To understand the observed characteristics of nuclear ground states, such as angular momentum and the high stability of magic-number nuclei, it is necessary to include *nuclear spin-orbit effects*. Unlike the spin-orbit interaction for an electron in an atom, which is magnetic in origin, the spin-orbit effect for nucleons in a nucleus is due to the nuclear force. It is much stronger than in the atomic case and has opposite sign, so that in nuclei, spin-orbit split states with higher angular momentum lie lower in energy. When the nuclear spin-orbit effect is added to a spherical finite potential, the magic numbers are predicted, because the spin-orbit potential produces especially large jumps between certain energy levels. In effect, the spin-orbit interaction substantially raises the energy levels containing 9, 21, 29, 51, 83, and 127 nucleons, thereby making the levels that contain 8, 20, 28, 50, 82, and 126 nucleons unusually stable.

Finally, it is possible to understand how individual nucleons can be considered to be moving in well-defined orbitals within the cramped confines of the nucleus, which is literally swarming with other nucleons. Under these circumstances, it would seem that a given nucleon would undergo many collisions and not move in a well-defined orbital. However, for the low-energy nuclear ground state, the exclusion principle inhibits energy-changing collisions by preventing colliding nucleons from occupying already filled low-lying energy states. In effect, the exclusion principle prevents nucleon collisions within the densely packed nucleus at low energy and justifies the shell-model approach.

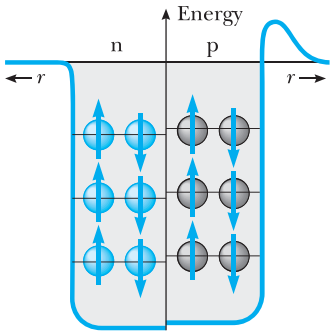

Figure 13.13 A square-well potential containing 12 nucleons. The gray circles represent protons, and the colored circles represent neutrons. The energy levels for the protons are slightly higher than those for the neutrons because of the Coulomb potential in the case of the protons. The difference in the levels increases as Z increases. Note that only two nucleons with opposite spin can occupy a given level, as required by the Pauli exclusion principle.

Collective Model

A third model of nuclear structure, known as the **collective model,** combines some features of the liquid-drop model and the independent-particle model. The nucleus is considered to have some "extra" nucleons moving in quantized orbits in addition to the filled core of nucleons. The extra nucleons are subject to the field produced by the core, as in the independent-particle scheme. Deformations can be set up in the core as a result of a strong interaction between the core and the extra nucleons, thereby initiating vibrational and rotational motions, as in the liquid-drop model. The collective model has been very successful in explaining many nuclear phenomena.

13.4 RADIOACTIVITY

In 1896 Henri Becquerel (1852–1908, French physicist) accidentally discovered that uranyl potassium sulfate crystals emitted an invisible radiation that could darken a photographic plate when the plate was covered to exclude light. After a series of experiments, he concluded that the radiation emitted by the crystals was of a new type, one that required no external stimulation and was so penetrating that it could darken protected photographic plates and ionize gases. This process of spontaneous emission of radiation by uranium was soon to be called **radioactivity.** Subsequent experiments by other scientists showed that other substances were even more powerfully radioactive. Marie (1867–1934) and Pierre Curie (1859–1906) conducted the most significant investigations of this type. After several years of careful and laborious chemical separation processes on tons of pitchblende, a radioactive ore, the Curies reported the discovery of two previously unknown elements, both radioactive, which they named polonium and radium. Subsequent experiments, including Rutherford's famous

B I O G R A P H Y

MARIE SKLODOWSKA CURIE
(1867–1934)

Marie Sklodowska Curie was born in Poland shortly after the unsuccessful Polish revolt against Russia in 1863. After high school, she worked diligently to help meet the educational expenses of her older brother and sister in Paris. At the same time, she managed to save enough money for her own move to Paris, where she entered the Sorbonne in 1891. Although she lived very frugally during this period (she once fainted from hunger in the classroom), she graduated at the top of her class.

In 1895 Marie Sklodowska married the French physicist Pierre Curie (1859–1906), who was already known for the discovery of piezoelectricity. (A piezoelectric crystal exhibits a potential difference under pressure.) Using piezoelectric materials to measure the activity of radioactive substances, Marie Curie demonstrated the radioactive nature of the elements uranium and thorium. In 1898 she and her husband discovered a new radioactive element contained in uranium ore, which they called polonium after her native land. By the end of 1898, the Curies succeeded in isolating trace amounts of an even more radioactive element, which they named radium. In an effort to produce weighable quantities of radium, they took on the painstaking job of isolating radium from pitchblende, an ore rich in uranium. After four years of purifying and repurifying tons of ore, and using their own life savings to finance their work, the Curies succeeded in preparing about 0.1 g of radium. In 1903, along with Henri Becquerel, they received the Nobel Prize in Physics for their studies of radioactive substances.

After her husband's death in an accident in 1906, Marie Curie assumed his professorship at the Sorbonne. Unfortunately, she experienced prejudice in the scientific community because she was a woman. For example, after being nominated to the French Academy of Sciences, she was refused membership after losing by one vote.

In 1911 Marie Curie was awarded a second Nobel prize, this one in chemistry, for the discovery of radium and polonium. She spent the last few decades of her life supervising the Paris Institute of Radium.

(Courtesy of AIP Niels Bohr Library / W. F. Meggers Collection)

work on alpha-particle scattering, suggested that radioactivity was the result of the decay, or disintegration, of unstable nuclei.

Three types of radiation can be emitted by a radioactive substance: alpha (α), in which the emitted particles are ^{4}He nuclei; beta (β), in which the emitted particles are either electrons or positrons; and gamma (γ), in which the emitted "rays" are high-energy photons. A **positron** is a particle like the electron in all respects except that the positron has a charge of $+e$. In this book, the symbol e^- is used to designate an electron, and e^+ designates a positron.

It is possible to distinguish these three forms of radiation using the scheme shown in Figure 13.14. The radiation from a radioactive sample is directed into a region in which there is a magnetic field. The beam splits into three components, two bending in opposite directions and the third experiencing no change in direction. From this simple observation, we can conclude that the radiation of the undeflected beam carries no charge (the gamma ray), the component deflected upward consists of positively charged particles (α particles), and the component deflected downward consists of negatively charged particles (e^-). If the beam includes a positron (e^+), it is deflected upward.

The three types of radiation have quite different penetrating powers. Alpha particles barely penetrate a sheet of paper, beta particles can penetrate a few

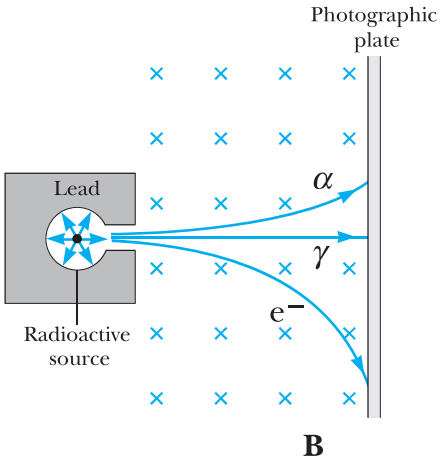

Figure 13.14 The radiation from a radioactive source can be separated into three components through the use of a magnetic field to deflect the charged particles. The photographic plate at the right records the events. The gamma ray is not deflected by the magnetic field.

millimeters of aluminum, and gamma rays can penetrate several centimeters of lead.

The rate at which a particular decay process occurs in a radioactive sample is proportional to the number of radioactive nuclei present (that is, those nuclei that have not yet decayed). If N is the number of radioactive nuclei present at some instant, the rate of change of N is

$$\frac{dN}{dt} = -\lambda N \tag{13.8}$$

where λ, called the **decay constant,** is the probability per unit time that a nucleus will decay. The minus sign indicates that dN/dt is negative because λ and N are both positive; that is, N is *decreasing* in time.

If we write Equation 13.8 in the form

$$\frac{dN}{N} = -\lambda dt$$

we can integrate the expression to give

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

or

$$N = N_0 e^{-\lambda t} \tag{13.9}$$

Exponential decay

where the constant N_0 represents the number of radioactive nuclei at $t = 0$. Equation 13.9 shows that **the number of radioactive nuclei in a sample decreases exponentially with time.**

The **decay rate** R, or the number of decays per unit time, can be obtained by differentiating Equation 13.9 with respect to time:

Decay rate

$$R = \left|\frac{dN}{dt}\right| = N_0\lambda e^{-\lambda t} = R_0 e^{-\lambda t} \tag{13.10}$$

where $R_0 = N_0\lambda$ is the decay rate at $t = 0$ and $R = \lambda N$. The decay rate of a sample is often referred to as its **activity.** Note that both N and R decrease exponentially with time. The plot of N versus t shown in Figure 13.15 illustrates the exponential decay law.

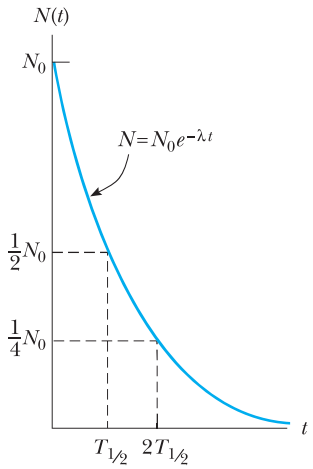

Figure 13.15 A plot of the exponential decay law for radioactive nuclei. The vertical axis represents the number of radioactive nuclei present at any time t, and the horizontal axis is time. The time $T_{1/2}$ is the half-life of the sample.

Another parameter that is useful in characterizing the decay of a particular nucleus is the **half-life,** $T_{1/2}$:

> The **half-life** of a radioactive substance is the time it takes half of a given number of radioactive nuclei to decay.

Setting $N = N_0/2$ and $t = T_{1/2}$ in Equation 13.9 gives

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

Writing this in the form $e^{\lambda T_{1/2}} = 2$ and taking the natural logarithm of both sides, we get

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{13.11}$$

This is a convenient expression for relating half-life to the decay constant. Note that after an elapsed time of one half-life, $N_0/2$ radioactive nuclei remain (by definition); after two half-lives, half of these have decayed and $N_0/4$ radioactive nuclei remain; after three half-lives, $N_0/8$ remain; and so on. In general, after n half-lives, the number of radioactive nuclei remaining is $N_0/2^n$. Thus we see that nuclear decay is independent of the past history of a sample.

A frequently used unit of activity is the **curie** (Ci), defined as

The curie

$$1 \text{ Ci} \equiv 3.7 \times 10^{10} \text{ decays/s}$$

This value was originally selected because it is the approximate activity of 1 g of radium. The SI unit of activity is the **becquerel** (Bq):

The becquerel

$$1 \text{ Bq} \equiv 1 \text{ decay/s}$$

Therefore, $1 \text{ Ci} = 3.7 \times 10^{10}$ Bq. The curie is a rather large unit, and the more frequently used activity units are the millicurie mCi (10^{-3} Ci) and the microcurie, μCi (10^{-6} Ci).

EXAMPLE 13.4 How Many Nuclei Are Left?

The isotope carbon-14, $^{14}_{6}\text{C}$, is radioactive and has a half-life of 5730 years (yr). If you start with a sample of 1000 carbon-14 nuclei, how many will still be around in 22,920 yr?

Solution In 5730 yr half the sample will have decayed, leaving 500 carbon-14 nuclei. In another 5730 yr (for a total elapsed time of 11,460 yr), the number will be reduced to 250 nuclei. After another 5730 yr (total time

17,190 yr), 125 will remain. Finally, after four half-lives (22,920 yr), only about 62 will remain.

These numbers represent ideal circumstances. Calculation of radioactive decay is an averaging process conducted with a very large number of atoms, and the actual outcome depends on statistics. Our original sample in this example contained only 1000 nuclei, certainly not a very large number. Thus, if we actually counted the number remaining from this small sample after one half-life, it probably would not be 500.

EXAMPLE 13.5 The Activity of Radium

The half-life of the radioactive nucleus $^{226}_{88}\text{Ra}$ is about 1.6×10^3 yr. (a) What is the decay constant of $^{226}_{88}\text{Ra}$?

Solution (a) We can calculate the decay constant λ by using Equation 13.11 and the fact that

$$\begin{aligned} T_{1/2} &= 1.6 \times 10^3 \text{ yr} \\ &= (1.6 \times 10^3 \text{ yr})(3.16 \times 10^7 \text{ s/yr}) \\ &= 5.0 \times 10^{10} \text{ s} \end{aligned}$$

Therefore,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5.0 \times 10^{10} \text{ s}} = 1.4 \times 10^{-11} \text{ s}^{-1}$$

Note that this result gives the probability that any single $^{226}_{88}\text{Ra}$ nucleus will decay in 1 s.

(b) If a sample contains 3.0×10^{16} such nuclei at $t = 0$, determine its activity at this time.

Solution (b) We can calculate the activity of the sample at $t = 0$ using $R_0 = \lambda N_0$, where R_0 is the decay rate at $t = 0$ and N_0 is the number of radioactive nuclei present at $t = 0$. Since $N_0 = 3.0 \times 10^{16}$, we have

$$\begin{aligned} R_0 = \lambda N_0 &= (1.4 \times 10^{-11} \text{ s}^{-1})(3.0 \times 10^{16}) \\ &= 4.2 \times 10^5 \text{ decays/s} \end{aligned}$$

Since $1 \text{ Ci} = 3.7 \times 10^{10}$ decays/s, the activity, or decay rate, at $t = 0$ is

$$R_0 = 11.1 \ \mu\text{Ci}$$

(c) What is the decay rate after the sample is 2.0×10^3 yr old?

Solution (c) We can use Equation 13.10 and the fact that $2.0 \times 10^3 \text{ yr} = (2.0 \times 10^3 \text{ yr})(3.15 \times 10^7 \text{ s/yr}) = 6.3 \times 10^{10}$ s:

$$\begin{aligned} R &= R_0 e^{-\lambda t} \\ &= (4.2 \times 10^5 \text{ decays/s})\, e^{-(1.4 \times 10^{-11} \text{ s}^{-1})(6.3 \times 10^{10} \text{ s})} \\ &= 1.7 \times 10^5 \text{ decays/s} \end{aligned}$$

EXAMPLE 13.6 The Activity of Carbon

A radioactive sample contains 3.50 μg of pure $^{11}_{6}\text{C}$, which has a half-life of 20.4 min. (a) Determine the number of nuclei in the sample at $t = 0$.

Solution The atomic mass of $^{11}_{6}\text{C}$ is approximately 11.0, and so 11.0 g contains Avogadro's number (6.02×10^{23}) of nuclei. Therefore 3.50 μg contains N nuclei, where

$$\frac{N}{6.02 \times 10^{23} \text{ nuclei/mol}} = \frac{3.50 \times 10^{-6} \text{ g}}{11.0 \text{ g/mol}}$$

$$N = 1.92 \times 10^{17} \text{ nuclei}$$

(b) What is the activity of the sample initially and after 8.00 h?

Solution Since $T_{1/2} = 20.4 \text{ min} = 1224 \text{ s}$, the decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1224 \text{ s}} = 5.66 \times 10^{-4} \text{ s}^{-1}$$

Therefore, the initial activity or decay rate of the sample is

$$\begin{aligned} R_0 = \lambda N_0 &= (5.66 \times 10^{-4} \text{ s}^{-1})(1.92 \times 10^{17}) \\ &= 1.08 \times 10^{14} \text{ decays/s} \end{aligned}$$

We can use Equation 13.10 to find the activity at any time t. For $t = 8.00 \text{ h} = 2.88 \times 10^4$ s, we see that $\lambda t = 16.3$, and so

$$\begin{aligned} R = R_0 e^{-\lambda t} &= (1.09 \times 10^{14} \text{ decays/s})\, e^{-16.3} \\ &= 8.96 \times 10^6 \text{ decays/s} \end{aligned}$$

Table 13.3 lists values for activity versus the time in hours for this situation.

Table 13.3 Activity Versus Time for the Sample Described in Example 13.6

t (h)	R (decays/s)
0	1.08×10^{14}
1	1.41×10^{13}
2	1.84×10^{12}
3	2.39×10^{11}
4	3.12×10^{10}
5	4.06×10^{9}
6	5.28×10^{8}
7	6.88×10^{7}
8	8.96×10^{6}