is an energy unit analogous to the Rydberg in atomic physics:

$$E_0 = \frac{ke^2}{2r_0} = \frac{14.40\text{ eV}\cdot\text{Å}}{(2)(7.25\times10^{-5}\text{ Å})} = 0.0993\text{ MeV}$$

For the alpha decay of radium, the daughter nucleus is radon with atomic number $Z = 86$ and mass number $A = 222$. Equation 13.1 predicts the radius R of the radon nucleus to be

$$R = (1.2\times10^{-5}\text{ fm})(222)^{1/3} = 7.27\text{ fm}$$

Then the decay probability for alpha disintegration at $E = 5$ MeV is

$$\exp\{-4\pi(86)\sqrt{(0.0993/5)} + 8\sqrt{86(7.27/7.25)}\}$$
$$= \exp\{-78.008\} = 1.32\times10^{-34}$$

This probability is quite small, but the actual number of disintegrations per second is much larger because of the many collisions the alpha particle makes with the nuclear barrier. This collision frequency f is the reciprocal of the transit time for the alpha particle crossing the nucleus; that is, $f = v/2R$, where v is the speed of the alpha particle inside the nucleus. Here, as in most cases, f is about 10^{21} collisions per second (see Problem 16 in Chapter 7), leading to a predicted decay rate $\approx 10^{-13}$ disintegrations per second, in reasonable agreement with the observed value of $\lambda = 1.4\times10^{-11}\text{ s}^{-1}$.

Beta Decay

When a radioactive nucleus undergoes beta decay, the daughter nucleus has the same number of nucleons as the parent nucleus, but the atomic number is changed by 1:

$$^A_Z\text{X} \longrightarrow {}^{A}_{Z+1}\text{Y} + e^- \qquad (13.20)$$

Beta decay

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$$^{A}_{Z}X \longrightarrow {}^{\;\;A}_{Z-1}Y + e^{+} \tag{13.21}$$

Again, note that the nucleon number and total charge are both conserved in these decays. However, as we shall see later, these processes are not described completely by such expressions. We shall explain this shortly.

Two typical beta decay processes are

$$^{14}_{6}C \longrightarrow {}^{14}_{7}N + e^{-}$$

$$^{12}_{7}N \longrightarrow {}^{12}_{6}C + e^{+}$$

Notice that in beta decay a neutron changes into a proton (or vice versa). It is also important to point out that the electron or positron in these decays is not present beforehand in the nucleus but is created at the moment of decay out of the rest energy of the decaying nucleus.

Now consider the energy of the system before and after the decay. As with alpha decay, we assume energy is conserved and that the heavy recoiling daughter nucleus carries off *negligible* kinetic energy. (See Problem 43.) Experimentally, it is found that beta particles from a single type of nucleus are emitted with a continuous range of kinetic energies up to some maximum value, K_{max} (Fig. 13.19). The kinetic energy of the system after the decay is equal to the decrease in mass–energy of the system—that is, the Q value. However, because all parent nuclei of a given type have the same initial mass, *the Q value must be the same for each decay.* In view of this and the fact that the daughter nucleus carries off very little kinetic energy, why do the emitted beta particles have different kinetic energies? The law of conservation of energy seems to be violated! Further analysis shows that the decay processes given by Equations 13.20 and 13.21 also violate the principles of conservation of angular momentum (spin) and linear momentum!

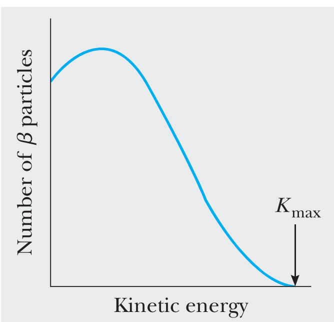

Figure 13.19 A typical beta decay curve. The maximum kinetic energy observed for the beta particles corresponds to the Q value for the reaction.

After a great deal of experimental and theoretical study, Pauli in 1930 proposed that a third particle must be present to carry away the "missing" energy and momentum. Fermi later named this particle the **neutrino** ("little neutral one") because it had to be electrically neutral and have little rest mass. Although it eluded detection for many years, the neutrino (symbolized by ν) was finally detected experimentally in 1956. It has the following properties:

Properties of the neutrino

- Zero electric charge.
- A rest mass that is much smaller than that of the electron. Recent experiments show that the mass of the neutrino is not 0 but is less than 2.8 eV/c^2.
- A spin of $\frac{1}{2}$, which satisfies the law of conservation of angular momentum when applied to beta decay.
- Very weak interaction with matter, which makes it very difficult to detect.

For the general form of the beta decays considered earlier, we can now write

$$^{A}_{Z}X \longrightarrow {}^{\;\;A}_{Z+1}Y + e^{-} + \overline{\nu} \tag{13.22}$$

$$^{A}_{Z}X \longrightarrow {}^{\;\;A}_{Z-1}Y + e^{+} + \nu \tag{13.23}$$

where the symbol $\overline{\nu}$ represents the **antineutrino,** the antiparticle to the neutrino. (We discuss antiparticles further in Chapter 15.) As in the case of alpha decay, the decays just listed are analyzed through conservation of energy and momentum, but we must use relativistic expressions because the kinetic energies of the electron and neutrino are not small compared to their rest energies.

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Enrico Fermi, an Italian-American physicist, received his doctorate from the University of Pisa in 1922, then did postdoctoral work in Germany under Max Born. He returned to Italy in 1924, and in 1926 became a professor of physics at the University of Rome. He received the Nobel Prize for Physics in 1938 for his work with the production of transuranic radioactive elements (those more massive than uranium) by neutron bombardment.

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Fermi first became interested in physics at the age of 14, after reading an old physics book in Latin. (He was an excellent scholar and could recite Dante's *Divine Comedy* and much of Aristotle's writings from memory.) His great ability to solve problems in theoretical physics and his skill for simplifying very complex situations made him somewhat of an oracle. Fermi was also a gifted experimentalist and teacher. During one of his early lecture trips to the United States, a car that he had purchased became disabled, and he pulled into a nearby gas station. After Fermi repaired the car with ease, the station owner offered him a job on the spot.

Fermi and his family emigrated to the United States, and he became a naturalized citizen in 1944. He taught first at Columbia University, then at the University of Chicago. As part of the Manhattan Project during World War II, Fermi was commissioned to design and build a structure, called an atomic pile, in which a self-sustained chain reaction might occur. The structure, built in a squash court under the stadium of the University of Chicago, contained uranium in combination with graphite blocks to slow the neutrons to thermal speeds. Cadmium rods inserted in the pile were used to absorb neutrons and control the reaction rate. History was made at 3:45 P.M. on December 2, 1942, as the cadmium rods were slowly withdrawn and a self-sustained chain reaction was observed. Fermi's earth-shaking achievement—the world's first nuclear reactor—marked the beginning of the atomic age.

Fermi died of cancer in 1954 at the age of 53. One year later, the 100th element was discovered and named *fermium* in his honor.

(Fermi National Accelerator Laboratory)

Electron capture

A process that competes with e^+ decay is called **electron capture.** This occurs when a parent nucleus captures one of its own orbital atomic electrons and emits a neutrino. The final product after decay is a nucleus whose charge is $Z - 1$:

$$^{A}_{Z}\mathrm{X} + e^- \longrightarrow {}^{A}_{Z-1}\mathrm{X} + \nu \tag{13.24}$$

In most cases it is an inner K-shell electron that is captured, and this is referred to as **K capture.** One example of this process is the capture of an electron by $^{7}_{4}\mathrm{Be}$ to become $^{7}_{3}\mathrm{Li}$:

$$^{7}_{4}\mathrm{Be} + e^- \longrightarrow {}^{7}_{3}\mathrm{Li} + \nu$$

Finally, it is instructive to mention the Q values for beta-decay processes. The Q values for e^- decay and electron capture are given by $Q = (M_X - M_Y)c^2$, while the Q values for e^+ decay are given by $Q = (M_X - M_Y - 2m_e)c^2$ where M_X and M_Y are the masses of neutral atoms. These relationships are useful for determining whether or not possible beta-decay processes are energetically allowed.

Carbon Dating

The beta decay of $^{14}\mathrm{C}$ is commonly used to date organic samples. Cosmic rays in the upper atmosphere cause nuclear reactions that create $^{14}\mathrm{C}$. In

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