12

Managing Uncertainty in a Supply Chain: Safety Inventory

PowerPoint presentation to accompany
Chopra and Meindl Supply Chain Management, 5e

Learning Objectives

1. Understand the role of safety inventory in a supply chain
2. Identify factors that influence the required level of safety inventory
3. Describe different measures of product availability
4. Utilize managerial levers available to lower safety inventory and improve product availability

The Role of Safety Inventory

- *Safety inventory* is carried to satisfy demand that exceeds the amount forecasted
  - Raising the level of safety inventory increases product availability and thus the margin captured from customer purchases
  - Raising the level of safety inventory increases inventory holding costs

The Role of Safety Inventory

- Three key questions
  1. What is the appropriate level of product availability?
  2. How much safety inventory is needed for the desired level of product availability?
  3. What actions can be taken to improve product availability while reducing safety inventory?

The Role of Safety Inventory

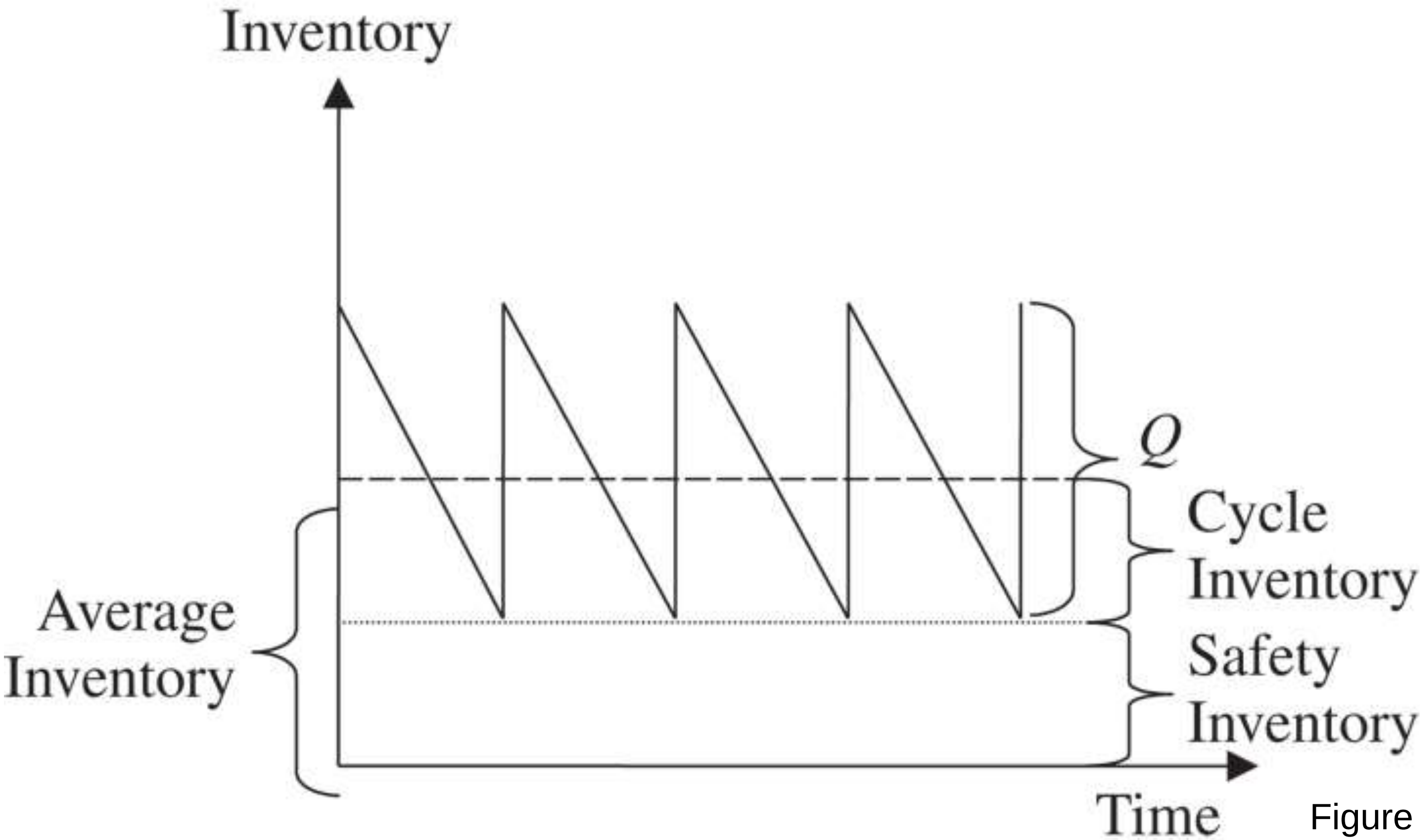


Figure 12-1

Determining the Appropriate Level

- Determined by two factors
  - The uncertainty of both demand and supply
  - The desired level of product availability
- Measuring Demand Uncertainty

  D = Average demand per period

  σ_D = Standard deviation of demand (forecast error) per period

  Lead time (L) is the gap between when an order is placed and when it is received

Evaluating Demand Distribution Over *L* Periods

$$D_L = \sum_{i=1}^{L} D_i \qquad \sigma_L = \sqrt{\sum_{i=1}^{L} \sigma_i^2 + 2\sum_{i>j} \rho_{ij}\sigma_i\sigma_j}$$

$$D_L = DL \qquad \sigma_L = \sqrt{L}\,\sigma_D$$

The *coefficient of variation*

$$cv = \sigma / \mu$$

Measuring Product Availability

1. *Product fill rate (fr)*
   - Fraction of product demand satisfied from product in inventory
2. *Order fill rate*
   - Fraction of orders filled from available inventory
3. *Cycle service level (CSL)*
   - Fraction of replenishment cycles that end with all customer demand being met

Replenishment Policies

1. *Continuous review*
   – Inventory is continuously tracked
   – Order for a lot size Q is placed when the inventory declines to the reorder point (ROP)
2. *Periodic review*
   – Inventory status is checked at regular periodic intervals
   – Order is placed to raise the inventory level to a specified threshold

Evaluating Cycle Service Level and Fill Rate

- Evaluating Safety Inventory Given a Replenishment Policy

Expected demand during lead time = DL

Safety inventory, ss = ROP – DL

Evaluating Cycle Service Level and Fill Rate

Average demand per week, D = 2,500

Standard deviation of weekly demand, σ_D = 500

Average lead time for replenishment, L = 2 weeks

Reorder point, ROP = 6,000

Average lot size, Q = 10,000

Safety inventory, $ss = ROP - DL$ = 6,000 – 5,000 = 1,000

Cycle inventory = Q/2 = 10,0002 = 5,000

Evaluating Cycle Service Level and Fill Rate

Average inventory = cycle inventory + safety inventory

= 5,000 + 1,000 = 6,000

Average flow time = average inventory/throughput

= 6,000/2,500 = 2.4 weeks

Evaluating Cycle Service Level and Fill Rate

- Evaluating Cycle Service Level Given a Replenishment Policy

$$CSL = Prob(\text{ddlt of } L \text{ weeks} \leq ROP)$$

$$CSL = F(ROP, D_L, \sigma_L) = NORMDIST(ROP, D_L, \sigma_L, 1)$$

(ddlt = demand during lead time)

Evaluating Cycle Service Level and Fill Rate

Q = 10,000, ROP = 6,000, L = 2 weeks

D = 2,500/week, σ_D = 500

$$D_L = DL = 2 \times 2{,}500$$

$$\sigma_L = \sqrt{L}\sigma_D = \sqrt{2} \times 500 = 707$$

$$CSL = F(ROP, D_L, \sigma_L) = NORMDIST(ROP, D_L, \sigma_L, 1)$$

$$= NORMDIST(6{,}000, 5{,}000, 707, 1) = 0.92$$

Evaluating Fill Rate Given a Replenishment Policy

- *Expected shortage per replenishment cycle* (ESC) is the average units of demand that are not satisfied from inventory in stock per replenishment cycle
- Product fill rate

$$fr = 1 – ESC/Q = (Q – ESC)/Q$$

Evaluating Fill Rate Given a Replenishment Policy

$$ESC = \int_{x=ROP}^{\infty} (x - ROP) f(x)\, dx$$

$$ESC = -ss\left[1 - F_s\left(\frac{ss}{\sigma_L}\right)\right] + \sigma_L f_s\left(\frac{ss}{\sigma_L}\right)$$

$$ESC = -ss\left[1 - NORMDIST\left(ss / \sigma_L, 0, 1, 1\right)\right] + \sigma_L NORMDIST\left(ss / \sigma_L, 0, 1, 0\right)$$

Evaluating Fill Rate Given a Replenishment Policy

Lot size, Q = 10,000

Average demand during lead time, D_L = 5,000

Standard deviation of demand during lead time, σ_L = 707

Safety inventory, $ss = ROP - DL$ = 6,000 – 5,000 = 1,000

$$ESC = -1{,}000[1 - NORMDIST(1{,}000/707, 0, 1, 1)]$$
$$+707\,NORMDIST(1{,}000/707, 0, 1, 0) = 25$$

$fr = (Q - ESC)/Q$ = 110,000 – 252/10,000 = 0.9975

Evaluating Fill Rate Given a Replenishment Policy

	A	B	C	D	E
1	*Inputs*				
2	*Q*	*D*	σ_D	*L*	*ss*
3	10,000	2,500	500	2	1,000
4	***Distribution of demand during lead time***				
5	D_L	σ_L			
6	5,000	707			
7	***Cycle Service Level and Fill Rate***				
8	*CSL*	*ESC*	*fr*		
9	0.92	25.13	0.9975		

Cell	Cell Formula	Equation
A6	=B3*D3	12.2
B6	=SQRT(D3)*C3	12.2
A9	=NORMDIST(A6+E3, A6, B6, 1)	12.4
B9	=-E3*(1-NORMDIST(E3/B6, 0, 1, 1)) + B6*NORMDIST(E3/B6, 0, 1, 0)	12.8
C9	=(A3-B9)/A3	12.5

Figure 12-2

Evaluating Safety Inventory Given Desired Cycle Service Level

Desired cycle service level = CSL

Mean demand during lead time = D_L

Standard deviation of demand during lead time = σ_L

Probability(demand during lead time ≤ $D_L + ss$) = CSL

- Identify safety inventory so that

$$F(D_L + ss, D_L, s_L) = CSL$$

Evaluating Safety Inventory Given Desired Cycle Service Level

$$D_L + ss = F^{-1}(CSL, D_L, \sigma_L) = NORMINV(CSL, D_L, \sigma_L)$$

or

$$ss = F^{-1}(CSL, D_L, \sigma_L) - D_L = NORMINV(CSL, D_L, \sigma_L) - D_L$$

$$ss = F_S^{-1}(CSL) \times \sigma_L = F_S^{-1}(CSL) \times \sqrt{L}\sigma_D$$

$$= NORMSINV(CSL) \times \sqrt{L}\sigma_D$$

Evaluating Safety Inventory Given Desired Cycle Service Level

Q = 10,000, CSL = 0.9, L = 2 weeks

D = 2,500/week, σ_D = 500

$$D_L = DL = 2 \times 2{,}500 = 5{,}000$$

$$\sigma_L = \sqrt{L}\,\sigma_D = \sqrt{2} \times 500 = 707$$

$$ss = F_s^{-1}(CSL) \times \sigma_L = NORMSINV(CSL) \times \sigma_L$$

$$= NORMSINV(0.90) \times 707 = 906$$

Evaluating Safety Inventory Given Desired Fill Rate

- Expected shortage per replenishment cycle is

$$ESC = (1 - fr)Q$$

- No equation for *ss*
- Try values or use *GOALSEEK* in Excel

Evaluating Safety Inventory Given Desired Fill Rate

Desired fill rate, fr = 0.975

Lot size, Q = 10,000 boxes

Standard deviation of ddlt, σ_L = 707

$$ESC = (1 - fr)Q = (1 - 0.975)10{,}000 = 250$$

Evaluating Safety Inventory Given Desired Fill Rate

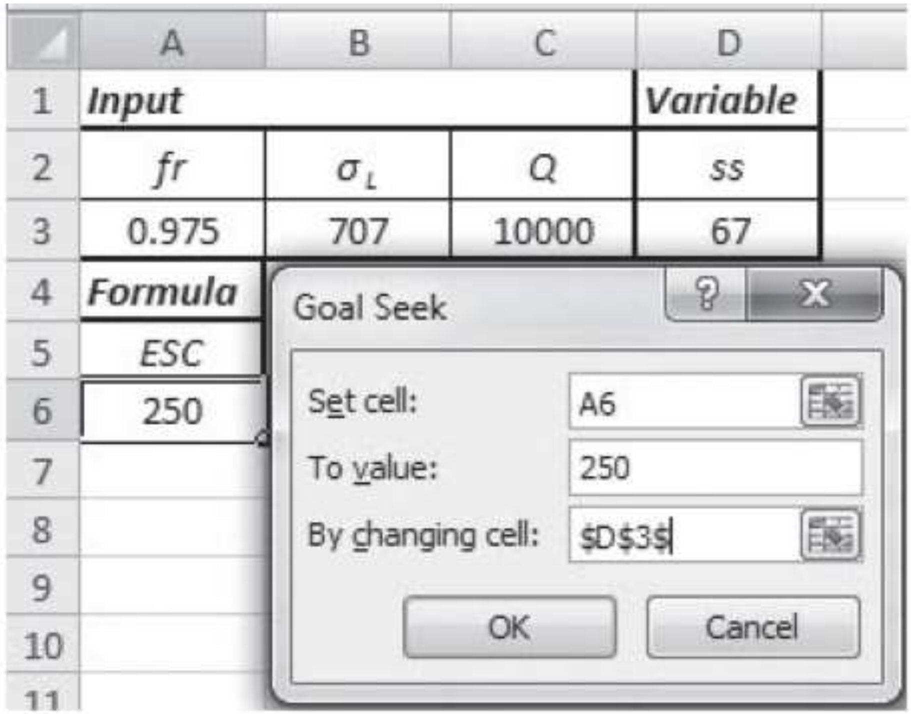
$$ESC = 250 = -ss\left[1 - F_s\left(\frac{ss}{\sigma_L}\right)\right] + \sigma_L f_s\left(\frac{ss}{\sigma_L}\right)$$

$$= -ss\left[1 - F_s\left(\frac{ss}{707}\right)\right] + 707 f_s\left(\frac{ss}{707}\right)$$

$$250 = -ss[1 - NORMDIST(ss/707,0,1,1)]$$
$$+707\,NORMDIST(ss/707,0,1,0)$$

- Use *GOALSEEK* to find safety inventory ss = 67 boxes

Evaluating Safety Inventory Given Desired Fill Rate



Cell	Cell Formula	Equation
A6	-D3*(1-NORMSDIST(D3/B3, 0, 1,1)) + B3*NORMDIST(D3/B3, 0, 1, 0)	12.10

Figure 12-3

Impact of Desired Product Availability and Uncertainty

- As desired product availability goes up the required safety inventory increases

Fill Rate	Safety Inventory
97.5%	67
98.0%	183
98.5%	321
99.0%	499
99.5%	767

Table 12-1

Impact of Desired Product Availability and Uncertainty

- Goal is to reduce the level of safety inventory required in a way that does not adversely affect product availability
  - Reduce the supplier lead time L
  - Reduce the underlying uncertainty of demand (represented by σ_D)

Benefits of Reducing Lead Time

D = 2,500/week, σ_D = 800, CSL = 0.95

$$ss = NORMSINV(CSL) \times \sqrt{L}\,\sigma_D$$

$$= NORMSINV(.95) \times \sqrt{9} \times 800 = 3{,}948$$

- If lead time is reduced to one week

$$ss = NORMSINV(.95) \times \sqrt{1} \times 800 = 1{,}316$$

- If standard deviation is reduced to 400

$$ss = NORMSINV(.95) \times \sqrt{9} \times 400 = 1{,}974$$

Impact of Supply Uncertainty on Safety Inventory

- We incorporate supply uncertainty by assuming that lead time is uncertain

D: Average demand per period

σ_D: Standard deviation of demand per period

L: Average lead time for replenishment

σ_L: Standard deviation of lead time

$$D_L = DL \qquad \sigma_L = \sqrt{L\sigma_D^2 + D^2\sigma_L^2}$$

Impact of Lead Time Uncertainty on Safety Inventory

Average demand per period, D = 2,500

Standard deviation of demand per period, σ_D = 500

Average lead time for replenishment, L = 7 days

Standard deviation of lead time, σ_L = 7 days

Mean ddlt, $D_L = DL$ = 2,500 x 7 = 17,500

Standard deviation of ddlt $s_L = \sqrt{L s_D^2 + D^2 s_L^2}$

$$= \sqrt{7 ´\ 500^2 + 2{,}500^2 ´\ 7^2}$$

$$= 17{,}500$$

Impact of Lead Time Uncertainty on Safety Inventory

- Required safety inventory

$$ss = F_S^{-1}(CSL) \times \sigma_L = NORMSINV(CSL) \times \sigma_L$$
$$= NORMSINV(0.90) \times 17{,}500$$
$$= 22{,}491 \text{ hard drives}$$

Table 12-2

s_L	σ_L	*ss* (units)	*ss* (days)
6	15,058	19,298	7.72
5	12,570	16,109	6.44
4	10,087	12,927	5.17
3	7,616	9,760	3.90
2	5,172	6,628	2.65
1	2,828	3,625	1.45
0	1,323	1,695	0.68

Impact of Aggregation on Safety Inventory

- How does aggregation affect forecast accuracy and safety inventories

D_i: Mean weekly demand in region i, $i = 1,\ldots, k$

σ_i: Standard deviation of weekly demand in region i, $i = 1,\ldots, k$

ρ_{ij}: Correlation of weekly demand for regions i, j, $1 \leq i \neq j \leq k$

Impact of Aggregation on Safety Inventory

Total safety inventory in decentralized option $= \sum_{i=1}^{k} F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_i$

$$D^C = \sum_{i=1}^{k} D_i; \qquad \text{var}\left(D^C\right) = \sum_{i=1}^{k} \sigma_i^2 + 2\sum_{i>j} \rho_{ij}\sigma_i\sigma_j;$$

$$\sigma_D^C = \sqrt{\text{var}\left(D^C\right)}$$

$$D^C = kD \qquad \sigma_D^C = \sqrt{k}\,\sigma_D$$

Impact of Aggregation on Safety Inventory

Require safety inventory on aggregation $= \sum_{i=1}^{k} F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D^C$

Holding-cost savings on aggregation per unit sold

$$= \frac{F_S^{-1}(CSL) \times \sqrt{L} \times H}{D^C} \times \left(\sum_{i=1}^{k} \sigma_i - \sigma_D^C \right)$$

Impact of Aggregation on Safety Inventory

- The safety inventory savings on aggregation increase with the desired cycle service level CSL
- The safety inventory savings on aggregation increase with the replenishment lead time L
- The safety inventory savings on aggregation increase with the holding cost H
- The safety inventory savings on aggregation increase with the coefficient of variation of demand
- The safety inventory savings on aggregation decrease as the correlation coefficients increase

Impact of Aggregation on Safety Inventory

- The Square-Root Law

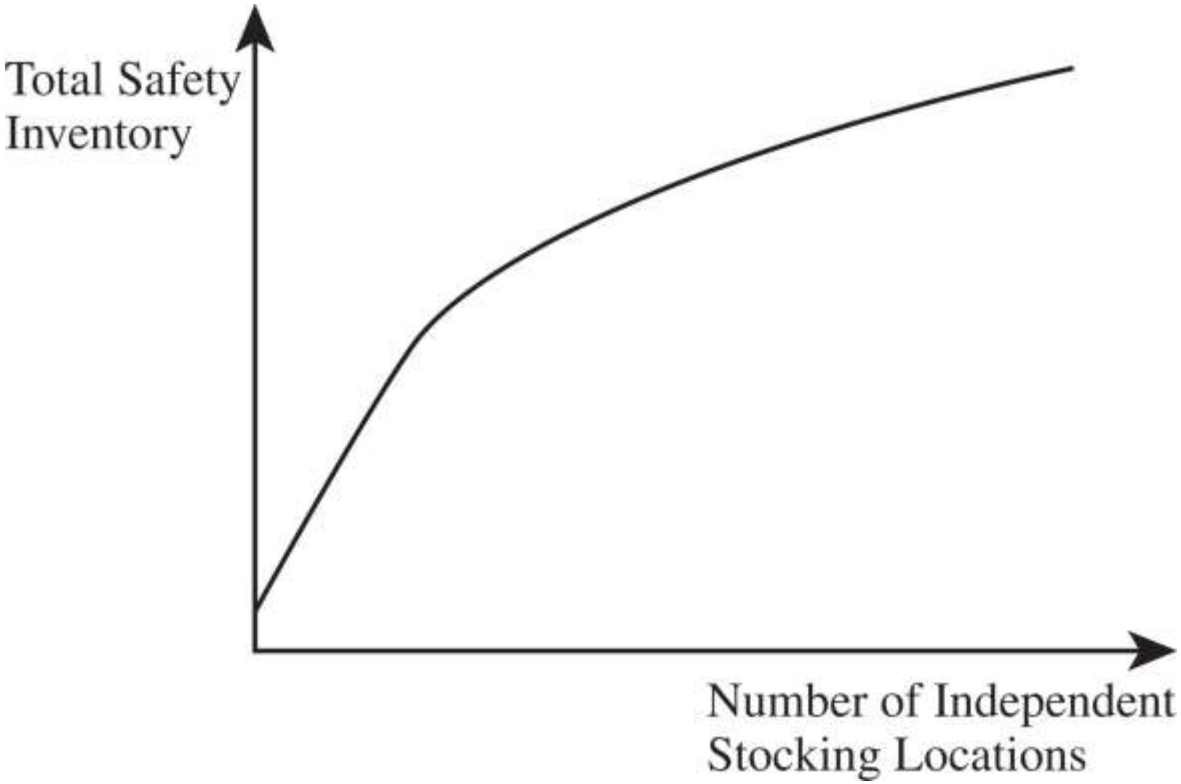


Figure 12-4

Impact of Correlation on Value of Aggregation

Standard deviation of weekly demand, $\sigma_D = 5$;

Replenishment, $L = 2$ weeks; Decentralized $CSL = 0.9$

Total required safety inventory, $ss = k \times F_s^{-1}(CSL) \times \sqrt{L} \times \sigma_D$

$$= 4 \times F_s^{-1}(0.9) \times \sqrt{2} \times 5$$

$$= 4 \times NORMSINV(0.9) \times \sqrt{2} \times 5 = 36.24 \text{ cars}$$

Aggregate $\rho = 0$

Standard deviation of weekly demand at central outlet, $\sigma_D^C = \sqrt{4} \times 5 = 10$

$$ss = F_s^{-1}(0.9) \times \sqrt{L} \times \sigma_D^C = NORMSINV(0.9) \times \sqrt{2} \times 10 = 18.12$$

Impact of Correlation on Value of Aggregation

ρ	Disaggregate Safety Inventory	Aggregate Safety Inventory
0	36.24	18.12
0.2	36.24	22.92
0.4	36.24	26.88
0.6	36.24	30.32
0.8	36.24	33.41
1.0	36.24	36.24

Table 12-3

Impact of Correlation on Value of Aggregation

- Two possible disadvantages to aggregation
  1. Increase in response time to customer order
  2. Increase in transportation cost to customer

Trade-offs of Physical Centralization

- Use four regional or one national distribution center

D = 1,000/week, σ_D = 300, L = 4 weeks, CSL = 0.95

- Four regional centers

Total required safety inventory, $ss = 4 \times F_s^{-1}(CSL) \times \sqrt{L} \times \sigma_D$

$$= 4 \times NORMSINV(0.95) \times \sqrt{4} \times 300 = 3{,}948$$

Trade-offs of Physical Centralization

- One national distribution center, $\rho = 0$

Standard deviation of weekly demand, $\sigma_D^C = \sqrt{4} \times 300 = 600$

$$ss = F_s^{-1}(0.95) \times \sqrt{L} \times \sigma_D^C$$
$$= NORMSINV(0.95) \times \sqrt{4} \times 600 = 1{,}974$$

Decrease in holding costs = (3,948 – 1,974) $1,000 x 0.2
= $394,765
Decrease in facility costs = $150,000
Increase in transportation = 52 x 1,000 x (13 – 10)
= $624,000

Information Centralization

- Online systems that allow customers or stores to locate stock
- Improves product availability without adding to inventories
- Reduces the amount of safety inventory

Specialization

- Inventory is carried at multiple locations
- Should all products should be stocked at every location?
  - Required level of safety inventory
  - Affected by coefficient of variation of demand
  - Low demand, *slow-moving items*, typically have a high coefficient of variation
  - High demand, *fast-moving items*, typically have a low coefficient of variation

Impact of Coefficient of Variation on Value of Aggregation

Table 12-4

	Motors	Cleaner
Inventory is stocked in each store		
Mean weekly demand per store	20	1,000
Standard deviation	40	100
Coefficient of variation	2.0	0.1
Safety inventory per store	132	329
Total safety inventory	211,200	526,400
Value of safety inventory	$105,600,000	$15,792,000
Inventory is aggregated at the DC		
Mean weekly aggregate demand	32,000	1,600,000
Standard deviation of aggregate demand	1,600	4,000
Coefficient of variation	0.05	0.0025
Aggregate safety inventory	5,264	13,159
Value of safety inventory	$2,632,000	$394,770
Savings		
Total inventory saving on aggregation	$102,968,000	$15,397,230
Total holding cost saving on aggregation	$25,742,000	$3,849,308
Holding cost saving per unit sold	$15.47	$0.046
Savings as a percentage of product cost	3.09%	0.15%

Product Substitution

- The use of one product to satisfy demand for a different product
  1. *Manufacturer-driven substitution*
     - Allows aggregation of demand
     - Reduce safety inventories
     - Influenced by the cost differential, correlation of demand
  2. *Customer-driven substitution*
     - Allows aggregation of safety inventory

Component Commonality

- Without common components
  - Uncertainty of demand for a component is the same as for the finished product
  - Results in high levels of safety inventor
- With common components
  - Demand for a component is an aggregation of the demand for the finished products
  - Component demand is more predictable
  - Component inventories are reduced

Value of Component Commonality

27 PCs, 3 components, 3 x 27 = 81 distinct components

Monthly demand = 5,000

Standard deviation = 3,000

Replenishment lead time = 1 month

CSL = 0.95

$$\begin{aligned}\text{Total safety inventory required} &= 81 \times NORMSINV(0.95) \times \sqrt{1} \times 3{,}000 \\ &= 399{,}699 \text{ units}\end{aligned}$$

$$\begin{aligned}\text{Safety inventory per common component} &= NORMSINV(0.95) \times \sqrt{1} \times \sqrt{9} \times 3{,}000 \\ &= 14{,}804 \text{ units}\end{aligned}$$

Value of Component Commonality

- With component commonality
- Nine distinct components

$$\text{Total safety inventory required} = 9 \times 14{,}804 = 133{,}236$$

Value of Component Commonality

Number of Finished Products per Component	Safety Inventory	Marginal Reduction in Safety Inventory	Total Reduction in Safety Inventory
1	399,699		
2	282,630	117,069	117,069
3	230,766	51,864	168,933
4	199,849	30,917	199,850
5	178,751	21,098	220,948
6	163,176	15,575	236,523
7	151,072	12,104	248,627
8	141,315	9,757	258,384
9	133,233	8,082	266,466

Table 12-5

Postponement

- Delay product differentiation or customization until closer to the time the product is sold
  - Have common components in the supply chain for most of the push phase
  - Move product differentiation as close to the pull phase of the supply chain as possible
  - Inventories in the supply chain are mostly aggregate

Postponement

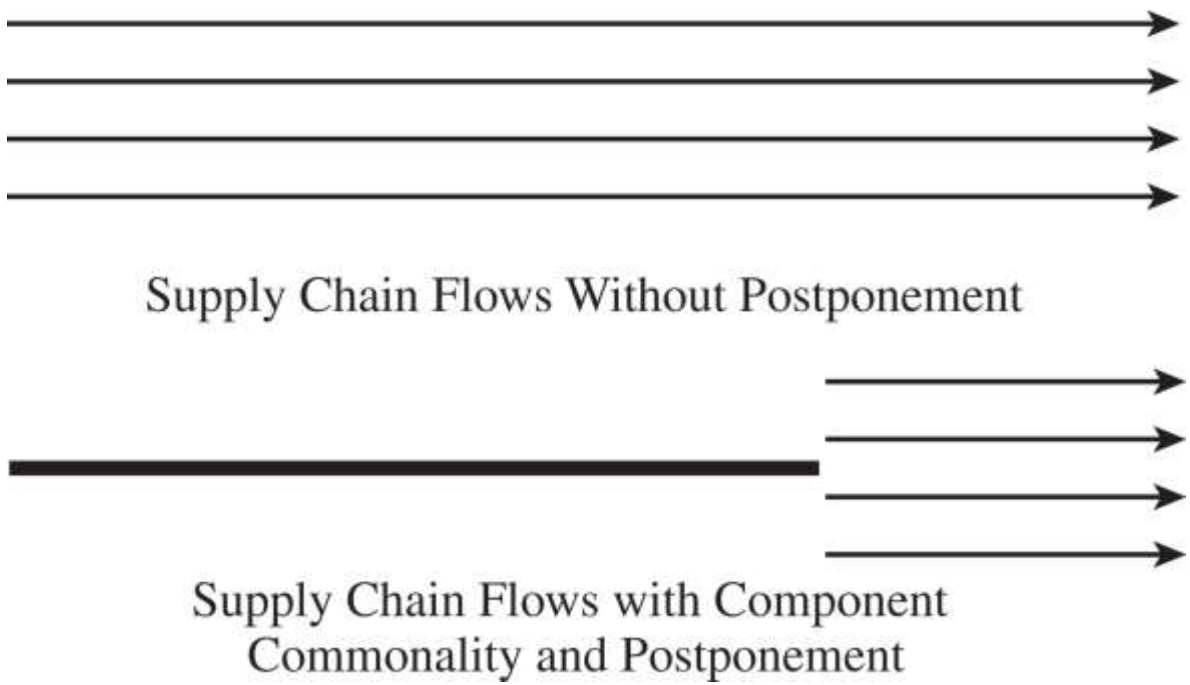


Figure 12-5

Value of Postponement

100 different paint colors, D = 30/week, σ_D = 10, L = 2 weeks, CSL = 0.95

Total required safety inventory, $ss = 100 \times F_s^{-1}(CSL) \times \sqrt{L} \times \sigma_D$

$$= 100 \times NORMSINV(0.95) \times \sqrt{2} \times 10 = 2{,}326$$

Standard deviation of base paint weekly demand, $\sigma_D^C = \sqrt{100} \times 10 = 100$

$$ss = F_s^{-1}(CSL) \times \sqrt{L} \times \sigma_D^C = NORMSINV(0.95) \times \sqrt{2} \times 100 = 233$$

Impact of Replenishment Policies on Safety Inventory

- Continuous Review Policies

D: Average demand per period

σ_D: Standard deviation of demand per period

L: Average lead time for replenishment

Mean demand during lead time, $D_L = DL$

Standard deviation of demand during lead time, $s_L = \sqrt{L} s_D$

$$ss = F_S^{-1}(CSL) ´ s_L = NORMSINV(CSL) ´ \sqrt{L} s_D, ROP = D_L + ss$$

Impact of Replenishment Policies on Safety Inventory

- Periodic Review Policies
  - Lot size determined by prespecified *order-up-to level* (OUL)

D: Average demand per period

σ_D: Standard deviation of demand per period

L: Average lead time for replenishment

T: Review interval

CSL: Desired cycle service level

Impact of Replenishment Policies on Safety Inventory

Probability(demand during $L + T \leq OUL$) = CSL

Mean demand during $T + L$ periods, $D_{T+L} = (T + L)D$

Std dev demand during $T + L$ periods, $s_{T+L} = \sqrt{T + L}\, s_D$

$$OUL = D_{T+L} + ss$$

$$ss = F_S^{-1}(CSL) ´\ s_{D+L} = NORMSINV(CSL) ´\ s_{T+L}$$

Average lot size, $Q = D_T = DT$

Impact of Replenishment Policies on Safety Inventory

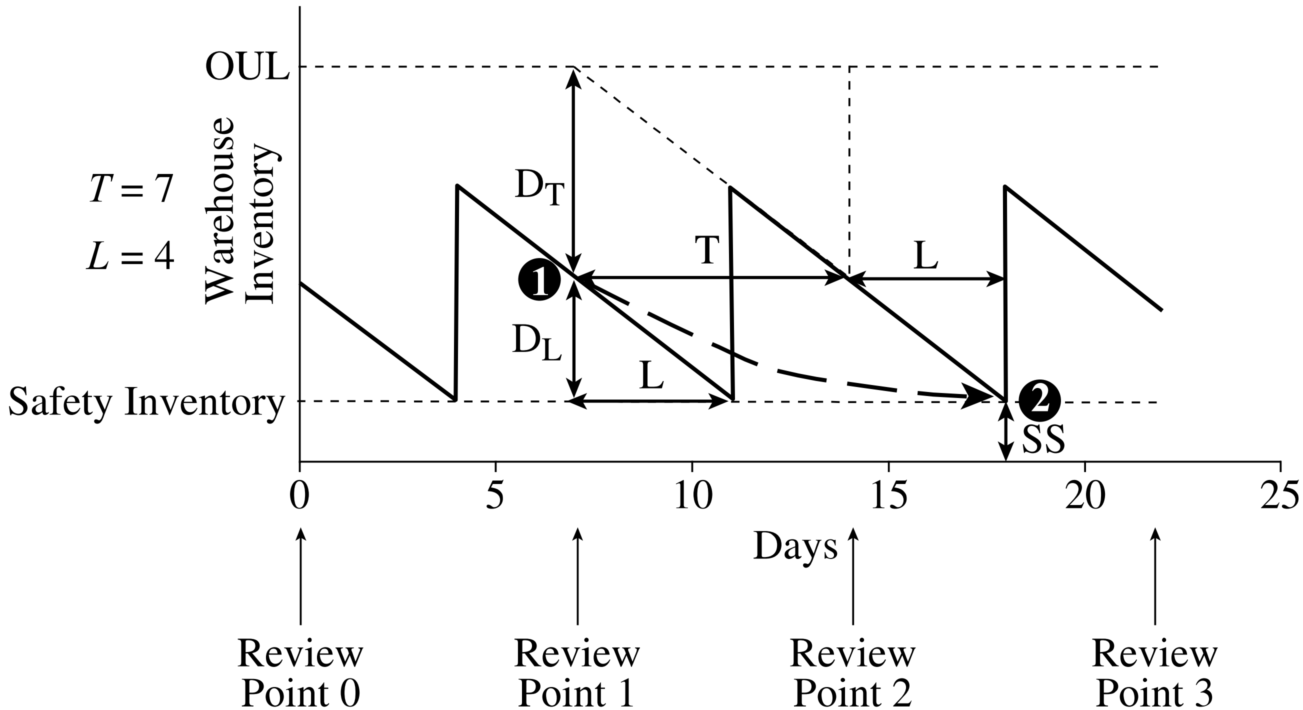


Figure 12-6

Evaluation Safety Inventory for a Periodic Review Policy

D = 2,500, σ_D = 500, L = 2 weeks, T = 4 weeks

Mean demand during $T + L$ periods, $D_{T+L} = (T + L)D$

$$= (2 + 4)2{,}500 = 15{,}000$$

Std dev demand during $T + L$ periods, $s_{T+L} = \sqrt{T + L}s_D$

$$= \left(\sqrt{4 + 2}\right)500 = 1{,}225$$

$$ss = F_S^{-1}(CSL)\ ´\ s_{D+L} = NORMSINV(CSL)\ ´\ s_{T+L}$$

$$= NORMSINV(0.90)\ ´\ 1{,}225 = 1{,}570 \text{ boxes}$$

$$OUL = D_{T+L} + ss = 15{,}000 + 1{,}570 = 16{,}570$$

Managing Safety Inventory in a Multiechelon Supply Chain

- In multiechelon supply chains stages often do not know demand and supply distributions
- Inventory between a stage and the final customer is called the *echelon inventory*
- Reorder points and order-up-to levels at any stage should be based on echelon inventory
- Decisions must be made about the level of safety inventory carried at different stages

The Role of IT in Inventory Management

- IT systems can help
  - Improve inventory visibility
  - Coordination in the supply chain
  - Track inventory (RFID)
- Value tightly linked to the accuracy of the inventory information

Estimating and Managing Safety Inventory in Practice

1. Account for the fact that supply chain demand is lumpy
2. Adjust inventory policies if demand is seasonal
3. Use simulation to test inventory policies
4. Start with a pilot
5. Monitor service levels
6. Focus on reducing safety inventories

Summary of Learning Objectives

1. Understand the role of safety inventory in a supply chain
2. Identify factors that influence the required level of safety inventory
3. Describe different measures of product availability
4. Utilize managerial levers available to lower safety inventory and improve product availability